Social Balance on Networks: The Dynamics of Friendship and Hatred

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Basic question:

How do social networks evolve when both friendly and unfriendly relationships exist?

Partial answers: *(Heider 1944, Cartwright & Harary 1956, Wasserman & Faust 1994)*

Social balance as a static concept: a state without frustration.

This work:

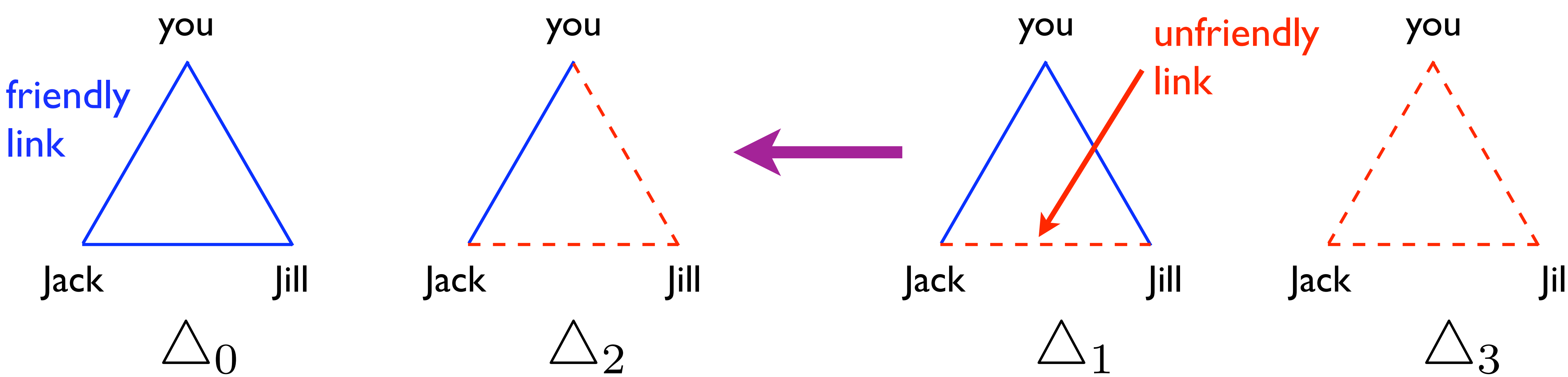
Endow a network with the simplest dynamics and investigate evolution of relationships.

Main questions:

Is balance ever reached? What is the final state like?
Or does the network evolves forever?

Socially Balanced States

Fritz Heider, 1946



unfrustrated / balanced

frustrated / imbalanced

Def: A network is balanced if all triads are balanced

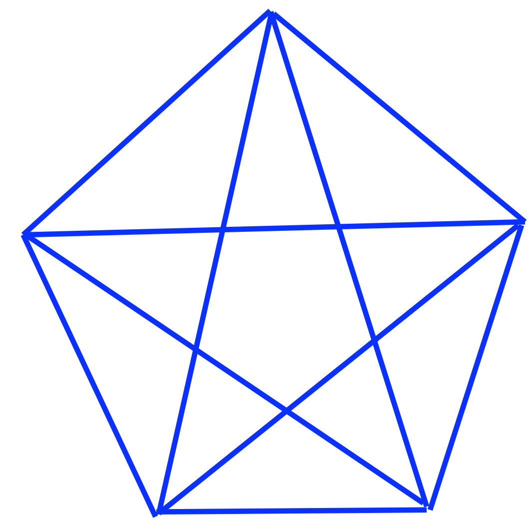
no $\triangle_1 \Rightarrow$ *a friend of my friend is my friend;*
a friend of my enemy is my enemy;
an enemy of my friend is my enemy;

no $\triangle_3 \Rightarrow$ *an enemy of my enemy is my friend.*

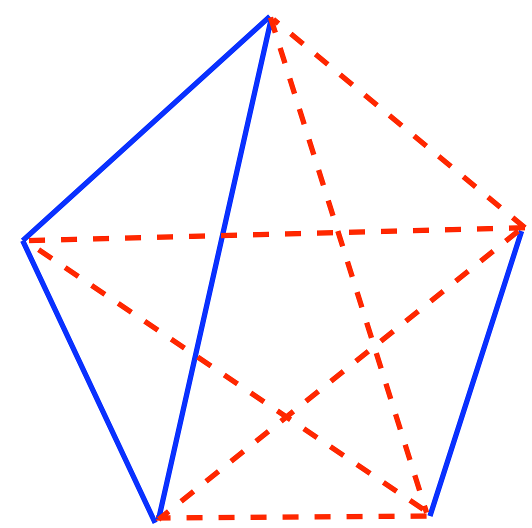
Arthashastra, 250 BCE

Theorem: on a complete graph a balanced state is

- either utopia:
  everyone likes each other



- or two groups of friends
  with hate between groups



(you are either with us or against us)

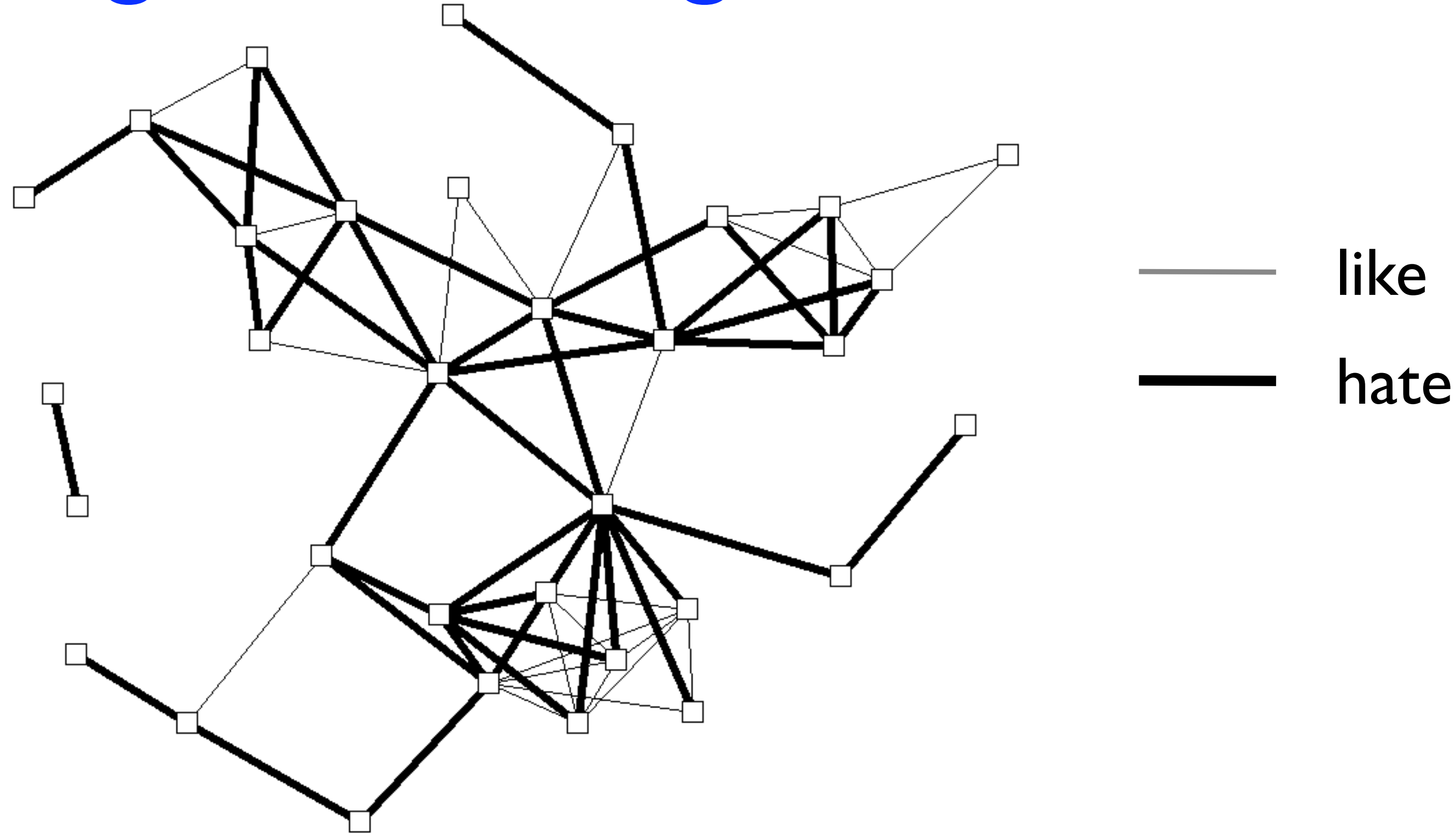
For not complete graphs there can be more than two cliques

