Optimal Placement of Suicide Bomber Detectors *

by

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Abstract

This paper develops upon the work by Kaplan and Kress [1], which considers the operational effectiveness of suicide bomber (SB) detector schemes. Here, we consider the optimal placement of detectors in a threat area where the potential targets are known. The threat area is divided into grids for the purpose of our analysis and can have several entrances. We assume that a SB would detonate at a potential explosive grid centroid. The number of individuals near every potential explosive grid is assumed to be given by a spatial Poisson process, with the density being a function of the specific potential explosive grid. It is assumed that the SB would take the shortest path from one of the entrances to the grid centroid where he/she intends to detonate. SB detectors are not perfectly reliable, with the probability of detection being a function of how long the SB would stay in the effective detection area. We choose the objective of minimizing the expected number of casualties. The problem is formulated as a nonlinear integer program and properties are derived to gain insights into the model as well as to develop efficient solution methods. Later, a greedy adding heuristic and a branch and bound algorithm are proposed. A base case is analyzed to illustrate the application of the model. We also perform a sensitivity analysis for a number of key factors as well as an investigation of the performance of the greedy heuristic procedure.

Keywords: suicide bomber, detector placement

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1 Introduction

Kaplan and Kress [1] analyze the operational effectiveness of suicide bomber (SB) detector schemes. There, they consider two urban environments where SBs attack. The first environment is a grid model which presumes a layout of blocks separated by street and sidewalk. The second environment is a plaza model where the potential circular targets with radius τ are distributed in accordance with a spatial Poisson process. They discuss the expected number of casualties under two different possible interventions. The first intervention is to instruct the individuals to flee and the second one is to hit-the-deck. They conclude that under some situations, intervention may even increase the expected number of casualties.

We develop upon the work of Kaplan and Kress by considering SB detector placement in an environment where these detectors are not assumed to be fully reliable. We draw upon constructs/concepts from geometrical probability (c.f. Larson and Odoni [2]) and from location science (c.f. Drezner and Hamacher [3]) to do this. The framework we adopt is a combination of the grid and plaza models in Kaplan and Kress. We assume that there are several entrances and that the whole area is divided into grids, with some of them blocked, i.e., representing obstacles. We assume that we know the list of potential targets, that is, the set of grids where a SB would prefer to detonate. A grid in this set is called a potential explosive grid. The distribution of individuals near every potential explosive grid is assumed to be spatial Poisson, with the density being a function of the specific potential explosive grid. We assume that the SB would walk directly from an entrance to the chosen potential explosive grid. While doing so, he/she can not cross any blocked grids.

We assume that a SB detector is not perfectly reliable and that the probability of detection depends on how long the SB would stay in its effective detection area. The problem we consider is how to place SB detectors such that the expected number of casualties is minimized.

The paper is organized as follows. In Section 2, we define the basic setting and develop the corresponding optimization model. Section 3 derives several properties of the model. Section 4 focuses on solution techniques, a greedy adding heuristic and a branch and bound algorithm. Section 5 contains a base case to illustrate the problem formulation. Computational analyses are presented in Section 6. Finally, Section 7 states our conclusions and future work directions.

2 Basic Modeling

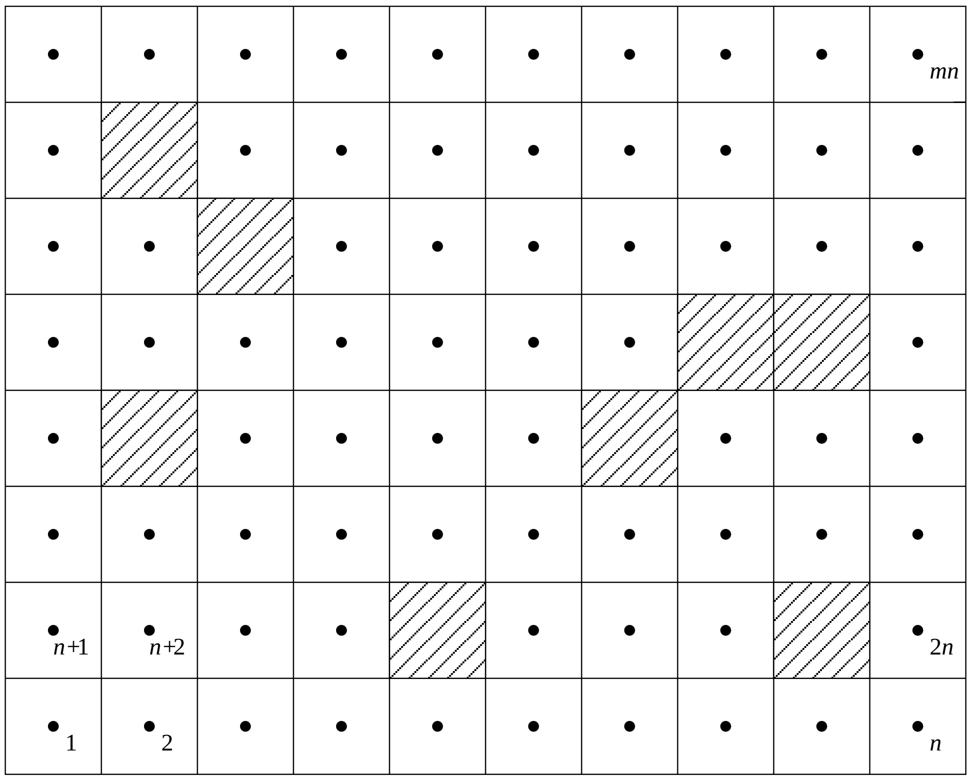

Figure 1: The area with $m \times n$ grids (The shadow grids are blocked)

We assume that the threat area is rectangular and is divided into $m \times n$ grids, given by the set $G = \{j : j = 1, 2, \ldots, mn\}$ and as shown in Figure 1. Entry into this threat area is possible through an entrance, where the set of entrances L is a subset of the set of boundary grids E. We let $l = |L|$ and label the entrances as $1, 2, \ldots, l$.

To model physical obstructions, we allow grids to be of two types. The first is a *blocked grid* through which the SB can not travel and in which no threatened individuals are present. The second is an *unblocked grid* through which SB travel is permitted and in which threatened individuals can also be present. Let $B \subset G$ be the set of all blocked grids, and $U = G \setminus B$ be the set of unblocked grids.

We assume knowledge of potential grids that the SB would like to attack. This set $S \subset U$

is referred to as the set of *potential explosive grids*. Near every potential explosive grid $j \in S$, threatened individuals are potentially present in accordance with a spatial Poisson process of density λ_j.

