A Theoretical Model to Predict Pool Boiling CHF Incorporating Effects of Contact Angle and Orientation

A theoretical model is developed to describe the hydrodynamic behavior of the vapor-liquid interface of a bubble at the heater surface leading to the initiation of critical heat flux (CHF) condition. The momentum flux resulting from evaporation at the bubble base is identified to be an important parameter. A model based on theoretical considerations is developed for upward-facing surfaces with orientations of 0 deg (horizontal) to 90 deg (vertical). It includes the surface-liquid interaction effects through the dynamic receding contact angle. The CHF in pool boiling for water, refrigerants and cryogenic liquids is correctly predicted by the model, and the parametric trends of CHF with dynamic receding contact angle and subcooling are also well represented. [DOI: 10.1115/1.1409265]

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Introduction

Critical heat flux (CHF) represents the limit of the safe operating condition of a system or a component employing boiling heat transfer under constant heat flux boundary condition. Loss of liquid contact with the heating surface at CHF leads to a significant reduction in the heat dissipation rate. In pool boiling application, such as in electronics equipment cooling, the drastic reduction in boiling heat flux after CHF may lead to devastating results. Therefore, a fundamental understanding of the mechanisms responsible for the initiation of this condition continues to be of great importance. The current work models this phenomenon by including the non-hydrodynamic aspect of surface-liquid interaction through the dynamic receding contact angle. The model is tested with a number of data sets available in literature.

Previous Work

Historical Perspective. As early as 1888, Lang [1] recognized through his experiments on high pressure water data that as the wall temperature increased beyond a certain point, the heat flux decreased dramatically. However, it was Nukiayama [2] who realized that the maximum boiling rate might occur at relatively modest temperature differences. An excellent summary of historical developments in this area was presented by Drew and Mueller [3].

Many researchers have considered various aspects of CHF. Some of the important milestones include Lang [1], Nukiayama [2], Bonilla and Perry [4], Cichelli and Bonilla [5], Kutateladze [6,7] Rohsenow and Griffith [8], Zuber [9], Costello and Frea [10], Gaertner [11,12], Katto and Yokoya [13], Lienhard and Dhir [14], Haramura and Katto [15], Liaw and Dhir [16], Ramilison and Lienhard [17], Elkassabgi and Lienhard [18], and Dhir and Liaw [19]. A brief overview of some of these and a few other important investigations is given in the following section. (The list provided here is not intended to be comprehensive, and the author is aware that he may have made some inadvertent omissions.)

Previous Models and Correlations. Although a number of early investigators reported the critical heat flux phenomenon, Bonilla and Perry [4] proposed the concept of flooding in obtaining a correlation from the experimental data. Using the column flooding theory, they derived four dimensionless groups. The basic concept seemed to work, but the approach was not pursued further. Cichelli and Bonilla [5] correlated their experimental data for organic liquids with $q_C^G p_C$ plotted against reduced pressure, $p/p_C$. Although their organic liquid data matched reasonably well, the predictions were considerably lower than the experimental data for water by Addoms [20].

Kutateladze [6,7] proposed that the meaning of bubble generation and departure were lost near the critical heat flux condition, and it was essentially a hydrodynamic phenomenon with the destruction of stability of two-phase flow existing close to the heating surface. Critical condition is reached when the velocity in the vapor phase reaches a critical value. Following a dimensional analysis, he proposed the following correlation.

$$q_C^G h_b p_{ig}^{0.5} \left[ \sigma g (p_i - p_g) \right]^{1/4} = K$$  \hspace{1cm} (1)

The value of $K$ was found to be 0.16 from the experimental data. Borishanskii [21] modeled the problem by considering the two-phase boundary in which liquid stream flowing coaxially with gas experiences instability. His work led to the following equation for $K$ in Kutateladze’s equation, Eq. (1).

$$K = 0.13 + 4 \left( \frac{p_i}{\mu g (p_i - p_g)} \right)^{0.4}$$  \hspace{1cm} (2)

Although viscosity appears in Eq. (2), its overall effect is quite small on CHF.