Static description

Long Beach Gangs

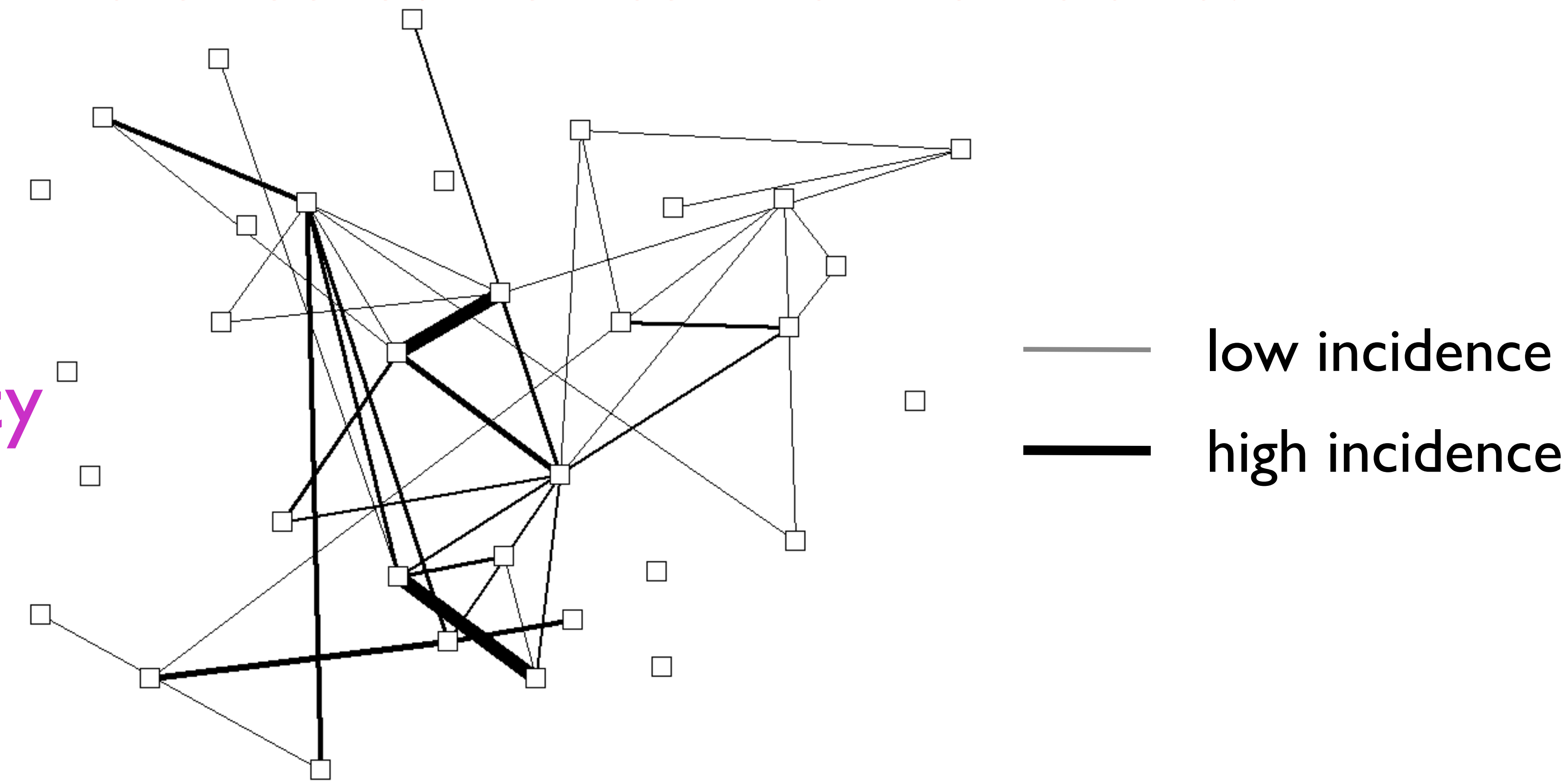
Nakamura, Tita, & Krackhardt (2007)

gang relations



how does violence correlate with relations?

violence frequency

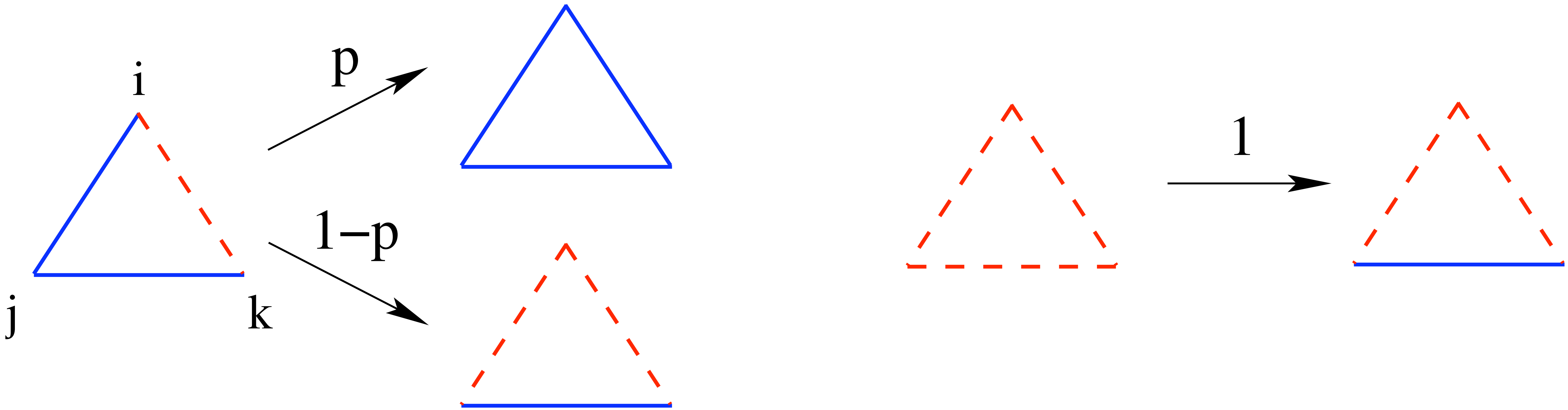


Local Triad Dynamics on Arbitrary Networks

(social graces of the clueless)