A SB uses entrance k and attacks potential explosive grid j with probability γ_{kj}. Hence, $\gamma_{kj} \geq 0, \forall k = 1, \ldots, l, j \in S$ and $\sum_{k=1}^{l} \sum_{j \in S} \gamma_{kj} = 1$. Our analysis assumes knowledge of these γ values. Later, we perform a robustness analysis across a range of γ values (Section 6). A fundamental assumption we make is that the SB can not detect a detector, i.e., the detectors are perfectly concealed.

The SB can walk directly from a grid center to another grid center provided that this straight line path does not intersect with a blocked grid. In other words, the SB attempts to travel on the straight line path from their chosen entrance k to their selected explosive grid j. If this straight line path intersects one or more blocked grids then they choose a shortest path, which consists of a sequence of straight line segments connecting grid centroids such that each such segment is nonintersecting with the set of blocked grids. To find such a path, we apply a shortest path algorithm (e.g. Dijkstra's Algorithm [4]) from k to j using an inter-grid distance matrix whose entries are either (i) infinity if the straight line path between the two grid points intersects one or more blocked grids, or (ii) straight line distance.

We assume for the sake of simplicity in presentation that the shortest path from k to j is unique and is labelled as P_{kj}. The analysis is readily extendable to the situation of non-unique paths provided we make the assumption that the SB randomly selects one of these paths.

We assume that grid centers for unblocked grids are potential detector placement points, and that the effective detection radius is τ, as determined by the National Research Council (NRC) panel [5]. As in Kaplan and Kress, we define timely detection as detection such that there are at least 10 seconds remaining before the SB reaches his/her targeted explosive grid. Assuming a walking speed of 1 m/sec, this converts to a distance of 10 m from the target.

Let $N_k(j)$ be the set of all non-blocked grids from which a detector can timely detect the SB while on path P_{kj}. To construct the set $N_k(j)$, we first identify all grids i such that the distance from i to the nearest point on path P_{kj} is less than τ. For each such i we then see if the first point of detection on path P_{kj} is at least 10 m away from j. If not, we remove

i from the set. This first point of detection is easily obtained by constructing a circle of radius τ centered in grid i and observing its intersection with path P_{kj}. For simplicity in presentation, for the rest of the paper we will write SB detection when we mean timely SB detection.

Now, we give an example to show how to obtain the set $N_k(j)$ geometrically. In Figure 2, we assume that every grid is 10 m $\times$ 10 m and $\tau = 10$ m. In this example, the entrance 1 is grid 3 and the explosive grid is grid 36. The shortest path from entrance 1 to grid 36 is $3 \rightarrow 9 \rightarrow 36$. From the graph, if the center of the grid is within the detection zone which is defined as the set of all unblocked points from which a detector can timely detect the SB while on the shortest path, it belongs to the set $N_k(j)$. Thus, in this example, $N_1(36) = \{3, 9, 15, 16, 22, 23, 29, 30, 35\}$.

We do not assume that the detectors are perfectly reliable. For every $i \in N_k(j)$, let p_{ikj} represent the probability of detecting the SB while traveling on path P_{kj}. To calculate p_{ikj}, we focus on the portion of path P_{kj} which is at least 10 m away from j. We label this sub-path as $\overline{P}_{kj}$. We now calculate the length of the section of $\overline{P}_{kj}$ which is within τ distance from i, l_{ikj}. These calculations are similar to those discussed in Batta and Chiu [6] for routing of a vehicle carrying hazardous materials. If the entrance k lies in the circle centered at grid i with radius τ, we will extend the first segment of $\overline{P}_{kj}$ to intersect with the circle when calculating l_{ikj}.

As in Przemieniecki [7], we assert that

$$p_{ikj} = 1 - e^{-\eta l_{ikj}}, \tag{1}$$

where η is the detector's instantaneous detection rate. For the example shown in Figure 2, given that $\eta = 0.06$, we have that $p_{15,1,36} = 1 - e^{-0.06 \times 15.833} = 0.613$.

By the definition of $N_k(j)$, we can restrict our detector placements to belong to the set $T = \bigcup_{k,j} N_k(j)$. Since the determination of detector placements is our primary motive, we define, for each $j \in T$, a binary variable x_j as follows:

$$x_j = \begin{cases} 1 & \text{if there is a detector placed in the center of grid } j, \\ 0 & \text{otherwise.} \end{cases}$$

We assume that there is at most one detector placed at a grid.

