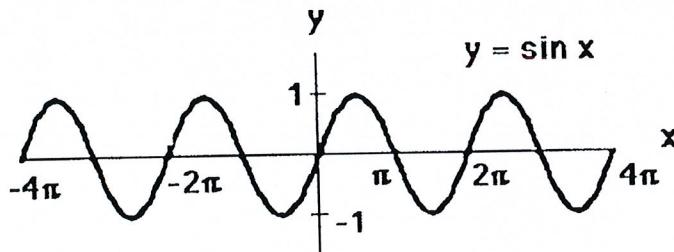


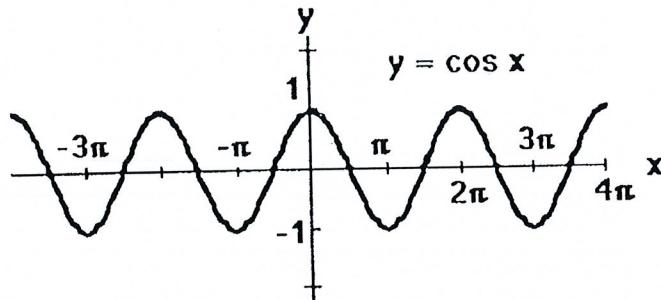
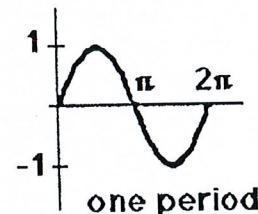
Graphs of Trig Functions

A. GRAPHS OF THE TWO BASIC FUNCTIONS: SINE and COSINE



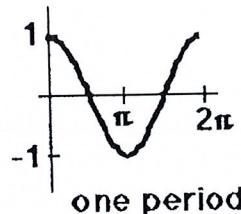
domain: $-\infty < x < \infty$

range: $-1 \leq y \leq 1$



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B. AMPLITUDE AND PERIOD

1. Definitions:

Amplitude - maximum ordinate (y) value

Period - length of the smallest domain interval which corresponds to a complete cycle of values of the function

Referring to previous graphs:

$$\begin{aligned} y &= \sin x \\ \text{amplitude} &= 1 \\ \text{period} &= 2\pi \end{aligned}$$

$$\begin{aligned} y &= \cos x \\ \text{amplitude} &= 1 \\ \text{period} &= 2\pi \end{aligned}$$

2. General graphs of the sine and cosine functions:

For functions of the form: $y = a \sin bx$ $y = a \cos bx$

$$\text{amplitude} = |a|$$

$$\text{period} = \frac{2\pi}{b}$$

Examples:

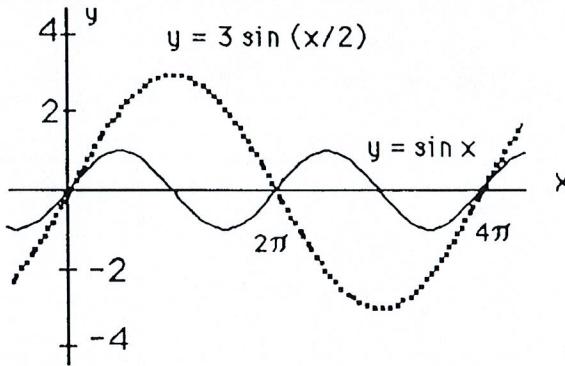
a. Graph $y = 3 \sin \frac{x}{2} = 3 \sin(\frac{1}{2}x)$

amplitude = $|a| = 3$

The function has a maximum y value of 3.

period = $\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 4\pi$

One complete cycle occurs between 0 and 4π .



