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# Statistical Models for Physically Derived Target Sub-Spaces

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# Statistical Models for Physically Derived Target Sub-Spaces

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## ABSTRACT

Traditional approaches to hyperspectral target detection involve the application of detection algorithms to atmospherically compensated imagery. Rather than compensate the imagery, a more recent approach uses physical models to generate target sub-spaces. These *radiance* sub-spaces can then be used in an appropriate detection scheme to identify potential targets.

The generation of these sub-spaces involves some *a priori* knowledge of data acquisition parameters, scene and atmospheric conditions, and possible calibration errors. Variation is allowed in the model since some parameters are difficult to know accurately. Each vector in the subspace is the result of a MODTRAN simulation coupled with a physical model. Generation of large target spaces can be computationally burdensome. This paper explores the use of statistical methods to describe such target spaces. The statistically modeled spaces can then be used to generate arbitrary radiance vectors to form a sub-space. Statistically modeled target sub-spaces, using limited training samples, were found to accurately resemble MODTRAN derived radiance vectors.

**Keywords:** Hyperspectral, Subpixel Target Detection, Invariant Subspace, Physics Based Modeling, Target Sub-Spaces, Statistical Models

## 1. INTRODUCTION

This paper investigates the use of statistical models to represent target sub-spaces, for use in target detection. The approach to target detection, as it applies in this paper, is one where the detection process is performed in the uncompensated, but calibrated, radiance domain. In order to perform such detection, estimates of how the target reflectance signature might appear to the sensor, through the atmosphere, have to be generated. These estimates are generated using an atmospheric propagation code and a physics-based, sensor-reaching, radiance model. A collection of these estimates produce what are called (radiance) *target spaces* or sub-spaces. The computational infrastructure to create such spaces can be quite complex and time consuming during run time. A possible alternative is to generate a sparse design scheme of a desired target space (minimum number of simulations or runs), and use this information as training for a statistical model. The statistical model can then be used to create an arbitrary number of radiance vectors to be used in the formation of a target space. This paper explores such an approach to target space modeling.

## 2. BACKGROUND AND THEORY

### 2.1. Detection: Traditional and Physics-Based

In target detection, we often seek to atmospherically compensate hyperspectral imagery so as to convert sensor reaching radiance to ground leaving spectral reflectance. Once the imagery has been compensated, detection algorithms are used to compare image reflectances to library or measured relfectances in search of a desired target (see Figure 1). Rather than compensate the imagery, an alternative is to estimate what the ground leaving spectral reflectance would look like as seen by the sensor in radiance space.[1] This approach entails modeling the propagation of a target reflectance spectrum through the atmosphere up to the sensor.

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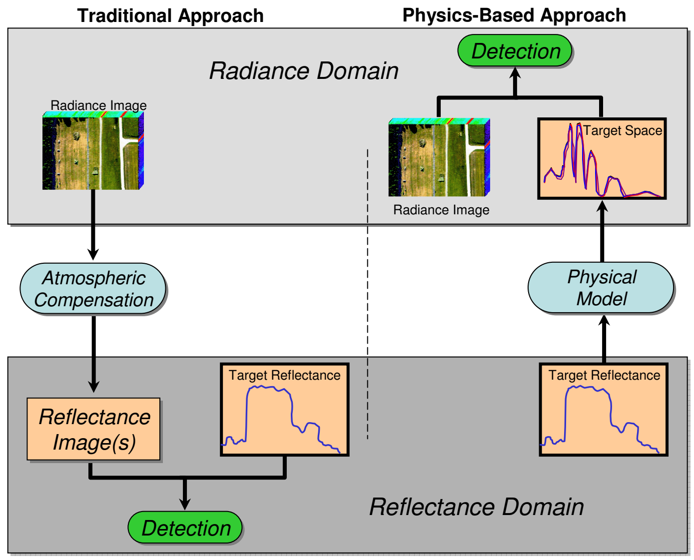


**Figure 1.** Illustrated are traditional and physics-based approaches to target detection. The traditional approach moves the imagery from the radiance domain the reflectance domain, where detection is performed. The physics-based approach moves the target reflectance to the radiance domain, where detection is performed.

The advantage this technique has over that of compensated imagery is that target illumination variations, for example, can be integrated into the process through use of a physical model thus making the approach *invariant* to illuminations effects. Since we are using a model, we can essentially incorporate any variation in the target signature such as target contamination or adjacency effects. If one wishes to implement variations on an atmospheric parameter, for example, using the compensation approach, we would need to generate multiple reflectance hypercubes. However, any variation of a sensor-reaching radiance spectrum, using a physical model, simply manifests itself as multiple spectra in a *target space*.