Rohsenow and Griffith [8] postulated that increased packing of the heating surface with bubbles at higher heat fluxes inhibited the flow of liquid to the heating surface. Plotting $q_C^G / (h_b p_g)$ versus $(p_i - p_g)/p_g$, they proposed the following correlation for CHF.

$$\frac{q_C^G}{h_b p_g} = C \left( \frac{g}{g_s} \right)^{1/2} \left[ \frac{p_i - p_g}{p_g} \right]^{0.6}$$  \hspace{1cm} (3)

The equation is dimensional with $C=0.012$ m/s, $g$ is the local gravitational acceleration, and $g_s$ corresponds to the standard $g$ value.

Zuber [9] further formalized the concept by considering the formation of vapor jets above nucleating bubbles and flow of liq-
uid between the jets toward the heating surface. As the heat flux increases, the vapor velocity increases causing vapor and liquid to compete for the same space; a condition for instability is thus created.

Zuber [9] also postulated that vapor patches form and collapse on the heater surface as CHF is approached. According to Zuber, “In collapsing, . . . [as] the vapor-liquid interface of a patch approaches the heated surface, large rates of evaporation occur and the interface is pushed violently back.” Zuber considered the dynamic effects of vapor jets to be important and proposed that the Taylor and Helmholz instabilities are responsible for the CHF condition. Using the stability criterion of a vapor sheet, they obtained an equation similar to that of Kutateladze [7], but the value of constant K ranged from 0.157 to 0.138. Simplifying the analysis further, Zuber proposed a value of $K = 0.131$.

Chang [22] considered the forces acting on the bubble and postulated that the CHF condition was attained when the Weber number (incorporating the velocity of liquid relative to the rising bubble) reached a critical value. The vapor continues to leave the heater surface until the critical velocity is reached, at which time some of the vapor is carried back to the heater surface. The analysis resulted in the following equation for vertical surfaces.

$$q^*_C = 0.998 \rho_1^{1/2} h_{lg} \sigma g (\rho_l - \rho_g)^{1/4}$$

For horizontal surfaces, he introduced a ratio $q^*_C, y / q^*_C = 0.75$, thereby changing the leading constant to 0.13 in Eq. (4) for horizontal surfaces.

Moissis and Berenson [23] developed a model based on the interaction of the continuous vapor columns with each other. The maximum heat flux is then determined by introducing Taylor-Helmholz instability for the counterflow of vapor flow in columns and liquid flow between them.

Haramura and Katto [15] refined an earlier model proposed by Katto and Yokoya [13] in which the heat transfer was related to the formation and evaporation of a macrolayer under a bubble. The presence of such macrolayer was reported by Kirby and Westwater [24] and Yu and Mesler [25]. Small vapor jets are formed in this macrolayer, and Kelvin-Rayleigh instability results in lateral coalescence of vapor stems. Using this approach, Haramura and Katto extended Zuber’s [9] analysis and arrived at the same equation, Eq. (1), derived by Kutateladze.

Lienhard and Dhir [14] critically evaluated the assumptions made in the Zuber’s [9] theory and modified the vapor velocity condition at which instability would set in. Consequently, they showed that Zuber’s equation would underpredict the CHF by 14 percent. Lienhard and Dhir modified Zuber’s theory to include the effect of size and geometry. They also noted the increase in the number of jets as CHF was approached. This aspect was further studied by Dhir and Liaw [19] for horizontal flat surfaces, and a detailed formulation was presented.

Van Outwerkerk [26] studied the stability of pool and film boiling mechanisms using high speed photographs of n-Heptane boiling on a glass surface coated with transparent heater film. Dry patches were observed at heat fluxes 20 percent below CHF. Although the fraction or sizes of dry patches did not change, there was a sudden transition to CHF from one of the patches. Simultaneous existence of nucleate and film boiling on two adjacent sections was observed on a thin wire which was partially removed and then immersed back into the liquid. The reason behind the transition mechanism was discussed.