1. Pick a random imbalanced (frustrated) triad
2. Reverse a single link so that the triad becomes balanced

   probability p: unfriendly → friendly;   probability 1-p: friendly → unfriendly



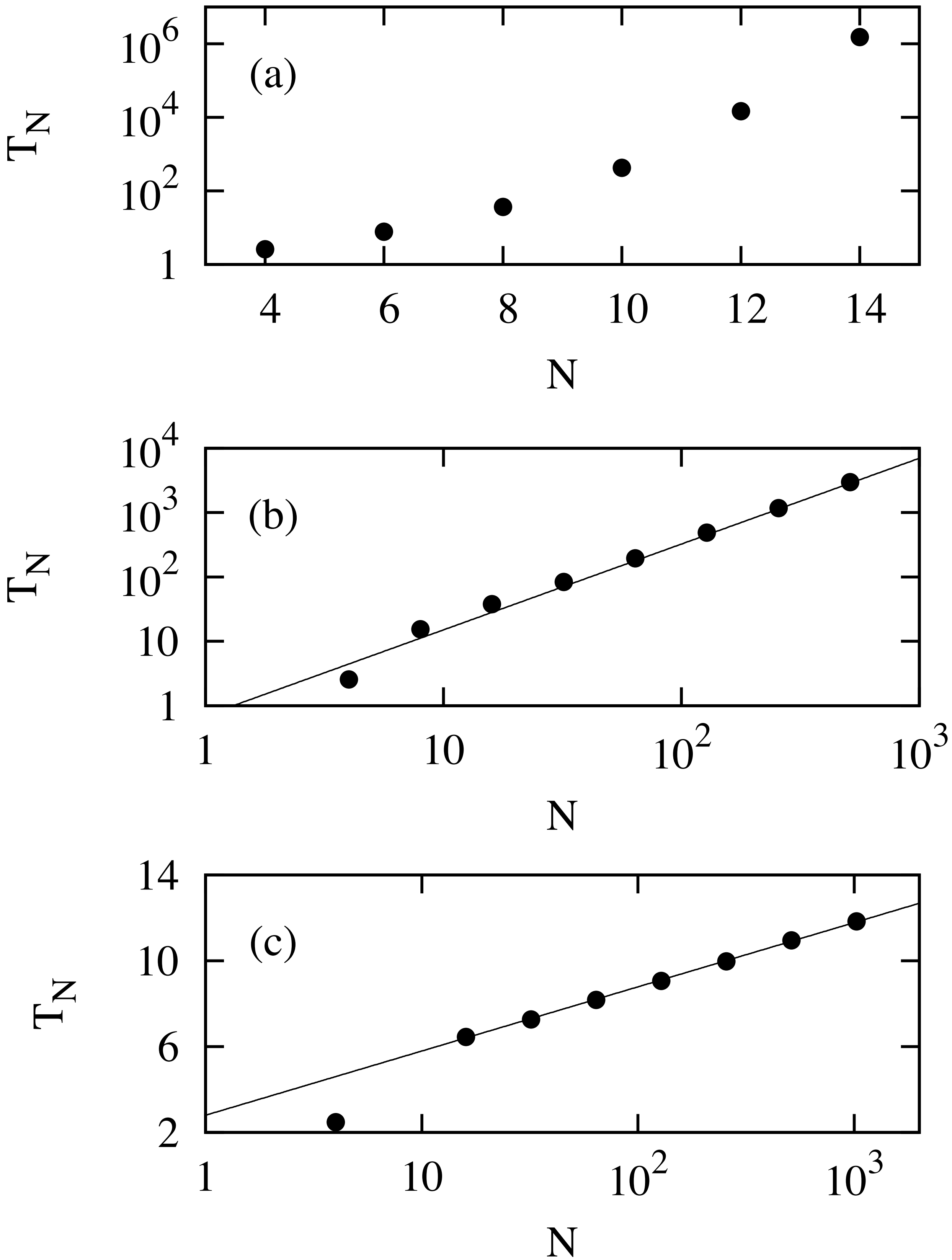
Fundamental parameter p:

p=1/3: flip a random link in the triad equiprobably

p>1/3: predisposition toward tranquility

p<1/3: predisposition toward hostility

Finite Society Reaches Balance (Complete Graph)



Two Cliques

$$p < \frac{1}{2}, \qquad T_N \sim e^{N^2}$$

$$p = \frac{1}{2}, \qquad T_N \sim N^{4/3}$$

$$p > \frac{1}{2}, \qquad T_N \sim \frac{\ln N}{2p-1}$$

Utopia

Triad Evolution (Infinite Complete Graph)

Basic graph characteristics:

N nodes

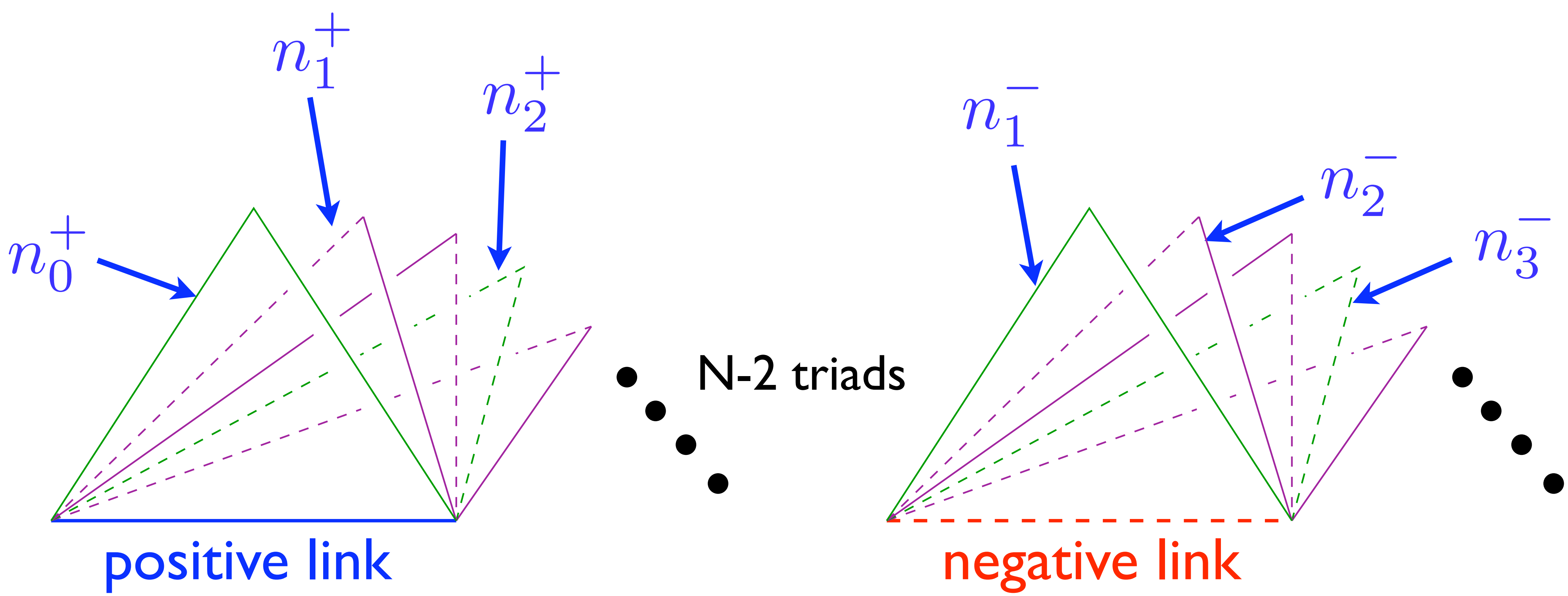
$\frac{N(N-1)}{2}$ links

$\frac{N(N-1)(N-2)}{6}$ triads

ρ = friendly link density

n_k = density of triads of type k

$n_k^{\pm}$ = density of triads of type k attached to a $\pm$ link



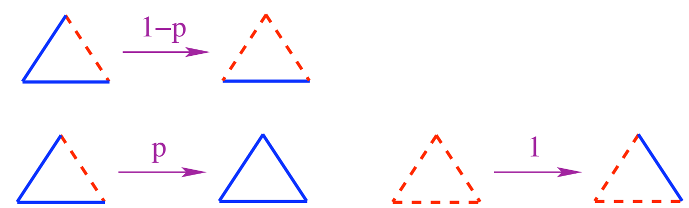
Triad Evolution on the Complete Graph

n_k = density of triads of type k

$n_k^{\pm}$ = density of triads of type k attached to a $\pm$ link

$\pi^+ = (1-p)\,n_1$ flip rate $+ \;\rightarrow\; -$

$\pi^- = p\,n_1 + n_3$ flip rate $- \;\rightarrow\; +$



Master equations:

$\triangle_1 \rightarrow \triangle_0$ $\triangle_0 \rightarrow \triangle_1$

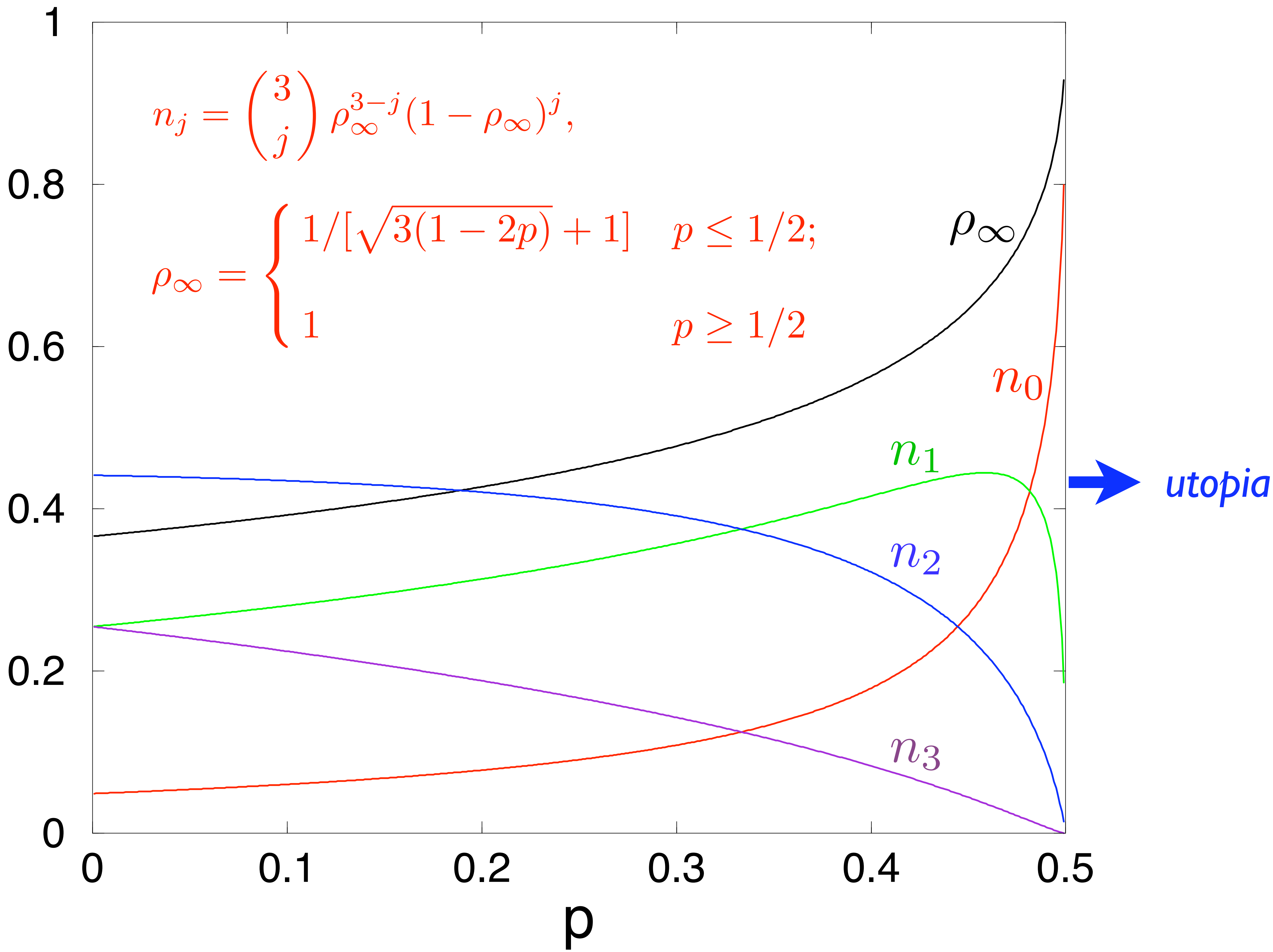
$$\frac{dn_0}{dt} = \pi^- n_1^- - \pi^+ n_0^+ \,,$$

$$\frac{dn_1}{dt} = \pi^+ n_0^+ + \pi^- n_2^- - \pi^- n_1^- - \pi^+ n_1^+ \,,$$

$$\frac{dn_2}{dt} = \pi^+ n_1^+ + \pi^- n_3^- - \pi^- n_2^- - \pi^+ n_2^+ \,,$$

$$\frac{dn_3}{dt} = \pi^+ n_2^+ - \pi^- n_3^- \,.$$

Steady State Triad Densities



Triad Evolution (Infinite Complete Graph)

rate equation for the friendly link density:

$- \to +$ in $\triangle_1$ $\quad + \to -$ in $\triangle_1$ $\quad - \to +$ in $\triangle_3$

$$\frac{d\rho}{dt} = 3\rho^2(1-\rho)[p-(1-p)] + (1-\rho)^3$$

$$= 3(2p-1)\rho^2(1-\rho) + (1-\rho)^3$$

$$\rho(t) \sim \begin{cases} \rho_\infty + Ae^{-Ct} & p < 1/2; \\ 1 - \dfrac{1-\rho_0}{\sqrt{1+2(1-\rho_0)^2 t}} & p = 1/2; \\ 1 - e^{-3(2p-1)t} & p > 1/2. \end{cases}$$

- $p < 1/2$: rapid onset of frustration
- $p = 1/2$: slow relaxation to utopia
- $p > 1/2$: rapid attainment of utopia