The physics-based model that has been used for this approach to detection is one derived by Schott.[2] In simplest form, the spectral radiance reaching an airborn or satellite sensor can be expressed as

$$L_p(\lambda) = \int_\lambda \beta_p(\lambda)\left[\Big(E_s'(\lambda)\tau_1(\lambda)\cos\theta + FE_d(\lambda)\Big)\tau_2(\lambda)\frac{r(\lambda)}{\pi} + L_u(\lambda)\right]d\lambda \tag{1}$$

where $L_p(\lambda)$ is the effective spectral radiance in the $p^{th}$ band in units of $[Wcm^{-2}sr^{-1}\mu m^{-1}]$, $E_s'(\lambda)$ is the exoatmospheric spectral irradiance from the Sun in units of $[Wcm^{-2}\mu m^{-1}]$, $\tau_1(\lambda)$ is the transmission through the atmosphere along the Sun-target path, $\theta$ is the angle from the surface normal to the Sun, $F$ is the fraction of the spectral irradiance from the sky ($E_d(\lambda)$), incident on the target (*i.e.,* not blocked by adjacent objects), sometimes called shape factor, $\tau_2(\lambda)$ is the transmission along the target-sensor path, $r(\lambda)$ is the spectral reflectance factor for the target of interest (*i.e.,* $r(\lambda)/\pi$ is the bidirectional reflectance $[sr^{-1}]$), $L_u(\lambda)$ is the spectral path radiance $[Wcm^{-2}sr^{-1}\mu m^{-1}]$, and $\beta_p(\lambda)$ is the normalized spectral response of the $p^{th}$ spectral channel of the sensor under study where

$$\beta_p(\lambda) = \frac{\beta_p'(\lambda)}{\int \beta_p'(\lambda)\, d\lambda} \tag{2}$$

with $\beta_p'(\lambda)$ being the peak normalized spectral response of the $p^{th}$ channel. Schott[2] also describes how the MODTRAN radiative transfer code[3] can be used to solve for each of the radiometric terms in Eq. (1) (*i.e.,* $E_s'(\lambda)$, $\tau_1(\lambda)$, $\tau_2(\lambda)$, $E_d(\lambda)$, and $L_u(\lambda)$) given a set of atmospheric and illumination descriptors. Once the terms are solved for, the spectral radiance target vector $\mathbf{x}$ observed by a $p$-channel sensor can be expressed as

$$\mathbf{x} = [L_1(\lambda), L_2(\lambda), \ldots, L_p(\lambda)]^T. \tag{3}$$

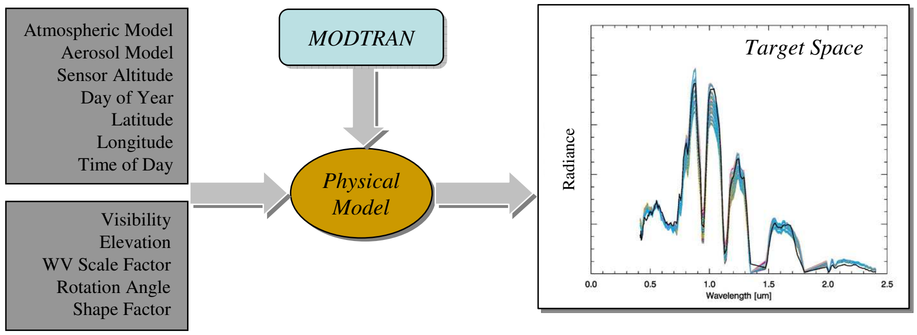


**Figure 2.** Example of how target spaces are created. The upper left input box corresponds to parameters that are usually known at image acquisition time. The lower left input box corresponds to parameters that are more difficult to estimate and are therefore varied.

## 2.2. Target Spaces

In practice a *family* of radiance vectors is usually generated to account for lack of knowledge about the atmospheric, illumination and viewing conditions. This is accomplished by varying the inputs to MODTRAN to span a range of variables. In doing so, a wide range of potential target spectral vectors spanning a *target space* can be generated from a single target reflectance spectrum. In general, many of the input parameters to MODTRAN are usually known at the time of image acquisition or can be reasonably estimated (*e.g.,* atmospheric and aerosol model, day of year, location, time of day, etc.). For this research, emphasis is placed on varying MODTRAN parameters that more likely to be unknown. These include visibility, water vapor scale factor and ground topography (see Figure 2). In the case of water vapor scale factor, a physics based atmospheric compensation algorithm can be used to estimate per pixel total column water vapor which can then be converted to an appropriate range of scale factors. In addition to classical MODTRAN input parameters, terms such as shape factor (F) and target orientation are also varied. Currently, the shape factor parameter is a simple scalar, that ranges from zero to one, in order to account for the variation in downwelled radiance.