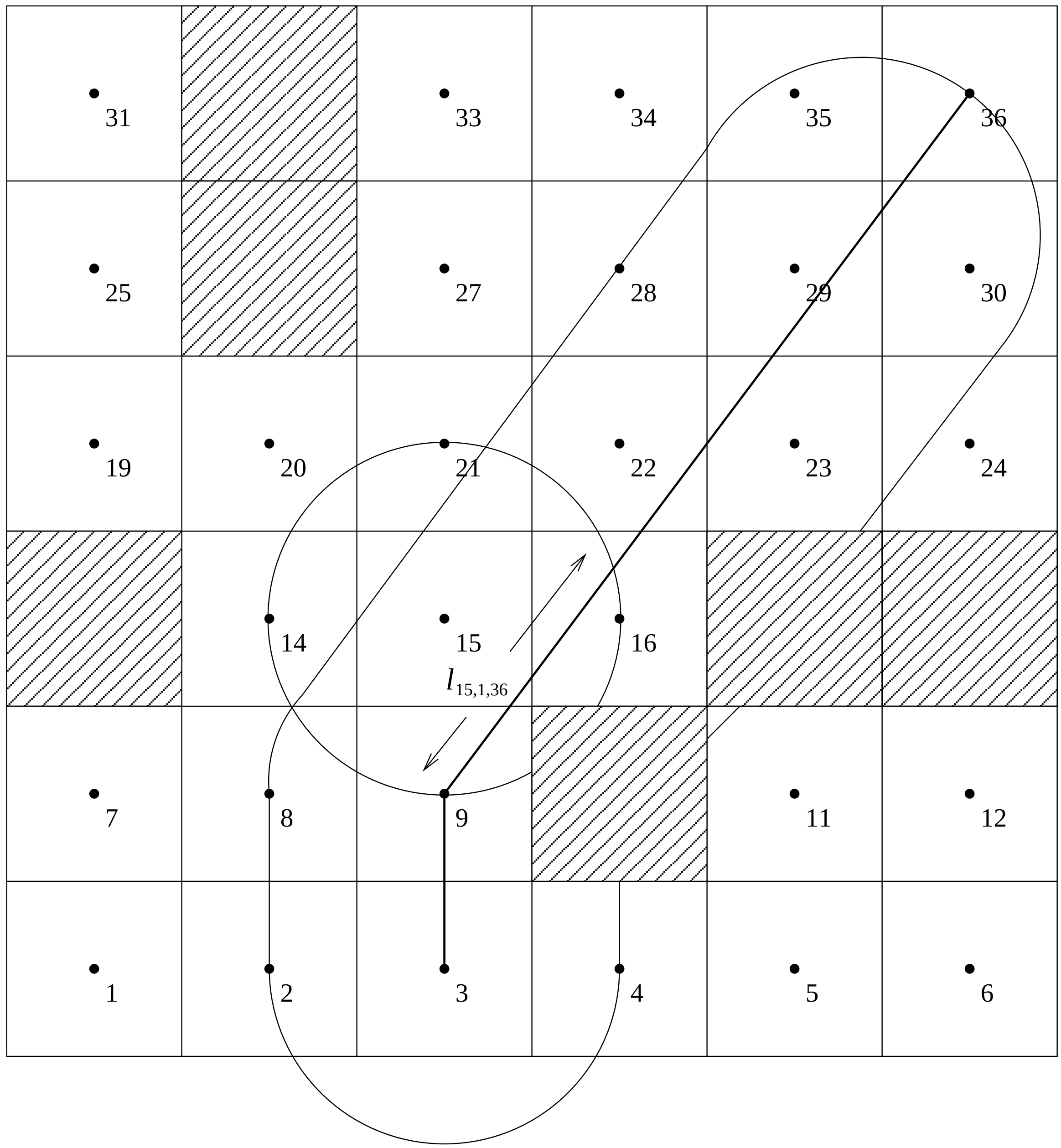

Figure 2: Illustration of $N_k(j)$

The expected number of casualties given that the SB detonates at grid j, C_j, is given by Equation 2 in Kaplan and Kress. This is needed for our objective function and is explained in Section 5.

Consideration of detectors allows us to perform a suitable intervention, i.e., when a detector alarm goes off, an action can be taken. The specific action we consider is that of neutralizing the SB. We assume a success probability of θ for this action. Other interventions (e.g., instructing individuals to flee and hit-the-deck) are not considered in this paper because these are demonstrated to be ineffective in many cases by Kaplan and Kress. Figure 3 gives a breakdown of possible events.

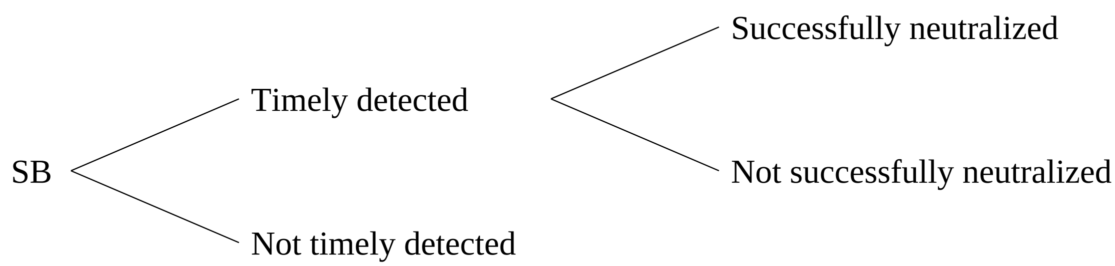

Figure 3: All possible events related to a SB

We now focus on constructing the objective function. Our fundamental assumption is that the detectors work independently. The probability of non-detection along path P_{kj} is

$$Pr\{ND_{kj}\} = \prod_{i \in N_k(j)} (1 - p_{ikj})^{x_i},$$

in which case the resultant number of expected casualties is C_j. If detection occurs (with probability $1 - Pr\{ND_{kj}\}$), there is still a chance of non-neutralization resulting in $(1-\theta)C_j$ expected number of casualties. Given the γ_{kj} values which specify the probabilities of using a specific entrance k and targeting a specific grid j, we get the total expected number of casualties as:

$$\sum_{k=1}^{l} \sum_{j \in S} [(1-\theta)C_j\gamma_{kj} + \theta \prod_{i \in N_k(j)} (1 - p_{ikj})^{x_i} C_j \gamma_{kj}]. \qquad (2)$$

We note that the first term of (2) is a constant and hence can be dropped from the perspective of optimization of detector placement. Furthermore, we let W_{kj} be equal to $\theta C_j \gamma_{kj}$.

For the optimization problem to be meaningful, we need to restrict the total number of detectors, M, to be placed. The problem can then be stated as the following nonlinear binary integer program:

$$
\begin{aligned}
(P) \quad \text{Min} \quad & \sum_{k=1}^{l} \sum_{j \in S} W_{kj} \prod_{i \in N_k(j)} (1 - p_{ikj})^{x_i} \\
\text{s.t.} \quad & \sum_{j \in T} x_j \leq M, \\
& x_j \in \{0, 1\}, \ \forall j \in T.
\end{aligned}
$$

3 Properties

In this section we develop a series of properties with the following goals in mind:

- Improved confidence level when using implicit enumeration;
- Better insight into the model and development of an alternative heuristic; and
- Dominance results that reduce the problem dimension/feasible set.

3.1 Implicit Enumeration Related Properties

The linear relaxation of (P) is:

$$
\begin{aligned}
(P_{LR}) \quad \text{Min} \quad & \sum_{k=1}^{l} \sum_{j \in S} W_{kj} \prod_{i \in N_k(j)} (1 - p_{ikj})^{x_i} \\
\text{s.t.} \quad & \sum_{j \in T} x_j \leq M, \\
& 0 \leq x_j \leq 1, \ \forall j \in T.
\end{aligned}
$$

Clearly, (P_{LR}) is a convex program, which implies that we can obtain an optimal solution for (P_{LR}) using well known algorithms (e.g., sequential quadratic programming (SQP) approach [8]).

3.2 Model Insight and Heuristic Development

Here, we just state two simple properties without proof. Our first property establishes that all detectors will be deployed, when possible. The second property focuses on the situation where we have just one entrance and one potential explosive grid. The result is used in the development of a greedy heuristic in Section 4.

Property 1. If $|T| \geq M$, $\sum_{j \in T} x_j^* = M$ in the optimal solution X^*.

Property 2. Suppose there is only one entrance k and one potential explosive grid j. For the situation where $|N_k(j)| \leq M$, we have $x_i^* = 1$ for every $i \in N_k(j)$ in the optimal solution X^*. For the situation $|N_k(j)| > M$, an optimal solution X^* is found by setting to 1 the first M elements in the set $N_k(j)$ when it is arranged in decreasing order of p_{ikj}.

3.3 Dominance Results

We first provide a grid dominance definition which allows us to eliminate some potential detector locations.

Definition 1. Consider two grids u and v. If $p_{ukj} \geq p_{vkj}$ for all k, j pairs and $p_{ukj} > p_{vkj}$ for at least one k, j pair, we say that grid u *dominates* grid v.