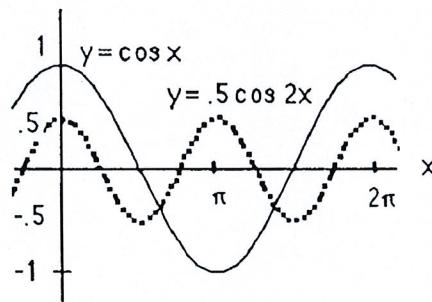
b. Graph $y = \frac{1}{2} \cos 2x$

amplitude = $|a| = \frac{1}{2}$

The function has extreme values of $y = \frac{1}{2}$ and $y = -\frac{1}{2}$.

period = $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

One complete cycle occurs between 0 and π .

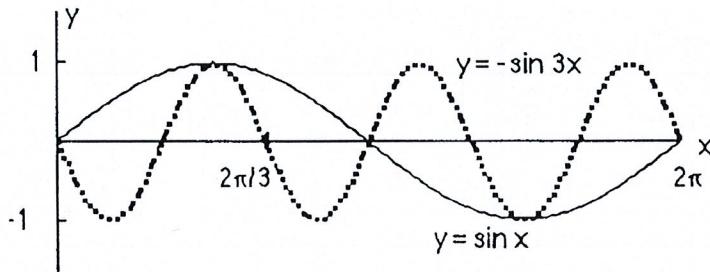


c. Graph $y = -\sin 3x$

amplitude = $|a| = |-1| = 1$ The function has extreme values of $y = 1$ and $y = -1$.

period = $\frac{2\pi}{b} = \frac{2\pi}{3}$

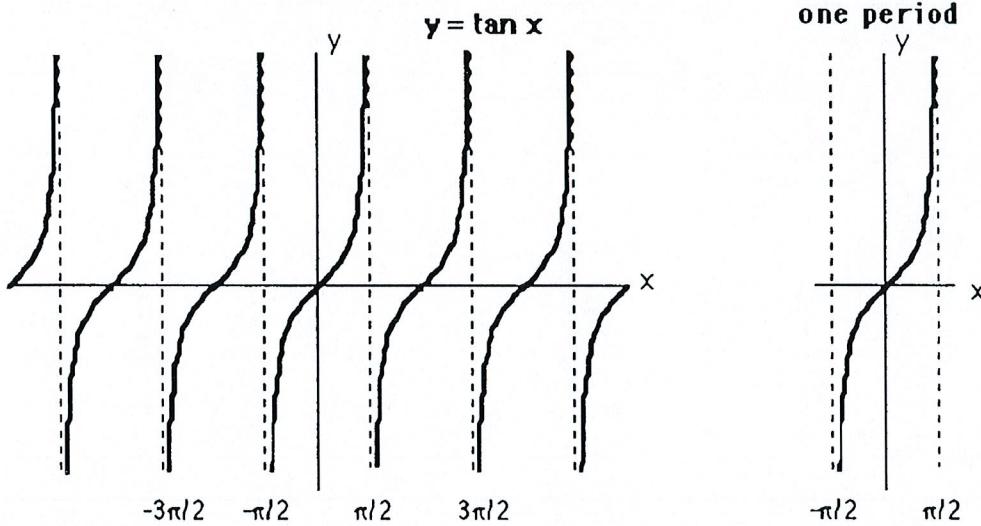
One complete cycle occurs between 0 and $\frac{2\pi}{3}$.



Note: Due to the minus sign, the graph of $y = -\sin 3x$ is the reflection about the x-axis of the graph of $y = +\sin 3x$.

C. TANGENT FUNCTION GRAPH AND PROPERTIES

1. The domain of $y = \tan x = \frac{\sin x}{\cos x}$ will not include values of x which make the denominator, $\cos x$, equal to zero (to avoid division by zero). Thus, for the graph of $y = \tan x$, $x \neq \dots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \dots$. Vertical asymptotes occur at these x values. The range of $\tan x$ is all real values of y .



