NUMERICAL STUDY OF AN EVAPORATING MENISCUS ON A MOVING HEATED SURFACE

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ABSTRACT

The present study is performed to numerically analyze an evaporating meniscus on a moving heated surface. This phenomenon is similar to the one observed at the base of a vapor bubble during nucleate boiling. The complete Navier-Stokes equations along with continuity and energy equations are solved. The liquid vapor interface is captured using the level set technique. A column of liquid is placed between two parallel plates with an inlet for water at the top to feed the meniscus. The location of water inlet at the top is kept fixed and the bottom wall is imparted with a velocity. Calculations are done in two-dimensions with a fixed distance between the plates. The main objective is to study the velocity and temperature fields inside the meniscus and calculate the wall heat transfer. The results show that the wall velocity creates a circulation near the meniscus base causing increased wall heat transfer as compared to a stationary meniscus. The local wall heat transfer is found to vary significantly along the meniscus base, the highest being near the advancing contact line.

INTRODUCTION

Nucleate pool boiling is typically characterized by the growth and departure of vapor bubbles on a heated surface. During its growth cycle, the bubble base expands initially, stays constant for some time and then contracts as the bubble departs. The high heat flux during nucleate boiling is often attributed to the wall heat transfer near the bubble base, which includes conduction to the liquid and its subsequent evaporation at the liquid-vapor interface. Investigators have extensively studied wall heat transfer near the bubble base by measuring the local transient surface temperatures and confirmed the existence of very high local wall heat flux.

In the present study, we are investigating an evaporating liquid meniscus on a moving heated plate. The meniscus profile is very similar to the contact region at the base of a growing vapor bubble as explained in Fig. 1. Figure 1(a) shows a typical nucleating bubble on a heated surface. Figure 1(b) shows an evaporating meniscus with no wall velocity, and Fig. 1(c) shows an evaporating meniscus with wall velocity. When the bubble base expands, it is similar to the receding contact line of the meniscus with wall velocity. When the bubble base remains constant during its growth cycle, the base resembles the meniscus with no wall velocity. Finally, when the bubble base shrinks, it is comparable to the advancing contact region of the meniscus with wall velocity.

Our calculations are done in two-dimensions since the predominant motion in the liquid would be in the plane that corresponds to the movement of the heated plate. We also neglected wall heat transfer to any evaporating thin film or microlayer at the end of the bulk meniscus. The main objective is to study the thermal and flow fields in the meniscus and calculate the wall heat transfer.

Calculations are carried out for three different cases, one with no wall velocity and the other two with wall velocities, with Reynolds number of 125 and 250 respectively.
Moore and Mesler [1] measured the surface temperatures during nucleate boiling with a special thermocouple having an extremely rapid response time. The fluid boiled was water at atmospheric pressure and saturated temperature. During nucleate boiling, the surface temperature was observed to fluctuate with time. Occasionally the surface temperature would drop rapidly and then return to its previous level. The temperature drop was as high as 200 F to 300 F in about 2 milliseconds. At low heat fluxes these dips occurred only infrequently; at higher heat fluxes the dips occurred more often. The authors argued that the theory of existence of a vaporizing microlayer at the base of the bubbles could only explain these observations.

Labunstov [2] proposed an analytical model explaining the mechanism of growth of vapor bubbles on a heating surface. He argued that the growth of bubbles developing on the heating surface must depend primarily on the heat flux supplied by the heating surface to the bubble surface close to the base of the bubble. His model was developed under these conditions:

a) there was a zone of intense evaporation around the base of the growing bubble

b) the heat spent on evaporation was brought directly to those elements of the bubble surface from the heating surface by heat conduction through the adjacent layers of liquid.

He also developed a vapor bubble growth expression and compared it with available experimental data. The model had limited success.

Mikic and Rohsenow [3] developed a correlation for pool boiling. The correlation assumed that the main
mechanism of heat transfer in nucleate boiling was transient heat conduction to, and subsequent replacement of, the superheated layer around nucleation sites associated with bubble departure. The transient conduction problem was modeled as conduction to a semi-infinite plate with a step change in temperature at the surface. The correlation was checked with available experimental data and the results were found to be satisfactory.

Koffman and Plesset [4] obtained microlayer thickness measurements in water, for small, short-lived bubbles characteristic of highly subcooled nucleate boiling. The detailed measurements were obtained using laser interferometry, combined with high-speed cinematography. The initial microlayer thickness for water was wedgelike with a thickness of 1.85 microns at a radius of 0.25 mm. The authors concluded that microlayer evaporation alone cannot account for the increased heat transfer rates observed in highly subcooled nucleate boiling, and microconvection must play at least an equal role.