Effect of contact angle was studied by very few investigators. Kirishenko and Cherniakov [27] developed the following correlation with contact angle as a parameter.

$$q^*_C = 0.171h_{lg} \sqrt{\gamma_g [\sigma g (\rho_l - \rho_g)]^{1/2} \left(1 + 0.324 \cdot 10^{-3} \beta^2 \right)^{1/4}}$$

Dieselhorst et al. [28] noted that this equation yielded much higher values of CHF for larger contact angles. This correlation was found to be quite inaccurate in representing the effects of contact angle for water. The correlation however exhibited the correct trend of decreasing heat flux with increasing contact angles.

Unal et al. [29] considered the existence of dry patches to lead the way toward CHF condition. The temperature reached at the center of the dry patch was considered to be an important parameter responsible for the rewetting behavior of the surface. Sadasivvan et al. [30] presented a good overview highlighting the need for new experiments to aid in the understanding of the CHF phenomenon.

**Parametric Effects.** Although CHF was identified as a hydrodynamic phenomenon, a number of researchers recognized the important role played by the surface characteristics. Tachibana et al. [31] studied the effect of heater thermal properties and found that CHF increased as (a) the thermal conductivity increased, and (b) as the heat capacity per unit surface area increased. However, thermal diffusivity, which is the ratio of thermal conductivity to the thermal mass, did not correlate well with CHF. For thin stainless steel heaters below 0.8 mm in thickness, the CHF was found to be lower than that for thick heaters (>0.8 mm thick). They also noted that the presence of aluminum oxide coating increased CHF, attributed to its affinity to water (wettability). More recently, Golobić and Bergles [32] presented a detailed literature survey on the effect of thermal properties on CHF for ribbon heaters. They developed a criterion for the asymptotic value of heater thickness for ribbon heaters beyond which the thickness effect on CHF was quite small. They recommended similar work for other geometries.

Contact angle is a key parameter affecting the bubble-wall interaction. Its effect on CHF was recognized by many investigators. An interesting set of experiments was performed by Costello and Frea [10] with submerged cylinders heated only on the top half. The heated surface was coated with mineral deposits resulting from boiling in tap water. The heat flux and duration of boiling was carefully regulated to achieve different levels of deposit thickness. The treated cylinders were then used with distilled water and the burnout heat fluxes were measured. They found that the presence of deposits on the heater surface resulted in over 50 percent increase in CHF in some cases over the freshly prepared clean cylinders. This non-hydrodynamic effect was not predicted by any of the existing correlations. Costello and Frea noted that, for cylindrical heaters, this effect was not as pronounced due to inadequate supply of liquid to the larger heated surface.

The effect of orientation is studied by many investigators. The horizontal and vertical geometries are of most practical interest. Howard and Mudawar [33] studied the effect of heater orientation in great detail and present three regions: upward-facing (0–60 deg), near vertical (60 to 165 deg), and downward facing (>165 deg). One of the factors that affect the CHF is the size of the heater, especially for near vertical and downward facing surfaces. The surfaces studied by Howard and Mudawar included heaters of different shapes with dimensions varying from 3 mm to 300 mm. Gaertner [12] conducted some highly illustrative experiments confirming the results reported by Costello and Frea. Another coated the inside bottom of a 3-inch diameter container with a thin non-wetting (close to 180 deg contact angle) fluorocarbon film, similar to Teflon. The container was then filled with distilled water and heated at the bottom. As the heater temperature increased, boiling was initiated, but the bubbles did not depart from the heater surface. Instead, they coalesced and covered the entire bottom surface with a film of vapor resulting in extremely low values of CHF. Another experiment demonstrated that the presence of grease in the system caused a gradual reduction in CHF as the heater surface was progressively coated with a non-wetting film of grease. Although the surface roughness itself was not directly responsible to any changes in CHF, if the contact angle changed as a result, the CHF was lowered. The experiments confirm that a low contact angle (highly wetting liquid) will result in...
a higher value of CHF, while a high contact angle, such as a non-wetting surface, will result in drastic reduction in CHF (also confirmed by Costello and Frey).

**Hydrodynamic and Non-Hydrodynamic Considerations in the Development of the Present Model.** The supply of liquid to and removal of vapor from a heated surface play a major role in reaching the CHF condition. If we consider an experiment in which a growing bubble is completely confined within side walls, the CHF will be reached immediately upon nucleation as the liquid is unable to reach the heater surface. This type of CHF is observed in microgravity environment with smooth heaters where a large stationary bubble envelops the heater surface.