The target orientation term is really a mechanism to account for illumination effects. In this approach to detection, the zenith angle is known (and fixed) because we assume the user has some *a priori* knowledge about when the imagery was collected (*i.e.,* time of day and location). Even if the spectral character of the target, as seen by the sensor through the atmosphere, is modeled correctly in MODTRAN, we may still have issues related to spectral magnitude. This can manifest itself as a time of day variation, target-to-sensor angle or orientation variation, or both. In this research we factor in an additional parameter that allows the user to vary the *orientation* of the target relative to the Sun, in a 2D sense.

This variation in target orientation, or more precisely illumination, is simply a modulation of the direct solar term to account for projected area effects. We implement this as a target rotation angle relative to the zenith angle determined in MODTRAN, for the time of day. If the rotation is zero, then there is no adjustment of the direct term. However, a positive or negative rotation angle implies that we have rotated the target toward or away from the Sun, respectively. The relation for this modulation of the direct term is

$$E_{s\_new}(\lambda) = \frac{E_s(\lambda)}{\cos\sigma'}\cos\sigma_{new} \tag{4}$$

where $E_s(\lambda) = E'_s(\lambda)\cos\sigma'$, $\sigma_{new} = \sigma' - \sigma_{rot}$ and $\sigma_{rot}$ is the user specified angle of rotation. The zenith angle $\sigma'$ comes from MODTRAN output. It is important to note here that $E_s(\lambda)$ in this notation comes directly from MODTRAN and already has the zenith angle attenuation applied to it. We differentiate this value from $E'_s(\lambda)$ which is the direct solar term with out any angular effects. As an example, if the zenith angle was 60 degrees and we specify a rotation angle of $\sigma_{rot} = -10$ degrees, then we have

$$E_{s\_new}(\lambda) = \frac{E_s(\lambda)}{\cos 60}\cos(60-(-10)) = 0.68E_s(\lambda) \tag{5}$$

which says we have effectively reduced the direct-term illumination onto the target by 32%. Additional information describing these parameters as they apply to target space generation can be found in the literature.[4]

The actual target spaces are created through a series of MODTRAN runs followed by a set of C-shell UNIX scripts used to parse the MODTRAN output. Since computation time can be burdensome, all the runs are submitted to a workload management system for compute intensive jobs. This management system provides a job queueing mechanism, scheduling policy, priority scheme, and resource management which makes the hundreds of MODTRAN runs, and this approach to target detection, possible. Upon completion of the submitted jobs, the MODTRAN tape6 and tape7 files are parsed so as to obtain the various terms in the sensor-reaching radiance equation. These spectral quantities are then resampled using a user supplied sensor response file. Finally, the resampled quantities are assembled into an ASCII look-up-table (LUT) to be used as input to the physics based model described in Eq. (1).

### 2.3. Modeling of Target Spaces

The end result of the process described in the previous section is a set of (estimated) sensor-reaching radiance vectors. Collectively, these form a target space for a *single* reflectance vector. Each vector is derived from a specific set of atmospheric and environmental conditions (*i.e,* water vapor, shape factor, visibility, etc.). In the event that one wishes to obtain a vector from a combination that is not in the target space, the user must re-run MODTRAN with the new set of specified parameters.

The goal of this research is to see whether a statistical model could adequately represent a radiance target space. This would prove useful by generating a statistical model as a surrogate to MODTRAN runs. That is, an initial set of MODTRAN runs would be created as training to the model. These initial runs would be selected so as to sparsely describe the desired target space. Once the model coefficients are generated from the estimated radiance vectors, a user could simply extract arbitrary radiance vectors, using the model, to form a target space with out additional MODTRAN runs. Since the model will not exactly represent the desired target space, we can expect some errors.

When formulating the statistical model used to describe the response (radiance) variable, $L$, the following notation, to denote each of the five predictors, will be used:

- Visibility (V)
- GND Topography (T)
- WV Scale Factor (W)
- Change in Target Illumination (IL)
- Shape Factor (SF)

It turns out that radiance is a linear combination of the IL and SF variables. However, we need to use a third degree polynomial with respect to the remaining variables. More specifically, for each spectral band, $p$, we have

$$L_p = F_1(log(V), T, W) + IL \cdot F_2(log(V), T, W) + SF \cdot F_3(log(V), T, W) + \varepsilon \qquad (6)$$

where $F_1(log(V), T, W)$, $F_2(log(V), T, W)$, and $F_3(log(V), T, W)$ are polynomials of the third degree with respect to the predictors $log(V)$, $T$, and $W$. For visibility, $V$, a logarithmic transformation was most appropriate. The last term $\varepsilon$ represents the modeling error. Note that the above model has 60 coefficients (20 for each of the third degree polynomials of three variables, including cross terms), which need to be estimated for each band separately. Other functional forms of the model were also explored through investigation of residuals, but the one shown in Eq. (6) produced the best results. All data fitting (estimation of statistical parameters) was performed using the least squares method.