Constrained (Socially Aware) Triad Dynamics

1. Pick a random link

2. Reverse it *only if*
the total number of balanced triads increases or stays the same

Outcome: Quick approach to a final static state
Typically: $T_N \sim \ln N$

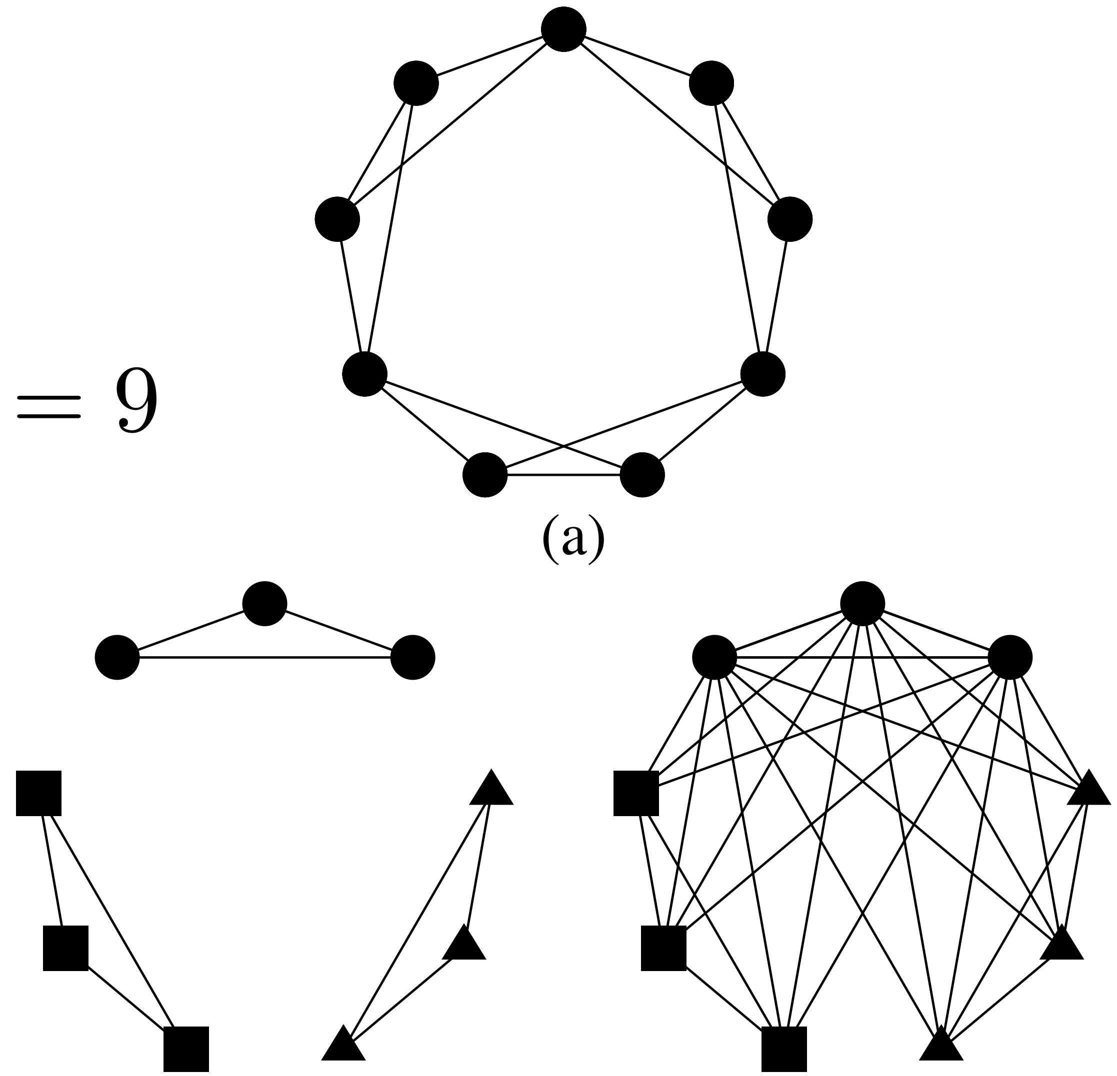
Final state is either balanced or *jammed*

Jammed state: Imbalanced triads exist, but any update only increases the number of imbalanced triads.

Final state is almost always balanced even though *jammed states* are much more numerous.

Jammed States

Examples for $N = 9$



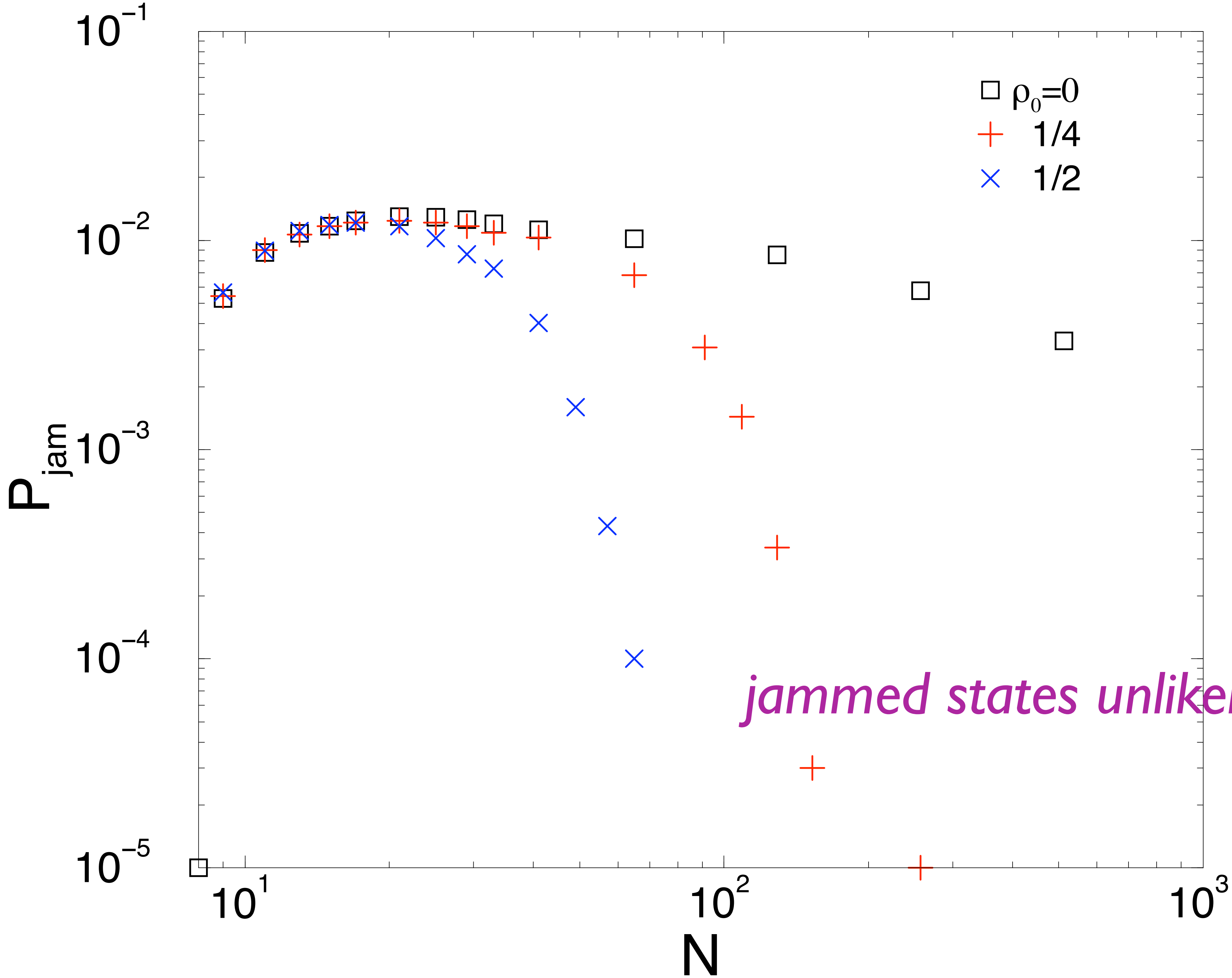
(a) (b) (c)