On the other hand, the photographs by Elkassabgi and Lienhard [18] just prior to the CHF show that, for saturated liquids, bubbles coalesce to form large vapor masses a short distance away from the heater surface. In subcooled pool boiling, bubbles are discrete, presenting little resistance to liquid flow toward the heater surface near CHF. In addition, the dependence on surface effects expressed through the contact angle near CHF. In microgravity environment with smooth heaters where the liquid is unable to reach the heater surface. This type of CHF is reached immediately upon nucleation as the liquid-vapor interface act in the radial direction, the movement of the heater surface.

The theoretical models available in literature do not incorporate the effect of contact angle. Its effect is considered indirectly through the changes in bubble size and number of nucleation sites in some of these models. In the present work, a theoretical model is presented which directly incorporates the effect of dynamic receding contact angle on CHF. The model is tested with data sets available in literature for different fluids.

**Development of the Model.** Departing from the earlier models on hydrodynamic instability, Chang [22] considered the force balance on a bubble in deriving the condition leading to CHF. The model of Haramura and Katto [15] also focuses on the bubble behavior by considering the presence of a thin macrolayer underneath a bubble. The strong effect of contact angle on CHF suggests that the interface conditions at the bubble base play an important role.

Figure 1(a) shows a bubble attached to a horizontal heating surface. The excess pressure inside the bubble under a quasi-static equilibrium state is able to sustain the necessary curvature of the interface at the departure condition. At low heat fluxes, the bubble would depart at certain size as governed by the forces due to inertia, pressure and buoyancy. Now consider the left half of the bubble and the forces acting on it in the direction parallel to the heating surface. The surface tension forces $F_{S,1}$ and $F_{S,2}$ act at the bubble base and the top surface of the bubble respectively.

The hydraulic pressure gradient (buoyancy) due to gravity is plotted as excess pressure due to hydrostatic head with the reference plane being at the top of the bubbles surfaces. It results in a triangular (excess) pressure distribution with $F_G$ as the resultant force. The evaporation occurring on sides of the bubble causes the vapor to leave the interface at a higher velocity due to the difference in specific volumes. At high evaporation rates near CHF, the force due to change in momentum, $F_M$, due to evaporation becomes larger than the sum of the gravitational and surface tension forces holding the bubble in place. This causes the liquid-vapor interface (side walls of the bubble) to move rapidly along the heater surface leading to the CHF condition. A detailed force balance is performed below to obtain the heat flux value at this condition (CHF). The present model is valid for upward-facing surfaces between horizontal and vertical orientations. For higher angles of orientations, the flow hydrodynamics and heater size effects become very important and need to be included in the model development.

The diameter $D_b$ corresponds to the departure condition at which the force balance is performed. Although the forces at the bubble interface act in the radial direction, the movement of the interface can be analyzed in a two-dimensional plane shown in Fig. 1(a). The surface tension forces act at the base and the top portion of the bubble. For a unit length in the direction normal to the plane, the forces $F_{S,1}$ and $F_{S,2}$ are given by

$$F_{S,1} = \sigma \cos \beta$$

and

$$F_{S,2} = \sigma,$$

where $\sigma$ is surface tension, N/m, and $\beta$ is the dynamic receding contact angle of the liquid-vapor interface with the solid heater surface.

The heat flux at the interface results in a force due to the change in momentum as vapor leaves the interface. For momentum analysis, the interface is represented as a plane with bubble height $H_b$ and a unit width normal to plane of Fig. 1(a). Expressing the heat flux due to evaporation per unit area of the interface as $q''_I$, the resulting force due to the momentum change is given by the product of the evaporation mass flow rate and the vapor velocity relative to the interface.

$$F_{M} = \frac{q''_I H_b}{h_{fg}} \frac{1}{h_{fg}} \rho_g = \left( \frac{q''_I}{h_{fg}} \right)^2 \frac{1}{\rho_g} H_b$$