Theorem 1. If grid v is dominated by grid u, $x_v^* \leq x_u^*$ in an optimal solution.

Proof. (By contradiction) Consider an optimal solution X^*, in which $x_v^* > x_u^*$. Since both x_v and x_u are binary variables, we have that $x_v^* = 1$ and $x_u^* = 0$. According to the definition of dominance, just by letting $x_u^{**} = 1$ and $x_v^{**} = 0$ and keeping values of other decision variables the same as X^*, we obtain another feasible solution X^{**} which has a smaller objective function value than that of X^*. That contradicts with the fact that X^* is an optimal solution. □

Corollary 1. If grid v is dominated by at least M grids, $x_v^* = 0$ in an optimal solution.

Proof. Suppose that $x_v^* = 1$ in an optimal solution X^*. According to Theorem 1, $x_u^* = 1$ for all those grids u which dominate grid v. That solution does not satisfy the constraint $\sum_{j \in T} x_j \leq M$, which is a contradiction. □

Corollary 2. If there exists one grid which dominates all other grids and $M \geq 1$, we will locate one detector in that grid in an optimal solution.

Proof. Suppose we do not locate one detector in that grid, according to Theorem 1, we have $\sum_{j \in T} x_j = 0$ in the optimal solution. That leads to a contradiction. □

As a special case of Corollary 2, if there is only one entrance and several explosive grids and the entrance grid dominates all other grids, we will place one detector in the entrance grid in the optimal solution, given that we will deploy at least one detector. This is intuitive.

4 Algorithms

In this section, we propose a greedy adding heuristic and a branch and bound algorithm to solve (P).

4.1 Greedy Adding Heuristic

We propose a greedy adding heuristic to obtain an approximate solution to this optimization problem. The idea of this heuristic is as follows: we place the first detector in the grid such that it will minimize the expected casualties if we are allowed to deploy only one detector. Then we choose the second placement such that the expected casualties is minimized given that the position of the first detector is fixed. We continue to follow this rule until we place the Mth detector.

Greedy Adding Heuristic (GAH) Procedure:

- Step 1: The initial potential placement set is $P = T$ and initial placement set is $Q = \emptyset$.
- Step 2: For every $j \in P$, compute the total expected casualties if we just place an additional detector at grid j. Then compute

$$i \in argmin\{j \in P : \begin{array}{l}\text{total expected casualties if an additional} \\ \text{detector is placed at grid } j\}, \end{array}$$

  and let

$$P = P \setminus \{i\} \text{ and } Q = Q \cup \{i\}.$$

- Step 3: Check if $|Q| < M$, if yes, then go to step 2, otherwise, Q represents the selected detector placements.

We now investigate a special case where the GAH procedure yields an optimal solution. This is for the situation where none of the sets $N_k(j)$ intersect. More formally, the condition is that for all possible pairs of (k, j), $\bigcap N_k(j) = \emptyset$.

Let g be the number of distinct $N_k(j)$, labeled as $1, 2, \ldots, g$; let $a_m = W_{kj}$ for the mth $N_k(j)$; let h_m be the number of elements in the mth $N_k(j)$ set; and q_{mn} be the p_{ikj} value for the nth element in the mth $N_k(j)$ set. With this notation in place, we recognize that (P) is reduced to the following optimization problem:

$$\begin{aligned} (A) \quad \text{Min} \quad & \sum_{m=1}^{g} a_m \prod_{n=1}^{h_m} (1 - q_{mn})^{x_{mn}} \\ \text{s.t.} \quad & \sum_{m=1}^{g} \sum_{n=1}^{h_m} x_{mn} \leq M, \\ & x_{mn} \in \{0, 1\}, \ \forall mn. \end{aligned}$$

We therefore focus our attention on showing that the GAH procedure solves (A) optimally. To do this, we need to establish two properties. The first is a greedy choice property and the second is an optimal substructure property [9].

Lemma 1. Greedy Choice Property: Let x_{rs} be the first decision variable to be set to be 1 by the GAH procedure. Then, there exists an optimal solution X^* to (A) where $x^*_{rs} = 1$.

Proof. (By construction) Let X^* be an optimal solution of problem (A). If x^*_{rs} happens to be 1, we are done. If $x^*_{rs} = 0$ in X^*, then we have two cases. In Case 1, there exists one rv such that $x^*_{rv} = 1$, where $v \in \{1, 2, \ldots, h_r\} \setminus \{s\}$. Case 2 considers the case when there does not exist such a rv such that $x^*_{rv} = 1$.

Since x_{rs} is the first decision variable set to be 1 by the GAH procedure, we have the following inequality

$$a_r(1 - q_{rs}) + \sum_{k \neq r} a_k \leq a_c(1 - q_{cd}) + \sum_{k \neq c} a_k, \quad \forall (c, d). \tag{3}$$

For Case 1, let us construct another feasible solution X^{**}. The only difference between X^{**} and X^* is that $x^{**}_{rs} = 1$ and $x^{**}_{rv} = 0$ in X^{**}. From Inequality (3), let $c = r$ and $d = v$, we have that $a_r(1 - q_{rs}) \leq a_r(1 - q_{rv})$. Thus, we have

$$a_r \prod_{n \notin \{s,v\}} (1 - q_{rn})^{x^*_{rn}} (1 - q_{rs}) \leq a_r \prod_{n \notin \{s,v\}} (1 - q_{rn})^{x^*_{rn}} (1 - q_{rv}),$$

which leads to the conclusion that the objective function value under X^{**} is no greater than that of X^*. Hence, X^{**} is an optimal solution with $x^{**}_{rs} = 1$.

For Case 2, suppose that in X^*, we have $x^*_{ut} = 1$, where $u \neq r$ and $t \in \{1, 2, \ldots, h_u\}$. let us construct another feasible solution $X^{'}$. The only difference between $X^{'}$ and X^* is that $x^{'}_{rs} = 1$ and $x^{'}_{ut} = 0$ in $X^{'}$. In order to prove that $X^{'}$ is also an optimal solution, what we need to prove is that

$$a_u \prod_{n \neq t} (1 - q_{un})^{x^*_{un}} + a_r(1 - q_{rs}) \leq a_u \prod_{n \neq t} (1 - q_{un})^{x^*_{un}} (1 - q_{ut}) + a_r,$$

that is,

$$a_u \prod_{n \neq t} (1 - q_{un})^{x^*_{un}} q_{ut} \leq a_r q_{rs}. \tag{4}$$

From Inequality (3), let $c = u$ and $d = t$, we have that $a_r(1 - q_{rs}) + a_u \leq a_u(1 - q_{ut}) + a_r$, that is, $a_u q_{ut} \leq a_r q_{rs}$. Since $\prod_{n \neq t}(1 - q_{un})^{x^*_{un}} \leq 1$, (4) is correct. Hence, in this case, $X^{'}$ is also an optimal solution with $x^{'}_{rs} = 1$. The result follows. □

