The physics of evolved stars is rich with open theoretical questions. In this talk, I will provide an overview of select topics for high-mass and low-mass evolved stars. For high-mass stars, core-collapse supernovae have posed a vexing puzzle for theorists for half a century despite being a major ingredient (and uncertainty) in fields ranging from stellar and galaxy evolution to the interstellar medium. In this area, I will discuss the viability of the delayed-neutrino mechanism for core collapse and the origin of pulsar kicks. For low-mass stars, planetary and stellar companions orbiting within ~5 AU of their host stars may strongly interact as the primary evolves off the main sequence. I will highlight a few of the physical processes that can occur when the companion becomes engulfed in a common envelope. In particular, I will touch on where to look for planetary companions around white dwarfs, the amplification of magnetic fields during common envelopes and the shaping of planetary nebulae.