The bubble height $H_b$ is related to the bubble diameter $D_b$ through the contact angle $\beta$ as follows:

$$H_b = \frac{D_b}{2} (1 + \cos \beta).$$

The heat flux at the interface $q''_I$ is an average value during the growth of the bubble since its inception. It is derived from the heat flux on the heater surface $q'''_I$ by approximating the interface as a cylinder with diameter $D_{avg}/2$ and height $H_{avg} = (D_{avg}/2)^2(1/2)$. The diameter $D_b$ corresponds to the departure condition at which the force balance is performed. Although the forces at the bubble interface act in the radial direction, the movement of the interface can be analyzed in a two-dimensional plane shown in Fig. 1(a). The surface tension forces act at the base and the top portion of the bubble. For a unit length in the direction normal to the plane, the forces $F_{S,1}$ and $F_{S,2}$ are given by

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The diameter $D_{avg}$ corresponds to the average diameter as the bubble grows. As a first approximation, it is taken as $D_{avg} = D_b/2$. Heat is removed by the bubble from an influence area considered to be a circle with diameter $2D_b$ as shown in Fig. 1(b), following the assumption made by Han and Griffith [34]. The heat flux on the heater surface is obtained by comparing the heat transfer rates over the influence area and over the interface area of the average bubble of diameter $D_{avg}$.

$$q'' = q''_I(D_{avg}/2)(1 + \cos \beta) \cdot (\pi D_{avg}/2) = (1 + \cos \beta) \frac{q''_I}{16}$$

(10)

The critical wavelength for initiating this instability, length of the Taylor instability of a vapor film over the heater surface inclined at an angle $\phi$ to the horizontal ($\phi = 0$ deg for a horizontal upward facing surface, $\phi = 90$ deg for a vertical surface), the component of the force parallel to the heater surface due to the hydrostatic head on a surface of height $H_b$, and unit width is given by

$$F_G = \frac{1}{2} g(\rho_l - \rho_g) H_b \cdot H_b \cdot 1 \cdot \cos \phi.$$ 

(11)

The critical heat flux or CHF occurs when the force due to the momentum change ($F_M$) pulling the bubble interface into the liquid along the heated surface exceeds the sum of the forces holding the bubble, $F_{S,1}$, $F_{S,2}$, and $F_G$. The bubble then expands along the heater surface and blankets it. At the inception of the CHF condition, the force balance yields

$$F_M = F_{S,1} + F_{S,2} + F_G.$$ 

(12)

Substituting Eqs. (6), (7), (8), and (11) into Eq. (12) and introducing Eq. (10), the heat flux at the heater surface, which corresponds to the CHF condition, $q''_c = q''_c^*$, is obtained.

$$q''_c = h_{js} p_s^{1/2} \left[ 1 + \cos \beta \frac{\sigma(1 + \cos \beta)}{H_b} + (\rho_l - \rho_g) g \frac{H_b}{2} \cos \phi \right]^{1/2}.$$ 

(13)

Substituting for $H_b$ from Eq. (9), Eq. (13) becomes

$$q''_c = h_{js} p_s^{1/2} \left[ \frac{\sigma(1 + \cos \beta)}{D_b} + (\rho_l - \rho_g) g \frac{D_b}{4} (1 + \cos \beta) \cos \phi \right]^{1/2}.$$ 

(14)

The diameter $D_b$ is obtained by assuming it to be half the wavelength of the Taylor instability of a vapor film over the heater surface. The critical wavelength for initiating this instability, derived earlier by Kelvin (referenced in Lamb [35]) and later used by Zuber [9] in the development of his hydrodynamic theory, is given by

$$\lambda_T = C_2 \frac{1}{2} \sigma \left[ \frac{g(\rho_l - \rho_g)}{1 + \cos \beta} \right]^{1/2}.$$ 

(15)

The value of $C_2$ ranges from 1 to 3. Using a value of 1 and substituting $D_b = \lambda_T/2$, with $\lambda_T$ from Eq. (15), (14) takes the following form:

$$q''_c = h_{js} p_s^{1/2} \left[ \frac{\sigma(1 + \cos \beta)}{D_b} + (\rho_l - \rho_g) g \frac{D_b}{4} (1 + \cos \beta) \cos \phi \right]^{1/2} \times \left[ \frac{\sigma g(\rho_l - \rho_g)}{1 + \cos \beta} \right]^{1/4}.$$ 