2. Amplitude = none, since there is no maximum value of y

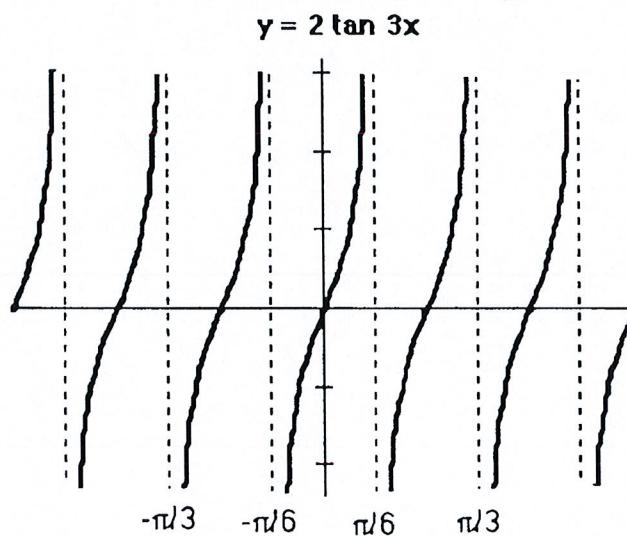
Period = π (a full period occurs between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$)

3. The period of $y = \tan bx$ is the regular period of $\tan x$ divided by b , that is $\frac{\pi}{b}$.

Example: Graph $y = 2 \tan 3x$

Solution: period = $\frac{\pi}{b} = \frac{\pi}{3}$, so a full period will occur between $-\frac{\pi}{6}$ and $\frac{\pi}{6}$

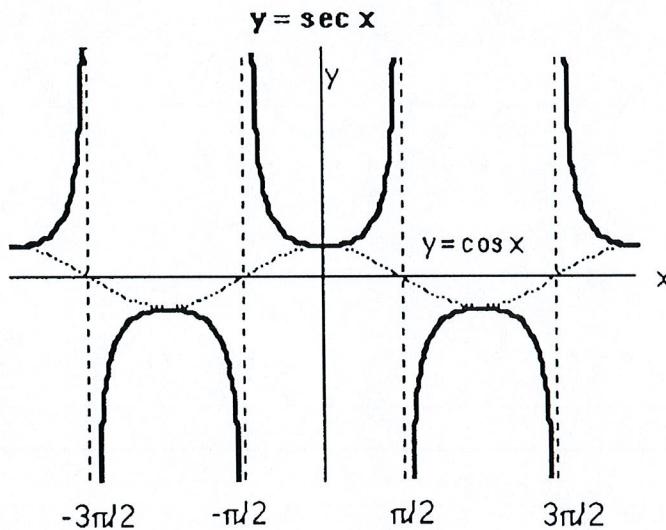
asymtotes will occur at $x = \frac{\pi}{6} \pm n \frac{\pi}{3}$, for integer values of n .



D. GRAPHS OF THE OTHER TRIGONOMETRIC FUNCTIONS

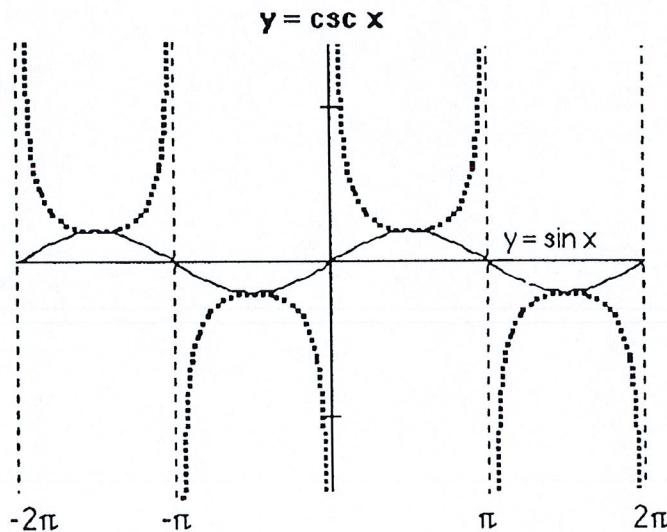
1. Both $\sec x = \frac{1}{\cos x}$ and $\tan x = \frac{\sin x}{\cos x}$ have vertical asymptotes at the same x values (at odd multiples of $\frac{\pi}{2}$), since both formulas have $\cos x$ in the denominator, and $\cos x = 0$ for these x values.