**Table 1.** Summary of scene parameters used in generating target space characteristics of an AVIRIS collection over Rochester, NY.

| Parameter | Value |
|---|---|
| Atmospheric Model | Midlatitude Summer† |
| Aerosol Model | Rural Extinction† |
| Sensor Altitude | 20.5 km |
| Day of the Year | 140 |
| Latitude | 43.1 N |
| Longitude | 77.6 W |
| Time of Day | 15.4 GMT |
| Visibility | Varied |
| Scene Topography | Varied |
| WV Scale Factor | Varied |

†An estimated parameter based on scene conditions and location.

# 3. RESULTS

## 3.1. Creating a Large Target Space

Section 2.2 described the theory as to what goes into the generation of a typical target space. In this section, we describe the generation of a large target space to be used in the analysis of statistical target space modeling.

In order to evaluate the performance of a *statically modeled* target space, a large target space was created which emulated what might be seen by the AVIRIS hyperspectral sensor. This target space modeled conditions during a collection campaign over Rochester, NY in the summer of 1995. A summary of the 'required' MODTRAN parameters used in the generation of the target space can be seen in Table 1.

The varied parameters listed in Table 1 (*i.e,* visibility, scene topography and water vapor), were chosen to reflect a wide range of possible levels. This included 11 levels of visibility, 7 levels of scene topography and 10 levels of water vapor (scale factor). The visibility parameter was chosen such that there were more samples below 25 km than above. This is because of the fact that there is typically little change in the sensor-reaching radiance when the visibility is greater than 25 to 30 km. However, below 20 km, the interactions due to scattering, for example, become increasingly important and tend to dominate the signal especially at very low values. The scattering algorithm used here was that of ISAAC in place of the more computational DISORT. The ISAAC algorithm was chosen simply because of its computational speed. The loss in accuracy versus speed was acceptable given the nature of this study. Future analysis could use the DISORT algorithm without out loss of generality or conclusions drawn in this research.

The ground variation was chosen so as to model the elevation associated with a small mountain (4000 feet). A summary of these varied parameters and their ranges can be seen in Table 2. Lastly, variations in target orientation and shape factor were implemented. The target orientation levels were selected such that the amount of illumination on the target varied from no direct illumination (-100%) to an overall increase in illumination of 15%, the most direct illumination the target could receive for the specified time of day. Similarly, the downwelled term was modulated to represent conditions where the target was in heavy shade (SF=0.2) versus open sky (SF=1). A summary of these levels can be seen in Table 2.

Accounting for all combinations and levels of parameters, the final target space contained a total of $11 \cdot 7 \cdot 10 \cdot 9 \cdot 5 = 34,650$ vectors. Using a single reflectance spectrum of a red painted asphalt surface (see Figure 3), the physics based model of Eq. (1) was implemented generating the target space shown in Figure 4. Additional, bands with low signal-to-noise or heavy water absorption were removed leaving a total of 152 (from 224) working spectral bands.

## 3.2. Statistical Target Space Evaluation

The first step in the evaluation of the model described in Section 2.3 was to fit it to the large target space described in Section 3.1. The model described in Eq. (6) was fitted for each of the 152 spectral bands. A