Jammed states are only for:

$$N = 9, 11, 12, 13, \ldots$$

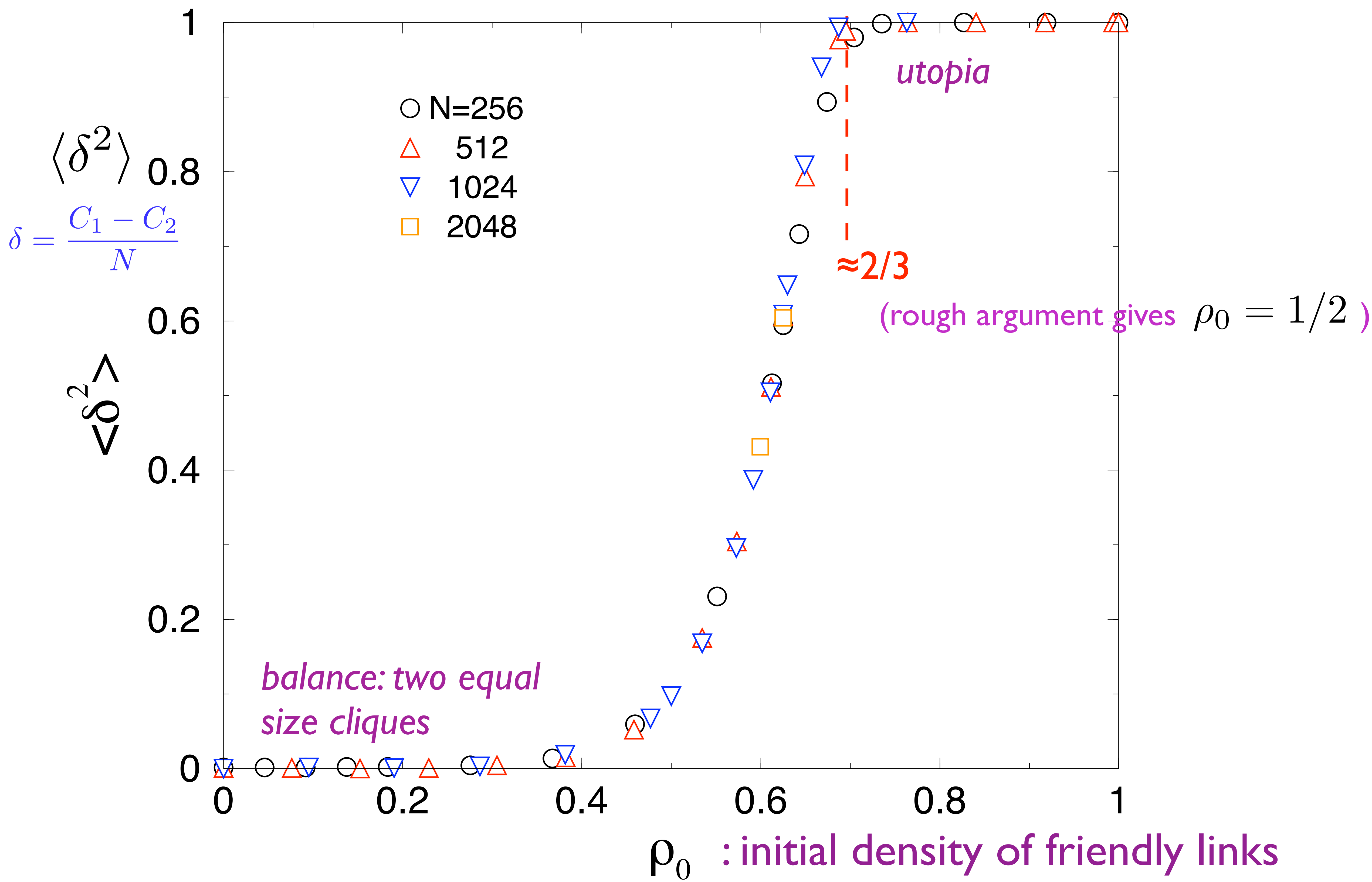
(unresolved situations)

Likelihood of Jammed States



jammed states unlikely for large N

Final Clique Sizes

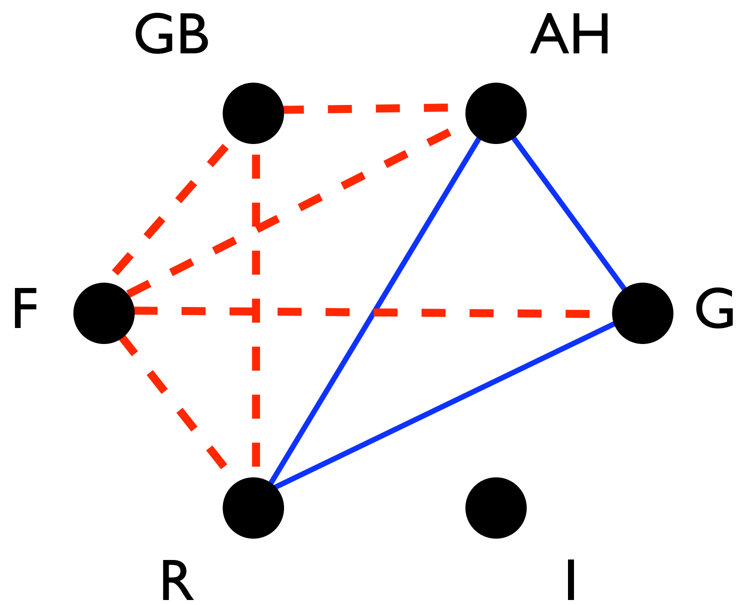


Related theoretical works

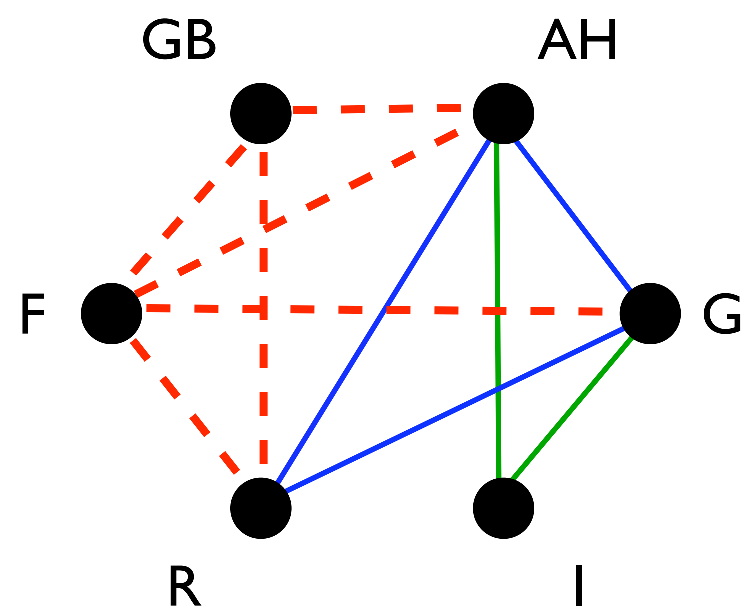
Kulakowsi et al. '05:
- Continuous measure of friendship