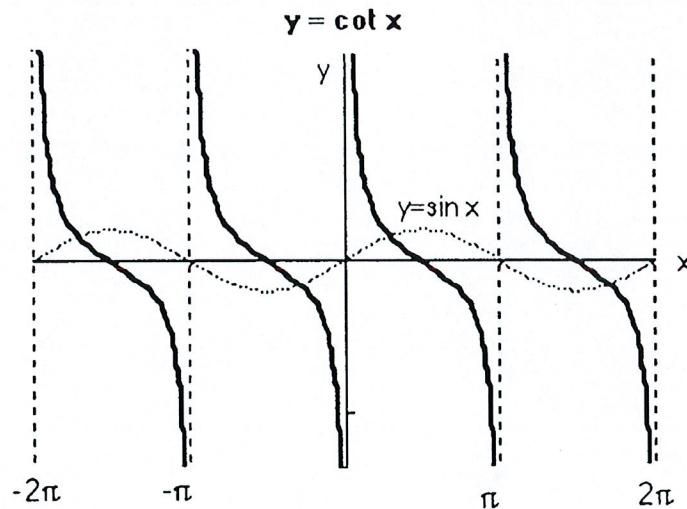
$y = \sec x$ period = 2π amplitude = none, since there is no maximum value of y



2. Both $\csc x = \frac{1}{\sin x}$ and $\cot x = \frac{\cos x}{\sin x}$ have the vertical asymptotes at the same x values (at integer multiples of π), since both have $\sin x$ in the denominator and $\sin x = 0$ for these x values.

$y = \csc x$ period = 2π amplitude = none, since there is no maximum value of y
 $y = \cot x$ period = π amplitude = none, since no maximum value of y





3. Periods of other general trig functions

$$y = \csc bx \quad y = \sec bx \quad y = \cot bx \quad \text{new period} = \frac{\text{regular period}}{b}$$

Examples:

a. Find the period of $y = \csc 4x$

regular period of $y = \csc x$ is 2π

$$\text{new period of } y = \csc 4x \text{ is } \frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$$

b. Find the period of $y = \cot \frac{2}{3}x$

regular period of $y = \cot x$ is π

$$\text{new period of } y = \cot \frac{2}{3}x \text{ is } \frac{\pi}{b} = \frac{\pi}{2/3} = \frac{3\pi}{2}$$

PROBLEMS:

1. Find the period of each of the following. Find the amplitude for the sine and cosine functions.

a. $y = \cos 4x$

b. $y = 4 \tan 6x$

c. $y = 3 \sin \frac{1}{2}x$

d. $y = \sec 7x$

2. Construct one period of the graph of each of the following:

a. $y = \sin 2x$

b. $y = 3 \cos x$

c. $y = 4 \sin 3x$

d. $y = 2 \cos \frac{2}{3}x$

Solutions:

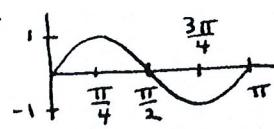
1. a. $\frac{2\pi}{4} = \frac{\pi}{2}$, amplitude = 1

c. $\frac{2\pi}{1/2} = 4\pi$, amplitude = 3

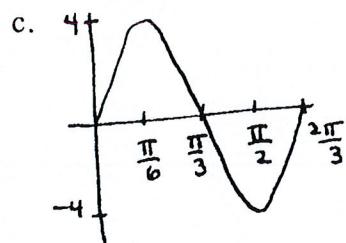
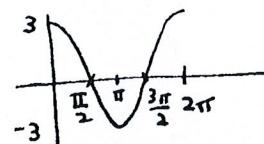
b. $\frac{\pi}{6}$

d. $\frac{2\pi}{7}$

2. a.



b.



d. period = $\frac{2\pi}{2/3} = 3\pi$