Lemma 2. Optimal Substructure Property: If X^* is an optimal solution to problem (A) containing $x^*_{rs} = 1$, then the remaining elements of X^* (with x^*_{rs} deleted) are optimal to the remaining optimization problem $(A^{'})$:

$$\begin{aligned} (A^{'}) \quad \text{Min} \quad & \sum_{m \neq r} a_m \prod_{n=1}^{h_m} (1 - q_{mn})^{x_{mn}} + a_r(1 - q_{rs}) \prod_{n \neq s} (1 - q_{rn})^{x_{rn}} \\ \text{s.t.} \quad & \sum_{mn \neq rs} x_{mn} \leq M - 1, \\ & x_{mn} \in \{0, 1\}, \ \forall mn \neq rs. \end{aligned}$$

Proof. (By contradiction) Suppose the remaining X^* with x^*_{rs} deleted is not optimal to problem $(A^{'})$. Then, there exists an optimal solution $\overline{X}$ to $(A^{'})$. By combining this optimal solution $\overline{X}$ with $x_{rs} = 1$, we obtain another feasible solution to (A) with objective function value less than that of X^*, which is a contradiction. □

Theorem 2. If for all possible pairs of (k, j), $\bigcap N_k(j) = \emptyset$, then the solution given by the GAH procedure is optimal.

Proof. By combining the results of Lemma 1 and Lemma 2, the theorem follows. □

4.2 Branch and Bound Algorithm

Since the relaxation problem is a convex nonlinear program, we can use a branch and bound solution algorithm and obtain an exact solution [10].

To enhance the performance of the solution method, we can use Corollary 1 of Theorem 1 to check if a grid is dominated by at least M other grids. If yes, we can eliminate the decision variable associated with that grid. Moreover, according to Theorem 1, we can add constraint $x_v \leq x_u$ to the constraint set if we know that grid v is dominated by grid u. By doing this, we decrease the feasible space without eliminating any optimal solutions.

5 Base Case

In this section, we will provide a base case to illustrate the problem formulation and to investigate the performance of the proposed GAH procedure which is measured by the relative error. Our case is based on a 80 m $\times$ 80 m study area, which is divided into 64 equal grids of size 10 m $\times$ 10 m (see Figure 4). In Table 1, we summarize the parameter values of our base case. The values of the parameters τ, r and b are chosen consistent with those in Kaplan and Kress.

Table 1: Base case parameter values

Parameter	Description	Value
B	Set of blocked grids	$B = \{13, 26, 31, 40, 43, 50\}$
L	Set of entrances (labeled as $1, 2, \ldots, 8$)	$L = \{3, 6, 17, 24, 41, 48, 59, 62\}$
S	Set of potential explosive grids	$S = \{28, 46\}$
M	Number of detectors to be placed	3
γ_{kj}	Probability that a SB enters from k and attacks j	0.0625
τ	Detector detection radius	10 m
η	Instantaneous detection rate	0.06
λ_j	Population density near grid j	$\lambda_{28} = \lambda_{46} = 0.4$ persons m^{-2}
θ	Probability of successful neutralization	0.6
r	Target-area radius	10 m
b	Individual base width	0.5 m
n	Number of effective fragments	∞
C_j	Expected casualties if explosion is at j	37.32

In this table, the target-area radius, individual base width and number of effective frag-

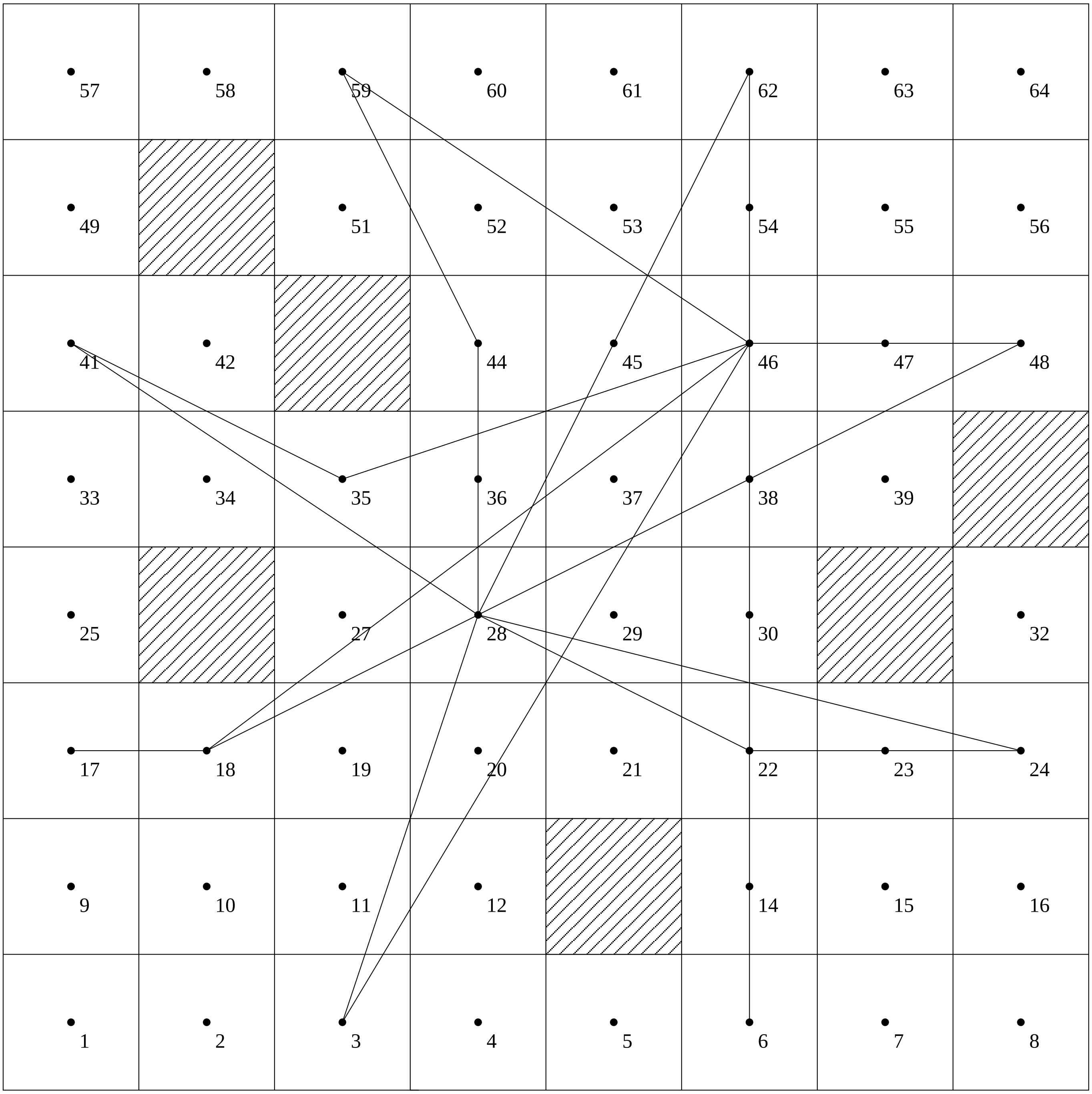

Figure 4: The base case with 8×8 grids

ments are needed to calculate the C_j values using Equation 2 in Kaplan and Kress which is restated as follows:

$$C_j = \frac{2\pi}{\lambda_j b^2}(1 - (1 + \lambda_j br)e^{-\lambda_j br}).$$

For this base case, we assume that the densities near two explosive grids are equal and that the probabilities γ_{kj} are equal for every entrance and explosive grid pair. From the parameter values, we can obtain W_{kj} for each (k, j) pair.

In the base case, the shortest paths from each entrance to each potential explosive grid are shown in Figure 4, which can be obtained via some standard shortest path algorithm (e.g. Dijkstra's Algorithm [4]). For example, the shortest path from entrance grid 41 to grid 46 is $41 \to 35 \to 46$.

After we obtain the shortest paths, we can obtain the corresponding $N_k(j)$ for each entrance and potential explosive grid combination. For example, here

$$N_1(46) = \{2, 3, 4, 11, 12, 20, 21, 28, 29, 37, 38, 45\}.$$

For each $i \in N_k(j)$, we can obtain the corresponding l_{ikj}. Using Equation (1), we can obtain all these p_{ikj} values. For example, $l_{21,1,46} = 14.552$. Thus, $p_{21,1,46} = 1 - e^{-\eta \times l_{21,1,46}} = 0.582$.

By using the GAH procedure, we will first choose grid 30, then grid 37 and grid 59. The corresponding objective function value is 27.98.

When we use the branch and bound algorithm to solve this base case, there are $|T| = |\bigcup_{k,j} N_k(j)| = 47$ decision variables if we do not use dominance properties. By using the Corollary 1 of Theorem 1, we can eliminate some number of decision variables. For example, grids $2, 4$ are dominated by grids $3, 11, 12$, grids $16, 32$ are dominated by grids $22, 23, 24$ and grids $61, 63$ are dominated by grids $53, 54, 62$. Therefore, we can eliminate decision variables $x_2, x_4, x_{16}, x_{32}, x_{61}, x_{63}$. Furthermore, we can use Theorem 1 to add some constraints to decrease the feasible space. For example, $51, 52, 58, 60$ are dominated by grid 59. Thus we can add the following constraints:

$$x_{51} \leq x_{59},\ x_{52} \leq x_{59},\ x_{58} \leq x_{59},\ x_{60} \leq x_{59}.$$

When solved by a branch and bound algorithm, the optimal solution is $x_{22}^* = x_{37}^* = x_{59}^* = 1$ and the corresponding objective function value is 27.86. For this example, the relative error of the GAH procedure is

$$RE = \frac{\text{GAH Value} - \text{Branch and Bound Value}}{\text{Branch and Bound Value}} = \frac{27.98 - 27.86}{27.86} = 0.43\%.$$

6 Computational Analysis

In this section, we will first perform sensitivity and robustness analyses on the base case. Later, we perform an experiment to illustrate the performance of the GAH procedure.

6.1 Sensitivity and Robustness Analyses

Here, we study the sensitivity due to base case parameter value settings. First, we investigate the effect of the number of detectors. As shown in Figure 5, when we employ more detectors, the expected casualties will decrease. Moreover, the marginal benefit associated with each additional detector is decreasing, which makes sense intuitively.

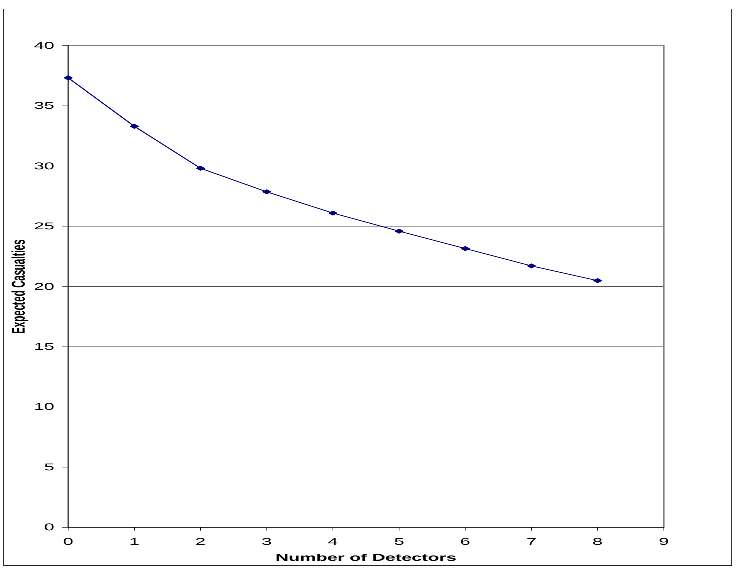

Figure 5: Effect of number of detectors

Figure 6 illustrates the effect of detection radius. The line shown in the figure is a piecewise convex function, where a detection radius 10 m represents a break point. When the detection radius increases from 6 m, the marginal benefit is decreasing. But when the detection radius hits 10 m, the marginal benefit increases and later decreases.

Figure 7 illustrates the effect of the instantaneous detection rate. The trend makes sense, since when the detection rate increases the detector is more accurate, hence the expected casualties will decrease. Figure 8 illustrates the effect of the target radius. When the target radius increases, the marginal destruction will actually decrease.