Meyer-Ortmanns et al. '07:
- Diluted graphs
- Spatial effects
- k-cycles
- ~Spin glasses ~Sat problems

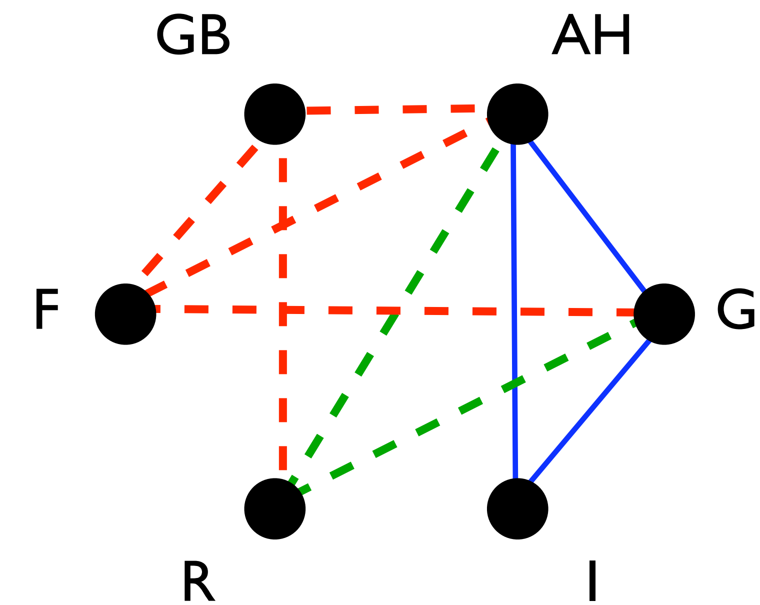
A Historical Lesson



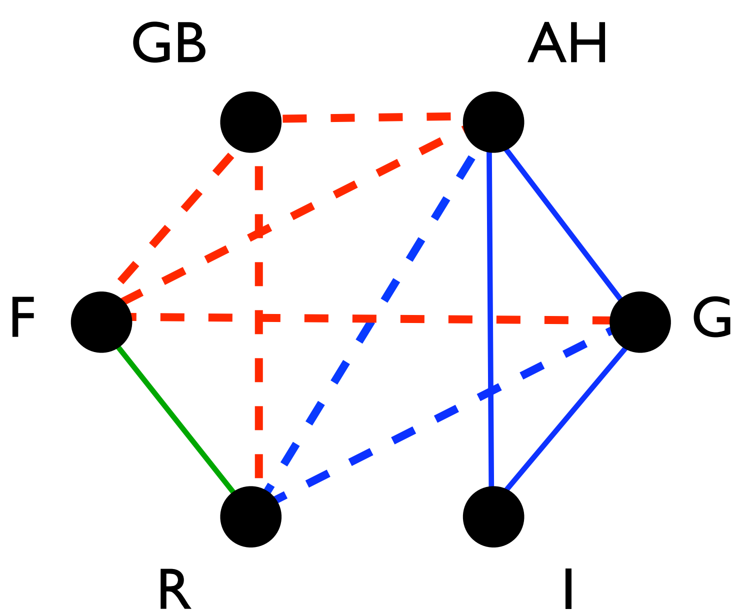
3 Emperor's League 1872-81



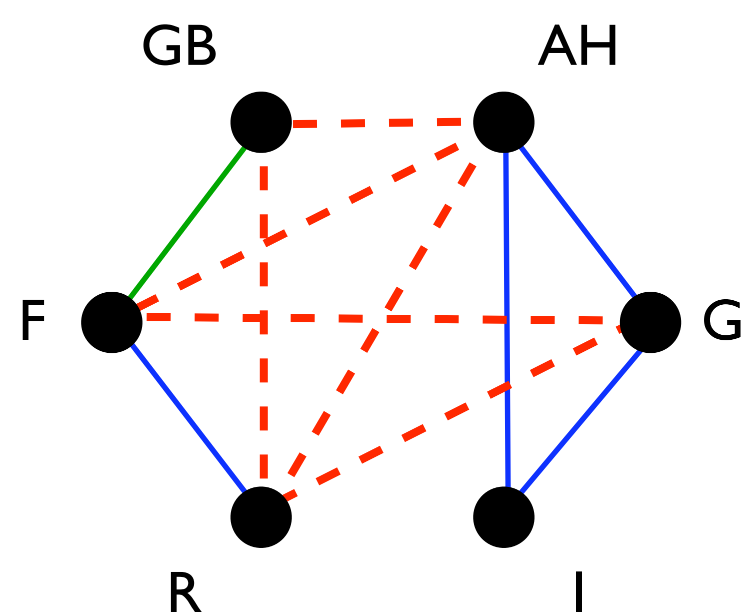
Triple Alliance 1882



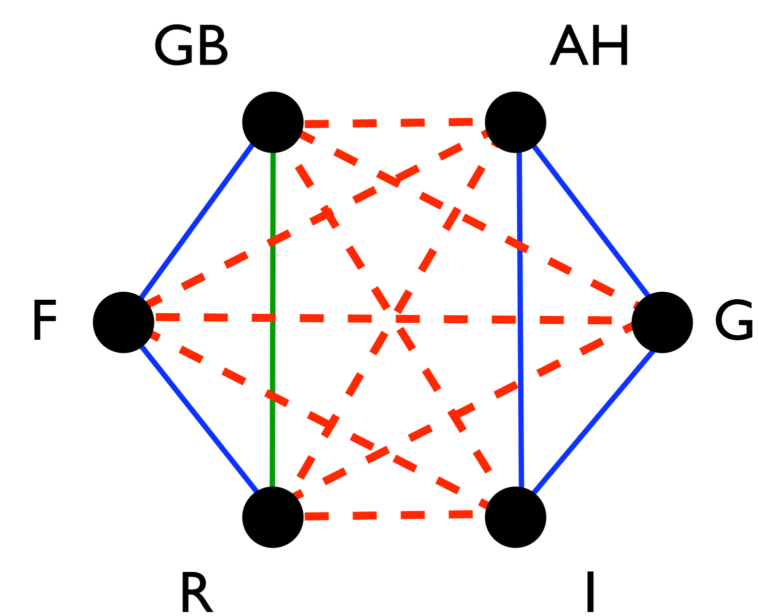
German-Russian Lapse 1890



French-Russian Alliance 1891-94



Entente Cordiale 1904

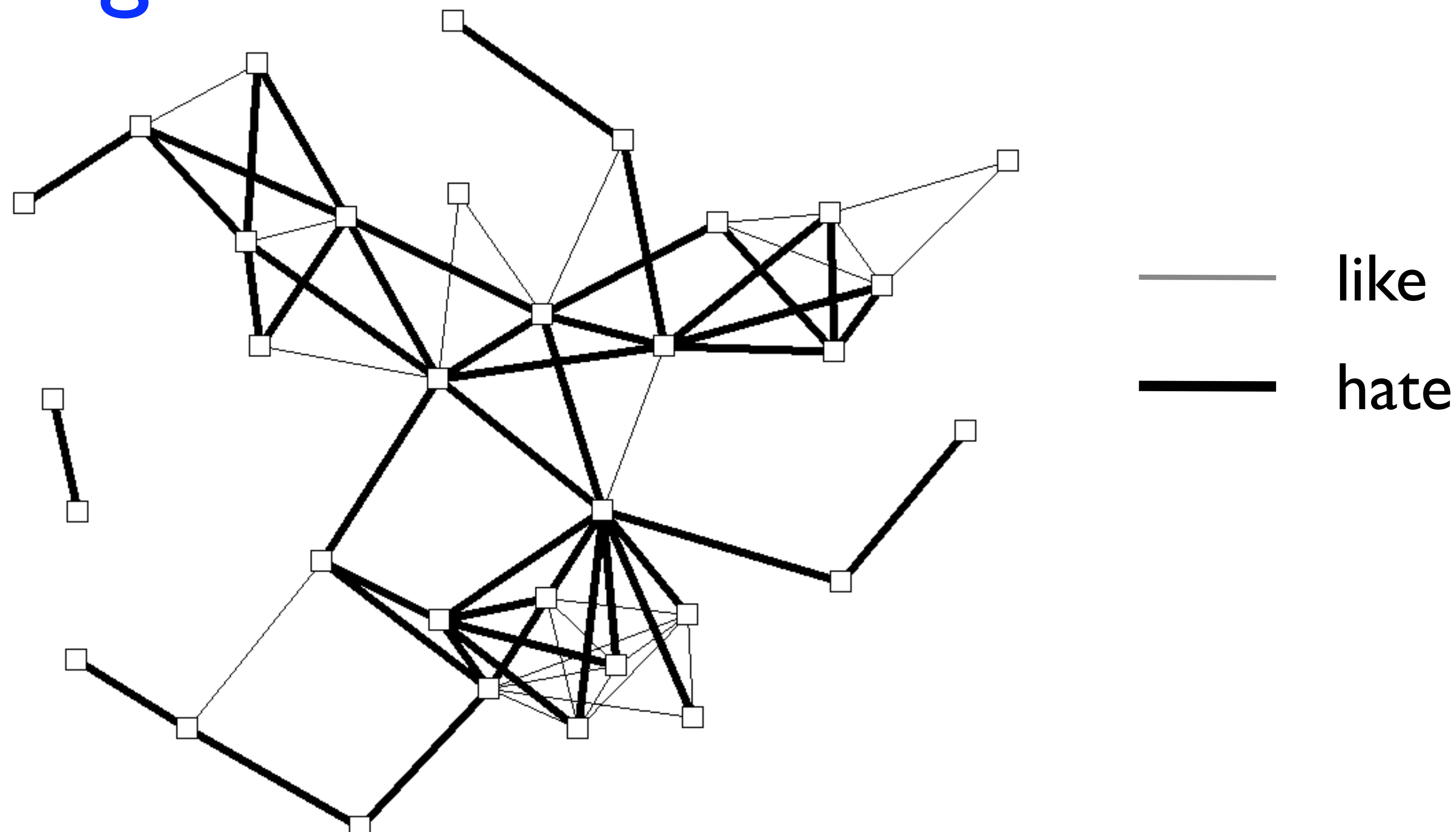


British-Russian Alliance 1907

Long Beach Gang Lesson

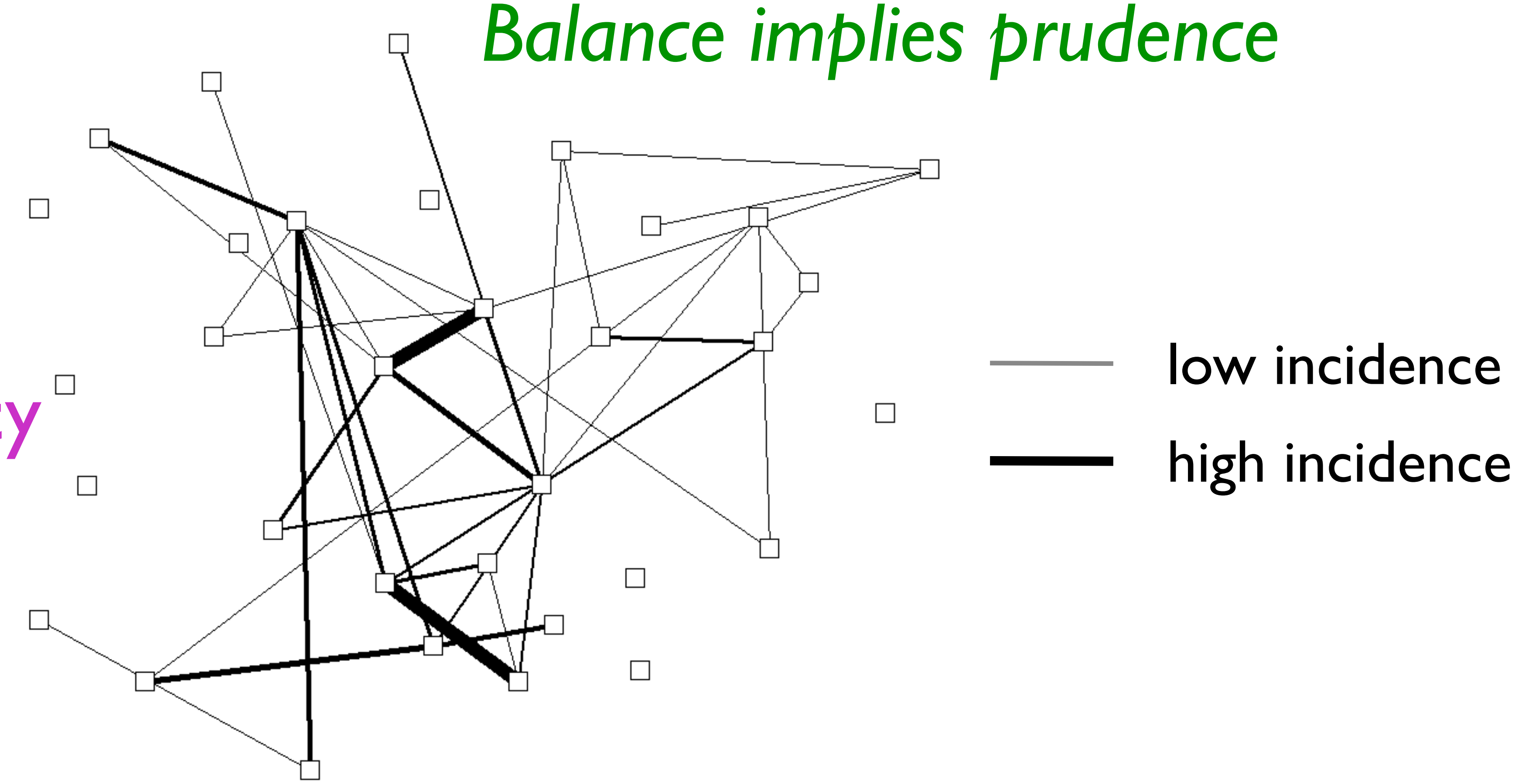
Nakamura, Tita, & Krackhardt (2007)

gang relations



Imbalance implies impulsiveness
Balance implies prudence

violence frequency



Summary & Outlook

Local triad dynamics:

finite network: social balance, with the time until balance strongly dependent on p

infinite network: phase transition at p=½ between utopia and a dynamical state

Constrained triad dynamics

jammed states possible but rarely occur

infinite network: two cliques always emerge, with utopia for $\rho_0 \gtrapprox 2/3$

Open questions:

allow  → several cliques in balanced state

asymmetric relations

dynamics in real systems, gang control?