(15)

Equation (16) predicts the CHF for saturated pool boiling of pure liquids. It includes the hydrodynamic as well as non-hydrodynamic (heater surface interaction) effects and the orientation of the heater surface. Since the liquid would start to recede at the onset of CHF, the dynamic receding contact angle is used in Eq. (16). A high speed photographic study is being conducted in the author’s lab to determine the contact angle at CHF from the exact shape of the interface.

**Effect of Subcooling**

The effect of subcooling has been investigated by Kutateladze and Schneiderman [36] who observed a linear relationship between the degree of subcooling and CHF for water, iso-octane, and ethyl alcohol over a graphite rod heater at various pressures. They proposed a model that considered the enhancement due to recirculation of subcooled liquid over the heater surface. Ivey and Morris [37] modified the Kutateladze and Schneiderman model by assuming that a portion of subcooled liquid first heats up to saturation point before evaporation occurs. Ivey and Morris conducted experiments with water boiling on horizontal wires, 1.22–2.67 mm diameter, and found a linear relationship with degree of subcooling. Zuber on the other hand considered transient heat conduction to the subcooled liquid to be responsible for the increase in CHF. Duke and Schrock [38] conducted experiments with a horizontal heater surface, and found that the pool boiling curve shifted to a lower wall superheat region, and the wall superheat at CHF also decreased as liquid subcooling increased. A linear relationship of CHF with subcooling was obtained by Jakob and Fritz [39] as reported by Rohsenov [40].

Elkassabgi and Lienhard [18] conducted experiments with subcooled pool boiling with R-113, acetone, methanol, and isopropanol and identified three regions depending on the level of subcooling. In region I at low subcooling levels, CHF increased linearly, while at very large subcooling in region III, CHF was insensitive to changes in subcooling. Region II represented transition between I and III.

It is proposed that the heat transfer mechanism leading to CHF involves heat transfer to the subcooled liquid. The heat transfer rate at the CHF condition is therefore related directly to the wall to bulk temperature difference. An increase in subcooling of the bulk liquid increases the transient conduction to the liquid. This approach is expected to represent Region I as defined by Elkassabgi and Lienhard. The CHF under subcooled condition is then given by

$$q''_{c, sub} = q''_{c, sat} \left( 1 + \frac{\Delta T_{sub}}{\Delta T_{sat sub}} \right).$$

(17)

However, as seen from the results of Duke and Schrock [38], the wall superheat at the CHF condition decreases with increasing subcooling. In Eq. (17) therefore, the actual wall superheat under subcooled conditions should be used. At this time there is no accurate model available for predicting the wall superheat (or the heat transfer coefficient under subcooled condition) at the CHF; the actual experimental data will therefore be used in verifying this model.

**Results**

The CHF model as given by Eq. (16) for saturated pool boiling, and Eqs. (16) and (17) in conjunction with the wall superheat data at CHF for subcooled pool boiling, is compared with the experimental data available in literature. Kutateladze correlation (Eq. 1) was chosen in this comparison since other correlations, such as Zuber’s, differ only in the value of the leading constant $K$ (which is 0.16 in Kutateladze correlation, and 0.131 in Zuber’s correlation). Both these earlier correlations do not include the contact angle as a parameter. Correlations such as Eq. (5) yield extremely high values of CHF especially for water.

Table 1 shows the details of the data sets and the results of the comparison with the Kutateladze correlation and the present model. It can be seen that the model predictions are quite good and are consistently better than the Kutateladze predictions.

The current model requires the knowledge of the dynamic receding contact angle for a given liquid-solid system. A majority of
data sets, with the exception of those, which specifically investigated the contact angle effect, does not report the contact angle. In view of this, following assumptions are made regarding the dynamic receding contact angle:

- water/copper system—$\theta = 45$ deg
- water/chromium coated surface—$\theta = 65$ deg
- cryogenic liquids/copper—$\theta = 20$ deg (no information is available on contact angles for cryogenic liquids)
- R-113/copper—$\theta = 5$ deg

An experimental work reporting the dynamic receding contact angles for water droplets impinging on hot surfaces is presented by Kandlikar and Steinke \cite{41}. They observed that for a smooth copper surface near saturation temperature of water, the dynamic receding contact angle is considerably lower than the dynamic advancing contact angle.