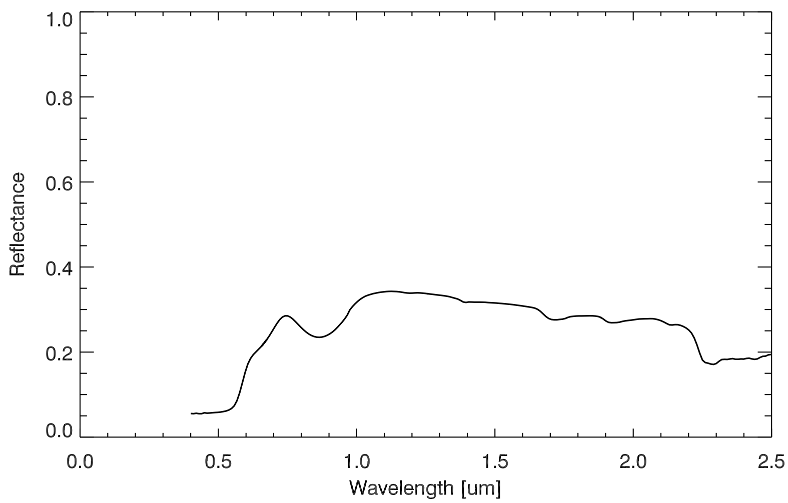


**Figure 3.** Reflectance spectrum of a red painted asphalt surface used in the target space generation process.

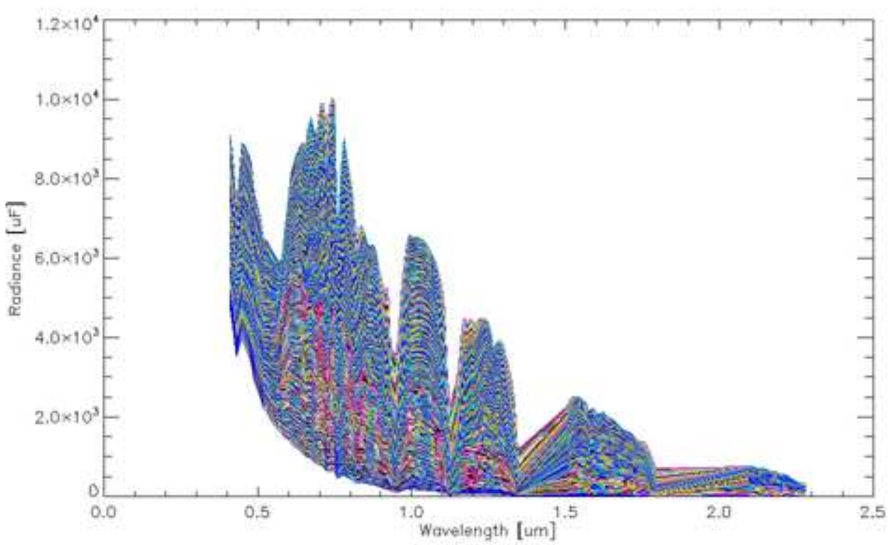


**Figure 4.** Sensor-reaching target space composed of 34,650 radiance vectors.

**Table 2.** Summary of varied parameters for the generation of a large target space.

| Parameter | Value |
|---|---|
| Visibility [km] | 5, 8, 10, 12, 15, 18, 20, 25, 30, 40, 50 |
| GND Topography [km] | 0.1, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2 |
| WV Scale Factor | 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2 |
| Change in Target Illumination‡ | -100%, -80%, -60%, -40%, -20%, -10%, 0%, 10%, 15% |
| Shape Factor (downwelled) | 0.2, 0.4, 0.6, 0.8, 1.0 |

‡Negative values indicate reduction in direct solar illumination term.

histogram of all 5,266,800 ($= 34,650 \cdot 152$ bands) residuals is shown in Figure 5(a). These residuals can be interpreted as the errors of approximation when using the statistical model.

We can see that most of the errors are under $\pm 20$ micro-flicks with a majority of them lower than $\pm 10$ micro-flicks, which manifests itself as an error of approximately one percent in the overall target space. This result is very encouraging and tells us that we can model a complicated, widely varying, target space with a reasonable degree of accuracy. Ultimately, however, the idea is that one would use the statistically modeled target space (with small residuals) in a detection scheme. These "additional" errors may or may not have an adverse impact on detection performance. Certainly this would depend on the magnitude of other errors or uncertainties in the process. This impact on detection performance will be the subject of future studies. Lastly, we can examine the residuals in terms of percent, since the magnitude of radiance varies significantly. Therefore, we can generate a histogram of *relative errors* by dividing residuals by the observed values. This can be seen in Figure 5(b). Here we see that the relative errors are on the order of a percent.

The above approximation errors refer to a situation, where the *whole large target space* is used to fit the model, and then the model is used to reconstruct the original data. In practice, this type of modeling can be used on much smaller target spaces. For example, one can generate a physics-based target space consisting of 123 spectra, for example. Then a model in the form of Eq. (6) would be fitted to this small training data set. The model coefficients could then be stored and used for the calculation of a radiance spectrum for *any* combination of the five predictors (parameters). In order to evaluate this methodology, we selected several small subsets (training sets) from the large target space introduced in Section 3.1, estimated models to those training subsets, and then calculated the approximation errors for all spectra in the large target space. This methodology is illustrated in Figure 6. The choice of these training subsets (we also call them "designs") was based on the principles of the statistical experimental design and the response surface methodology.

Before we can explain the specific designs that were used, we need to introduce the concept of the cube $2^3$ design. Let us assume that we deal with three parameters, and each parameter has two levels. The cube $2^3$ design consists of 8 runs created by all possible combinations of the values of the three parameters. These 8 runs can be represented geometrically as vertices of a cube in a 3-dimensional space of the three parameters.

We now explain a 41-run design that was used for the three parameters (Visibility (V), GND Topography (T), and WV Scale Factor (W)). This design requires seven levels of each of the three parameters. It consists of five cubes defined before. The first cube uses the first and the seventh levels of all three parameters as the two levels for the the cube $2^3$ design. In a similar fashion the second cube uses the second and the sixth levels of all three parameters. The third cube uses the third and the fifth levels of Visibility and the first and the seventh levels of the remaining two parameters. The forth cube uses the third and the fifth levels of GND Topography and the first and the seventh levels of the remaining two parameters. The fifth cube uses the third and the fifth levels of WV Scale Factor and the first and the seventh levels of the remaining two parameters. The five cubes defined above give a total of 40 combinations (runs). We also add a central point, with all three parameters at the middle (forth) level, for a total of 41 combinations.