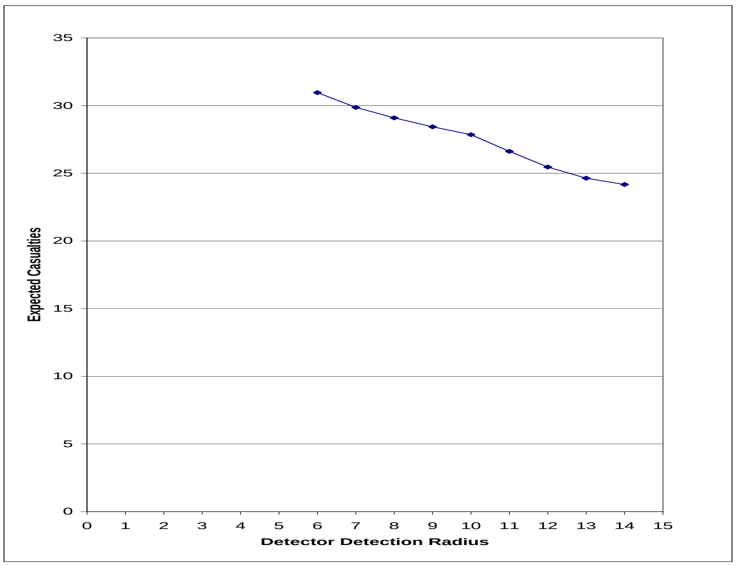

Figure 6: Effect of detection radius

We now consider a sample robustness analysis for the base case. In the base scenario, we assume that all the γ_{kj} values are the same and are equal to 0.0625. The optimal solution is: $x^*_{22} = x^*_{37} = x^*_{59} = 1$. Here, we do one perturbation on γ_{kj}. We assume that $\gamma_{kj} = 0.055 + \epsilon_{kj}$, where ϵ_{kj} is a random term. The summation of ϵ_{kj} over all (k, j) pairs equals to $1 - 0.055 \times 16 = 0.12$. For the perturbation, we consider ten randomly generated situations. The corresponding γ_{kj} values and the optimal solution for each situation are summarized in Table 2. In this table, Situ represents situation. From this table, we can see that 9 out of 10 situations have grids $22, 37, 59$ as optimal placements. Thus we can conclude that this placement set is a good choice when deploying three detectors.

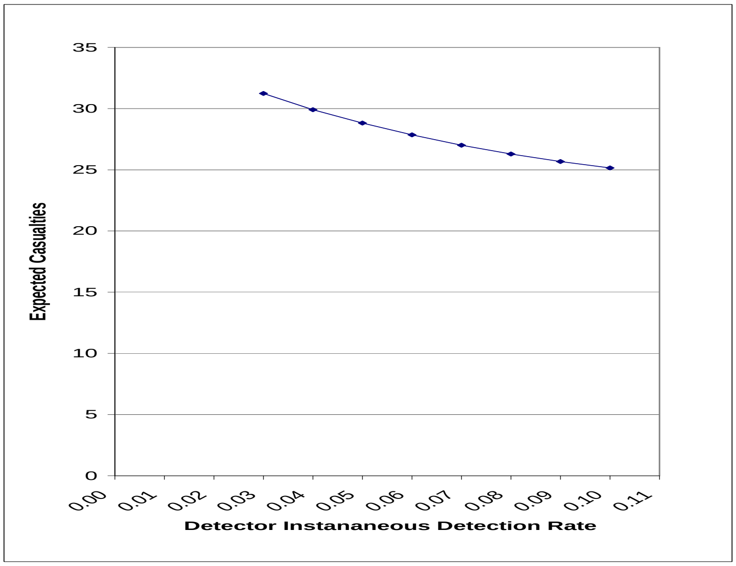

Figure 7: Effect of instantaneous detection rate

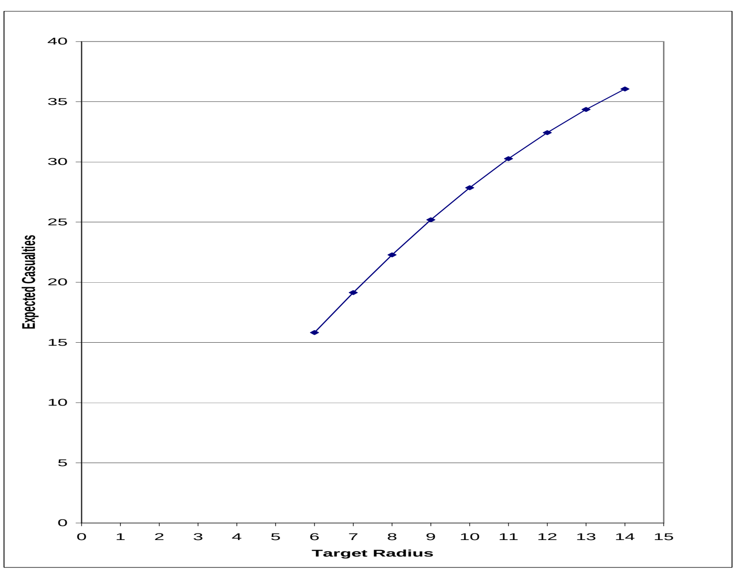

Figure 8: Effect of target radius

Table 2: Robustness analysis with $\gamma_{kj} = 0.055 + \epsilon_{kj}$

γ_{kj}	Situ1	Situ2	Situ3	Situ4	Situ5	Situ6	Situ7	Situ8	Situ9	Situ10
$\gamma_{1,28}$	0.063	0.067	0.071	0.067	0.058	0.063	0.058	0.058	0.060	0.066
$\gamma_{1,46}$	0.068	0.063	0.066	0.060	0.058	0.055	0.055	0.062	0.059	0.058
$\gamma_{2,28}$	0.061	0.067	0.069	0.066	0.060	0.055	0.069	0.069	0.064	0.063
$\gamma_{2,46}$	0.058	0.060	0.071	0.066	0.062	0.061	0.065	0.066	0.066	0.059
$\gamma_{3,28}$	0.068	0.064	0.063	0.063	0.063	0.064	0.063	0.061	0.064	0.060
$\gamma_{3,46}$	0.061	0.064	0.062	0.058	0.061	0.058	0.058	0.057	0.060	0.061
$\gamma_{4,28}$	0.068	0.064	0.056	0.065	0.059	0.072	0.060	0.056	0.067	0.058
$\gamma_{4,46}$	0.066	0.056	0.062	0.063	0.064	0.057	0.061	0.063	0.060	0.059
$\gamma_{5,28}$	0.062	0.066	0.060	0.063	0.063	0.070	0.064	0.065	0.063	0.066
$\gamma_{5,46}$	0.062	0.068	0.055	0.061	0.063	0.055	0.062	0.062	0.058	0.066
$\gamma_{6,28}$	0.059	0.057	0.057	0.067	0.065	0.064	0.063	0.067	0.067	0.065
$\gamma_{6,46}$	0.056	0.068	0.061	0.059	0.064	0.060	0.068	0.056	0.064	0.068
$\gamma_{7,28}$	0.056	0.062	0.068	0.065	0.066	0.073	0.061	0.061	0.057	0.061
$\gamma_{7,46}$	0.067	0.056	0.061	0.056	0.065	0.058	0.066	0.069	0.064	0.068
$\gamma_{8,28}$	0.068	0.062	0.061	0.059	0.065	0.072	0.057	0.063	0.059	0.059
$\gamma_{8,46}$	0.057	0.057	0.057	0.062	0.065	0.062	0.070	0.067	0.066	0.062
Optimal Sol	22,37,59	22,36,37	22,37,59	22,37,59	22,37,59	22,37,59	22,37,59	22,37,59	22,37,59	22,37,59

To further explore the robustness of the sensor placements suggested in the base case, we consider two other cases where the values of γ_{kj} are randomly generated. For both these cases, we perform a perturbation analysis and assume that the summation of the random terms is still 0.12. The results show that in all 10 situations the same optimal placements for sensors are suggested.