For this combination, they measured the dynamic receding contact angle to be between 45–80 deg for a copper surface depending on the surface roughness and temperature. In the CHF experiments, it is therefore recommended that the dynamic receding contact angles be measured using the impinging droplet technique preferably in a vapor atmosphere.

A small variation of 10 deg in estimating the dynamic receding contact angle below 60 deg results in less than 5–7 percent difference in the CHF predictions for all liquids investigated. However, changes in dynamic receding contact angles beyond 60 degrees have a significant effect of the CHF.

One of the major motivations behind the current work is to include the effect of dynamic receding contact angle, which is known to influence the CHF. Liaw and Dhir \cite{16} specifically investigated this effect by changing the equilibrium contact angle with different surface finish on the same heater. Figure 2 shows the results of the comparison. Although the differences at the two extremes are somewhat higher, it should be recognized that as the contact angle increases, the CHF value decreases dramatically as demonstrated by Gaertner \cite{11}. Present model correctly depicts this trend. Kutateladze model does not include the contact angle as a parameter and is seen to significantly overpredict the results at higher contact angles. The discrepancy between the present model and the data at higher values of contact angles is suspected because of the use of equilibrium contact angles as reported by Liaw and Dhir. The dynamic receding contact angles are always lower than the equilibrium contact angles. Using somewhat lower contact angle values further improves the agreement with the present model.

Similar results are obtained from the comparison with the R-113 data by Ramilison and Lienhard \cite{17} as shown in Fig. 3. However only a small range of contact angles was investigated. The results of Kutateladze model are close to the data in this case.

Deev et al. \cite{42} and Bewilogua et al. \cite{43} conducted experiments with cryogenic liquids—nitrogen, hydrogen and helium. Deev et al. conducted experiments with helium on horizontal and vertical plates. Figures 4 and 5 show the results of comparison.
with the present model. The agreements between the data and both the present model and Kutateladze model are good for the horizontal plate, while only the present model is able to represent the vertical plate data well. Similar observations can be made with the nitrogen, hydrogen, and helium data of Bewilogua et al. from Figs. 6–9. Their vertical plate data is somewhat scattered between the Kutateladze and the present model predictions.

Figure 10 shows the results of comparison with R-113 data by Abuaf and Staub [44]. The agreement in the mid and high-pressure ranges is good, but significant differences are noted at low pressures with both the present model and the Kutateladze correlation. Further investigation is warranted before the cause for this behavior can be explained.

Figure 11 shows the data obtained by Lienhard and Dhir [14].
For this case, the Kutateladze correlation considerably overpredicts the results. Changing the leading constant from 0.16 to 0.085 makes it overlap with the present model predictions, which are excellent. Similar observations can be made from Fig. 12 comparing the data by Bonilla and Perry. In this case, the Kutateladze correlation again overpredicts the results. Changing the leading constant to 0.11 improves the agreement in the high pressure region, but it still overpredicts the CHF in the low pressure region. The present model, on the other hand, is in excellent agreement with the data except for the lowest pressure data point.

One of the reasons why the present correlation and the Kutateladze correlation both yield almost the same results for cryogens is because the predictions from the present model for a contact angle of 20 degrees for cryogens are almost the same as those from Kutateladze correlation. The product of the terms in the two parenthesis on the right hand side of Eq. (16) represents the constant $K$ in Kutateladze correlation ($K=0.16$ in Kutateladze correlation). The variation of the corresponding term in the present model with the dynamic receding contact angle is shown in Table 2. It can be seen that for the assumed dynamic receding contact angle value of 20 deg for cryogens, this term is 0.178, which is very close to the Kutateladze constant $K=0.16$.