We now explain our approach to the remaining two parameters, that is, Change in Target Illumination (IL) and Shape Factor (SF). Since radiance is a linear combination of the IL and SF variables (see Eq. (6)), we need only three combinations in order to estimate the three coefficients (the intercept and the two slopes). The combinations were created as the two combinations of $IL = 0\%$ with $SF = 0.4$ and $SF = 1$ and one combination of $IL = -60\%$ with $SF = 1$.

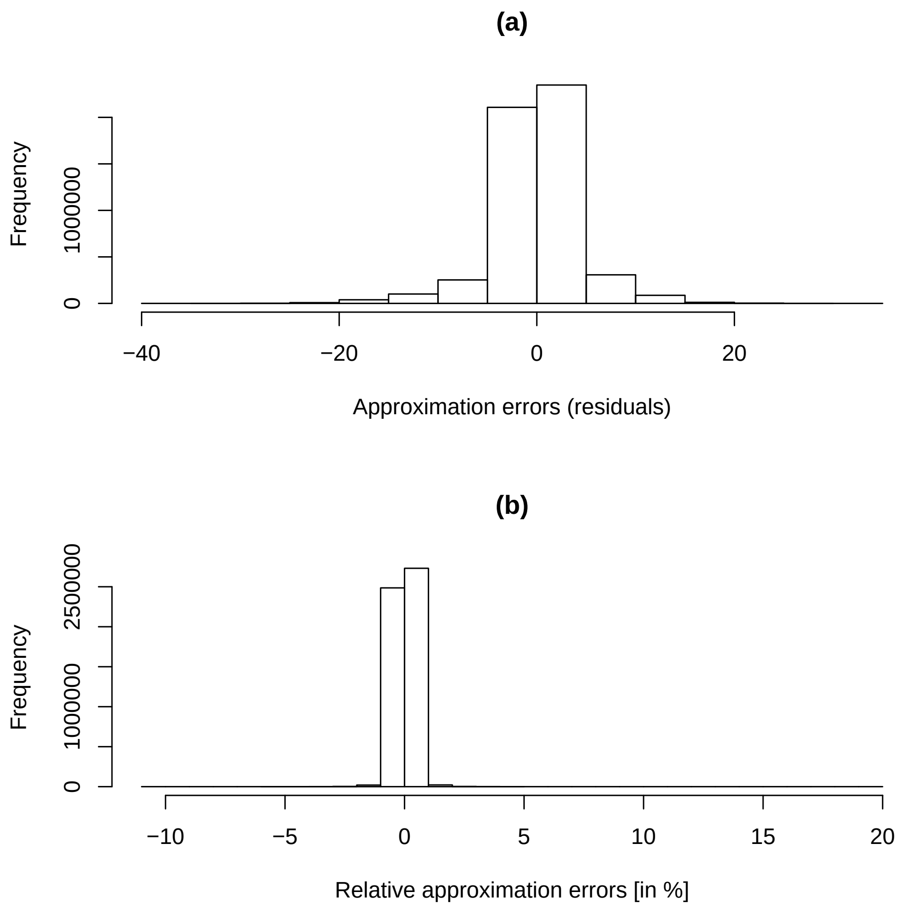


**Figure 5.** (a) Approximation (residuals), in radiance units [micro-flicks] and (b) relative approximation errors [in %] for the model of Eq. (6) fitted to the large target space.

