A PDE for Inferring the Free Energies of Ternary Mixtures from Light Scattering Data

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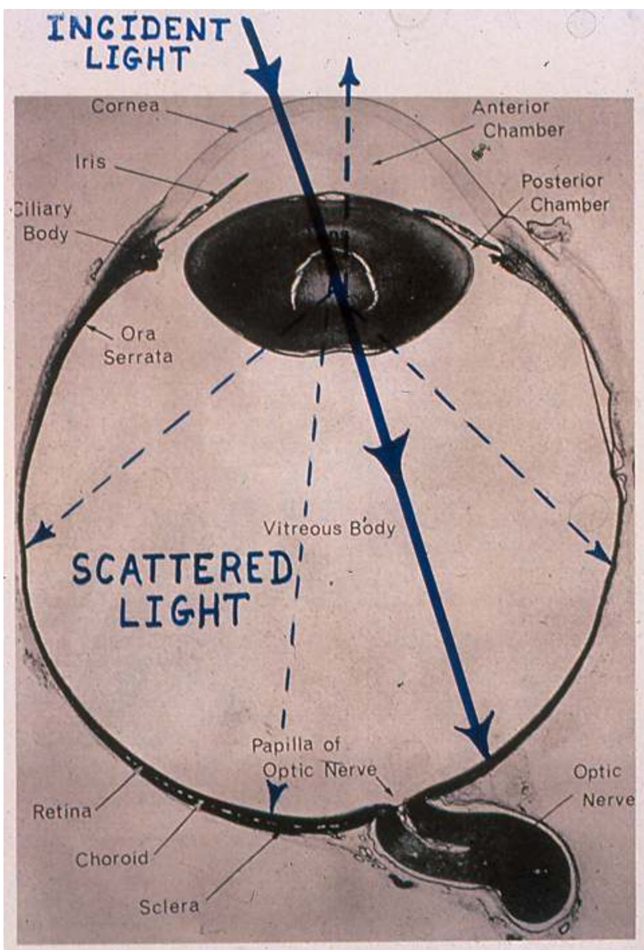
One cause of cataract disease is phase separation of the crystalline protein solution in the eye lens. This is our primary motivation for studying the general problem of determining free energies of ternary mixtures from scattering data.

George Thurston, building on the work of Kirkwood and Goldberg, derived a formula for the Rayleigh scattering of light from a mixture in terms of the free energy of the mixture regarded as a function of composition. We consider the problem of regarding this formula as the basis of a PDE for the free energy given measured values of the Rayleigh scattering ratio. This basic PDE has the form

$$\frac{\left(b(x,y)\right)^2 g_{xx} - 2b(x,y)a(x,y)g_{xy} + \left(a(x,y)\right)^2 g_{yy}}{g_{xx}g_{yy} - g_{xy}^2} = R(x,y)$$



In this talk will we discuss the theory of well-posed problems for this PDE. Interesting aspects of this theory include the equation's being fully nonlinear, the fact that both $R(x,y)$ and $g(x,y)$ can be singular, and the fact that the physical problem does not provide boundary conditions. Perhaps the most interesting aspect of the theory is that $R(x,y)$ can be measured only where $g(x,y)$ is convex; so the basic PDE must be replaced with the fully nonlinear elliptic PDE for a convex hull in certain regions.

We will discuss a hodograph analysis of this problem, which allows us to characterize the PDE and which allows us to interpret convex hulls as shocks.

We will also discuss numerical methods for solving this problem.