The effect of subcooling is shown in Fig. 13. Here, the data of Sakurai and Shiotu is used in the comparison. The data is for cylindrical heating surface. Although the present model is applicable only for flat surfaces, this comparison is shown to verify the trends predicted by the present model. The values for wall superheat at CHF are available from the data. The agreement with the present model for the horizontal plate is within ten percent, except for the data point at the highest subcooling of 40°C. For the vertical plate, the model underpredicts by about 18 percent, but the trend is well represented. The data exhibits a trend that is contrary to that observed by Elkassabgi and Elkassabgi and Lienhard. Their data tends to level off as subcooling increases, whereas Sakurai and Shiotu data seems to increase rapidly at higher de-
gree of subcooling. Although the agreement between the data and the model is quite good, extension of the present model to cylindrical geometry is recommended as future work.

A word on the accuracy of contact angle measurement is in order. The contact angles used in the correlation are obtained by previous investigators from the static measurements. The dynamic value of the receding contact angle under evaporating film condition is different than that obtained from static observations. Since the difference between the dynamic receding contact angle and the static receding contact angle is small as shown by Kandlikar and Steinke [41], the use of static receding contact angle is recommended when the dynamic values are not available.

Conclusions

The following conclusions are made from the present study:

1. After reviewing the existing models for pool boiling CHF, a need for including contact angle, surface orientation, and subcooling effects in modeling is identified.

2. A theoretical model to predict the CHF in saturated pool boiling is developed. The force parallel to the heater surface resulting from the evaporation at the liquid-vapor interface of a bubble near the heater surface is identified to be an important factor. As this force due to exiting vapor momentum exceeds the retarding forces due to gravity and surface tension, the vapor in the bubble spreads along the heater surface, blankets it, and initiates the CHF condition. The receding contact angle plays an important role in arriving at the CHF condition.

3. The model predicts the experimental data for water, refrigerants, and cryogenic liquids as well as seen from Table 1. The model is valid for orientations from 0 deg (horizontal surface) to 90 deg (vertical surface).

4. The model correctly predicts the effect of dynamic receding contact angle and subcooling on CHF for low values of subcooling.

5. Further work is suggested to develop a model to predict the wall superheat at CHF under highly subcooled pool boiling conditions.

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Nomenclature

\[ C = \text{constant in Eq. (3), m/s} \]
\[ D_b = \text{bubble diameter at departure, m} \]

\[ D_{avg} = \text{average diameter of a bubble during growth period, m} \]
\[ F_G = \text{force due to gravity, parallel to the heater surface, N} \]
\[ F_M = \text{force due to change in momentum due to evaporation, parallel to the heater surface, N} \]
\[ F_{S,1} = \text{force due to surface tension at the bubble base, parallel to the heater surface, N} \]
\[ F_{S,2} = \text{force due to surface tension at the top of the bubble, parallel to the heater surface, N} \]
\[ g = \text{gravitational acceleration, m/s}^2 \]
\[ g_s = \text{standard acceleration due to gravity, m/s}^2 \]
\[ h_f, \lambda = \text{latent heat of vaporization, J/kg} \]
\[ K = \text{constant in Kutateladze correlation, eq. (1)} \]
\[ p = \text{pressure, Pa} \]
\[ p_c = \text{critical pressure, Pa} \]
\[ q_f = \text{heat flux, W/m}^2 \]
\[ q_f^c = \text{critical heat flux, W/m}^2 \]
\[ q_i = \text{heat flux at the interface, W/m}^2 \]

Greek Symbols

\[ \beta = \text{dynamic receding contact angle, degrees} \]
\[ \phi = \text{heater surface angle with horizontal, degrees} \]
\[ \lambda = \text{critical wavelength for Taylor instability, m} \]
\[ \mu = \text{viscosity, Pa s} \]
\[ \rho = \text{density, kg/m}^3 \]
\[ \sigma = \text{surface tension, N/m} \]

Subscripts

\[ \text{avg} = \text{average} \]
\[ b = \text{departure bubble condition} \]
\[ C = \text{critical heat flux condition} \]
\[ g = \text{vapor} \]
\[ I = \text{interface} \]
\[ l = \text{liquid} \]
\[ I_q = \text{latent quantity} \]

References