6.2 Performance of the GAH Procedure

In this subsection, we conduct a computational experiment to illustrate the performance of the GAH procedure, while considering three factors. These are: (i) the number of entrances; (ii) the number of potential explosive grids; and (iii) whether or not we have blocked grids. For the number of entrances and the number of explosive grids, we consider 3 cases each. So, in all we have $3 \times 3 \times 2 = 18$ combinations.

The basic setting is shown in Table 3. In this table, NE represents the number of entrances, NEx represents the number of explosive grids, EGs represents the set of explosive grids, W/Wo represents with or without blocked grids, BGs represents the set of blocked grids, GSol represents the solution of GAH procedure, GVal represents the objective function value from the GAH procedure, GT represents the running time (in CPU seconds) of the GAH procedure, BBSol represents the solution from the branch and bound algorithm, BBVal represents the objective function value from the branch and bound algorithm, BBT represents the running time (in CPU seconds) of the Branch and Bound algorithm, and RE represents the relative error of the GAH procedure.

From this experiment, we see that for 13 out of 18 combinations, the GAH procedure obtains the optimal solution. For the other five combinations, the relative error is very small, never more than 1.5%. Furthermore, the running time of the heuristic is 3 or more orders of magnitude smaller than that of the exact branch and bound algorithm. We note that this significantly reduced running time is especially helpful to conduct robustness analysis of the type discussed earlier in this section.

We now consider two situations where the grid sizes are 5 m $\times$ 5 m and 2.5 m $\times$ 2.5 m, respectively. For the first situation which has 256 grids, the GAH procedure only needs 0.52 seconds to obtain the solution. Even for the second case with 1024 grids, the running time

Table 3: Details of computational experiment

NE	Entrances	NEx	EGs	W/Wo	BGs	GSol	GVal	GT	BBSol	BBVal	BBT	RE
8	3,6,17,24,41,48,59,62	3	22,28,46	W	13,26,31,40,43,50	20,30,36	28.14	0.13	20,30,36	28.14	164	**0.00%**
8	3,6,17,24,41,48,59,62	3	22,28,46	Wo	N/A	21,30,36	30.20	0.08	21,30,36	30.20	1146	**0.00%**
8	3,6,17,24,41,48,59,62	2	28,46	W	13,26,31,40,43,50	30,37,59	27.98	0.07	22,37,59	27.86	172	0.43%
8	3,6,17,24,41,48,59,62	2	28,46	Wo	N/A	30,37,44	30.00	0.08	30,37,44	30.00	646	**0.00%**
8	3,6,17,24,41,48,59,62	1	28	W	13,26,31,40,43,50	19,30,44	27.29	0.04	19,30,44	27.29	28	**0.00%**
8	3,6,17,24,41,48,59,62	1	28	Wo	N/A	19,30,44	28.76	0.07	19,30,44	28.76	92	**0.00%**
6	3,6,24,33,59,62	3	22,28,46	W	13,26,31,40,43,50	3,30,36	27.83	0.02	20,30,44	27.74	234	0.32%
6	3,6,24,33,59,62	3	22,28,46	Wo	N/A	21,44,62	29.21	0.05	3,6,44	29.09	435	0.42%
6	3,6,24,33,59,62	2	28,46	W	13,26,31,40,43,50	3,22,44	26.30	0.02	3,22,44	26.30	163	**0.00%**
6	3,6,24,33,59,62	2	28,46	Wo	N/A	21,44,62	29.12	0.06	20,24,44	28.80	743	1.13%
6	3,6,24,33,59,62	1	28	W	13,26,31,40,43,50	3,22,44	25.52	0.03	3,22,44	25.52	26	**0.00%**
6	3,6,24,33,59,62	1	28	Wo	N/A	3,21,44	27.11	0.03	3,21,44	27.11	82	**0.00%**
4	4,24,33,61	3	22,28,46	W	13,26,31,40,43,50	4,30,33	25.93	0.03	24,34,61	25.59	97	1.35%
4	4,24,33,61	3	22,28,46	Wo	N/A	4,32,33	25.59	0.07	4,33,61	25.59	79	**0.00%**
4	4,24,33,61	2	28,46	W	13,26,31,40,43,50	30,33,61	24.56	0.01	30,34,61	24.56	90	**0.00%**
4	4,24,33,61	2	28,46	Wo	N/A	4,33,61	25.59	0.07	4,33,61	25.59	101	**0.00%**
4	4,24,33,61	1	28	W	13,26,31,40,43,50	4,32,61	25.59	0.02	12,34,61	25.59	195	**0.00%**
4	4,24,33,61	1	28	Wo	N/A	4,32,61	25.59	0.01	4,24,61	25.59	126	**0.00%**

for the GAH procedure is only 19.96 seconds. The branch and bound algorithm is not able to find an optimal solution for either of these situations even after several hours of effort. When grid size gets smaller, the model is a closer representation of reality but the dimension of the problem increases dramatically. For such cases, we can use the GAH procedure to obtain a good approximate solution.

7 Conclusions and Future Work

In this paper, we considered how to deploy SB detectors in a threat area where the potential targets are known. Based on a grid model, we proposed an optimization model where the objective function is the total expected casualties. We derived a series of properties to gain a better understanding of the model. Later, we developed two algorithms (one heuristic, one exact) to solve the corresponding model. We also presented a base case study. Using this base case, we illustrated both sensitivity and robustness analyses of the model. Computational experiments to verify the effectiveness of the heuristic procedure were also developed.

In our model development we considered just one type of detector. Actually there are several kinds of detectors available in the market with different characteristics and costs. One possible future direction is to consider how to choose from different kinds of detectors, how many for each kind to employ, and where to site the different detector types.

In this paper, we assumed that the detectors work independently. This assumption is reasonable if the effective detection areas of each placed detector do not substantially intersect. For the case where these areas have significant intersection, the joint detection probability should be considered. In particular we note that the case of multiple sensors at a grid would fall in this category. The use of data fusion techniques to study the benefit of fused reports from multiple detectors is suggested as a way to address this modeling enhancement.

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