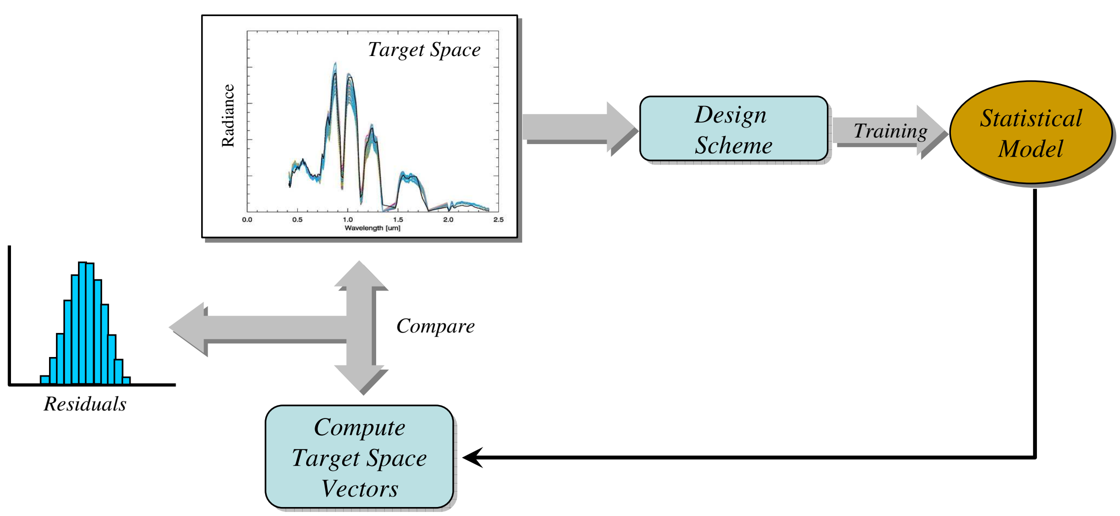


**Figure 6.** Statistical target space model evaluation methodology.

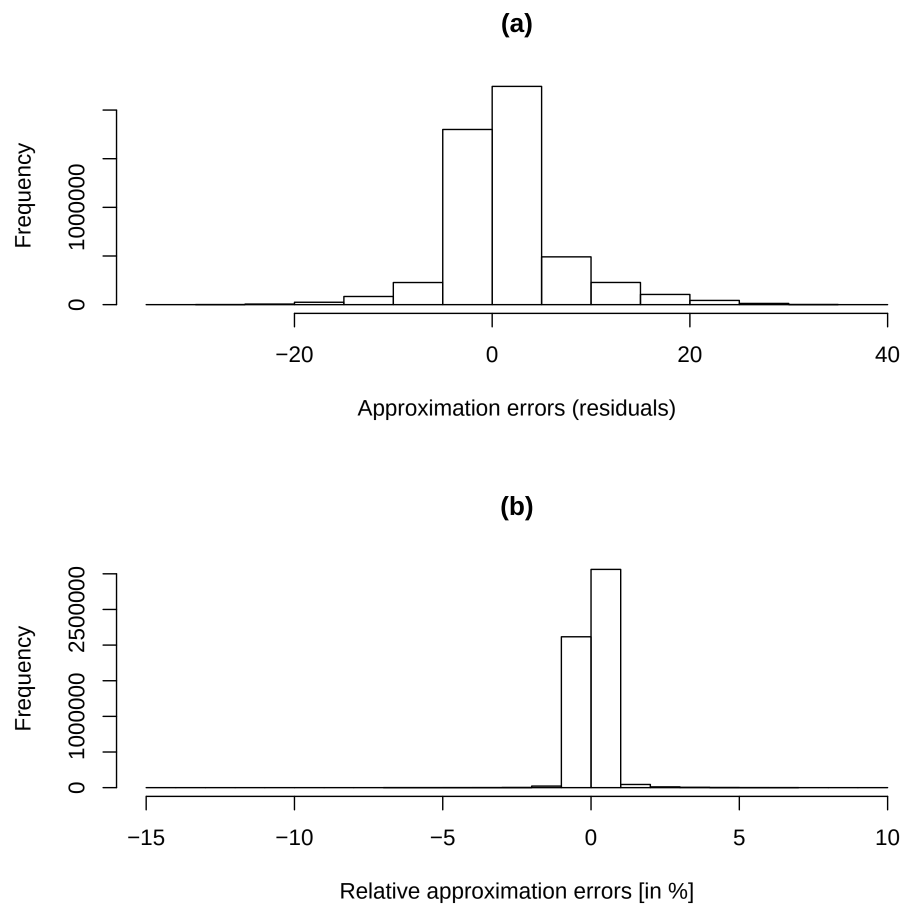


**Figure 7.** (a) Approximation errors (residuals), in radiance units [micro-flicks] and (b) relative approximation errors [in %] for the model of Eq. (6) fitted to the "small target space" consisting of 123 spectral curves.

The final design for all five parameters consists of 123 runs created by combining the 41 runs for the three parameters (V, T, and W) and the the three runs for IL and SF. The training set of the resulting 123 spectra will be referred to as the "small target space". The evaluation of this small target space for the purpose of fitting the statistical model (as described in Figure 6) is summarized in Figure 7. We can see that the residuals are not much larger than those shown in Figure 5 representing the case of training the model on the large target space.

We have also tried several other designs (training sets)—some larger ones consisting of more than 123 runs and some smaller ones. It turns out that using more runs does not reduce residuals in any significant way. On the other hand, when using less than 123 runs, the magnitude of residuals starts to increase significantly. For example, a training set consisting of 99 runs was constructed in the way described above, except that the second cube was not used. The resulting residuals are shown in Figure 8. One can see that the absolute values of residuals are larger than 20 more often than those in Figure 7. This smaller training set of 99 runs might still be acceptable, if one were concerned about the computational speed related to running the additional runs of MODTRAN. However, in most cases, the training set of 123 runs seems to be optimal in the sense of a balance between sufficiently small residuals and still relatively small number of runs, which is computationally efficient.

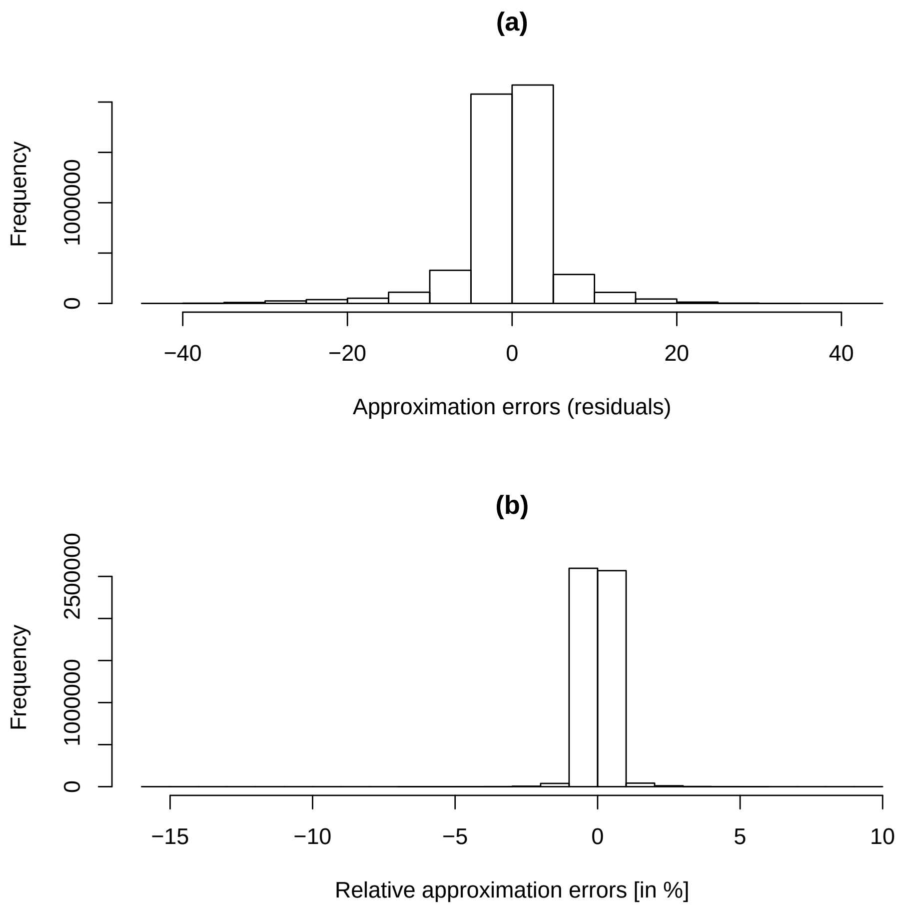


**Figure 8.** (a) Approximation errors (residuals), in radiance units [micro-flicks] and (b) relative approximation errors [in %] for the model of Eq. (6) fitted to the a target space consisting of 99 spectral curves.

## 4. CONCLUSIONS

This paper explored the use of a statistical model to represent target spaces for use in target detection applications. A large target space (34,650 vectors), encompassing a very wide range of conditions, was created based on a typical high altitude collection campaign utilizing MODTRAN and a physics-based model. This target space was then used as training to a third order polynomial. Model coefficients were derived in a least squares sense. The model was then used to "re-create" the original target space where comparisons were made based on residuals between the two spaces. The bulk of the residuals were less than 10 micro-flicks, which corresponds to an error on the order of one percent.

A sparse set of samples (123 vectors) from the large 34,650 vector space, were also used as training. Again, the model was used to re-create the original large target space. The residuals here were on the order of those seen when using all 34,650 vectors for training. However, it was found that using a set of training samples less than 123 vectors increased the overall residuals associated with comparing the statistically modeled target space to the actual MODTRAN derived target space. Additionally, it was found that there was no significant added value to training with more than 123 vectors. This was determined through analysis of residuals, which did not vary much when 123 vectors versus 34,650 vectors were used for training.

These finding tell us that it is possible to represent a very large (34,650) vector target space with a limited number of MODTRAN runs (123, for example) with a fair amount of accuracy (one percent, for the given parameters set forth in this study).

Future analysis will study this approach to statistical target space representation by varying the target signature used in the physics model. That is, only one target reflectance was used in this study. Future studies will vary the spectral nature of the target reflectance to see if there is any adverse impacts on target space representation. The same can be said for the type of atmospheric model. For this study, only one type of atmospheric model was investigated (*i.e.,* rural extinction). Future studies may look at the impact of using urban and maritime extinction models.

Finally, since this type of model is to be used in a target detection scheme, the impact of using a statistical model in conjunction with a detector needs to be addressed. Since the statistically represented target space is an approximation to a set of sensor-reaching target vectors (which are also estimates), one needs to consider the propagation of radiance vector errors and their impact on detection performance.

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