Kandlikar and Kuan [5] developed a new meniscus configuration to visualize the advancing and receding fronts of a liquid vapor interface. It consists of a circular evaporating meniscus on a smooth heated copper surface. They obtained the meniscus shape and heat transfer rates as functions of heater temperature, liquid flow rate and surface velocity. For a stationary meniscus, they found the contact angle to be almost independent of the water flow rate and the heat flux in the range of parameters investigated. In case of a moving meniscus, the receding contact angle dropped to a lower value and remained almost constant at higher velocities, whereas the advancing contact angle was found to be independent of the surface velocity.

Kandlikar and Kuan [6] studied the heat transfer from a moving and evaporating meniscus on a heated surface. The water flow rate, heater surface temperature and the speed of rotation were controlled to provide a stable meniscus. They found that at lower velocities, the heat flux was relatively insensitive to velocity, but it increased almost linearly with velocity at higher velocities. The results indicated that transient heat conduction played a major role in the heat transfer process to a moving meniscus.

Son and Dhir [7] developed a two-dimensional numerical model of growth and departure of single vapor bubbles during nucleate pool boiling. They used the level-set technique to implicitly capture the liquid vapor interface. Mukherjee and Dhir [8] extended the model to three-dimensions and studied merger and departure of multiple bubbles during nucleate pool boiling. The present calculations are done using a similar model, to study the evaporating meniscus on a moving heated plate.

**NUMERICAL MODEL**

**Method**

The numerical analysis is done by solving the complete incompressible Navier-Stokes equations and using the SIMPLER method [9], which stands for Semi-Implicit Method for Pressure-Linked Equations Revised. A pressure field is extracted from the given velocity field. The continuity equation is turned into an equation for the pressure correction. During each iteration, the velocities are corrected using velocity-correction formulas. The computations proceed to convergence via a series of continuity satisfying velocity fields. The algebraic equations are solved using the line-by-line technique, which uses TDMA (tri-diagonal matrix algorithm) as the basic unit. The speed of convergence of the line-by-line technique is further increased by supplementing it with the block-correction procedure [10]. Multi-grid technique is used to solve the pressure fields.

Sussman et al. [11] developed a level set approach where the interface was captured implicitly as the zero level set of a smooth distance function. The level set function was typically a smooth function, denoted as $\phi$. This formulation eliminated the problems of adding/subtracting points to a moving grid and automatically took care of merging and breaking of the interface. The present analysis is done using this level set technique.

The liquid vapor interface is identified as the zero level set of a smooth distance function $\phi$. The level set function $\phi$ is negative outside the meniscus and positive inside the meniscus. The interface is located by solving the level set equation. A 5th order WENO (weighted, essentially non-oscillatory) scheme is used for left sided and right sided discretization of $\phi$ [12]. While $\phi$ is initially a distance function, it will not remain so after solving the level set equation. Maintaining $\phi$ as a distance function is essential for providing the interface with a width fixed in time. This is achieved by reinitialization of $\phi$. A modification of Godunov’s method is used to determine the upwind directions. The reinitialization equation is solved in fictitious time after each fully complete time step. With $\Delta t = \frac{d}{2u_0}$, ten $\tau$ steps are taken with a 3rd order TVD (total variation diminishing) Runge Kutta method.

**Governing Equations**

**Momentum equation** -

$$
\rho \left( \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} \right) = -\nabla p + \rho \vec{g} - \rho \beta_T (T - T_{sat}) \vec{g} - \sigma \kappa \nabla H + \nabla \cdot \mu \nabla \vec{u} - \nabla \cdot \mu \nabla \vec{u}^T
$$

$$
\text{Energy equation} -
\rho C_p \left( \frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T \right) = \nabla \cdot \kappa \nabla T \quad \text{for } \phi > 0
$$

$$
T = T_{sat} \quad \text{for } \phi \leq 0
$$

**Continuity equation** -

$$
\nabla \cdot \vec{u} = \frac{\dot{m}}{\rho \varepsilon \rho} \nabla \rho
$$
The curvature of the interface is defined as -
\[ \kappa(\phi) = \nabla \cdot \frac{\nabla \phi}{|\nabla \phi|} \]  \hspace{1cm} (4)

The mass transfer rate of liquid evaporating at the interface -
\[ \dot{m} = k_Y \nabla T \]  \hspace{1cm} (5)

The vapor velocity at the interface due to evaporation –
\[ \dot{u}_{evp} = \frac{\dot{m}}{\rho_v} = \frac{k_Y \nabla T}{\rho_v h_{fg}} \]  \hspace{1cm} (6)

To prevent instabilities at the interface, the density and viscosity are defined as -
\[ \rho = \rho_v + (\rho_l - \rho_v)H \]  \hspace{1cm} (7)
\[ \mu = \mu_v + (\mu_l - \mu_v)H \]  \hspace{1cm} (8)

H is the Heaviside function -
\[ H = 1 \text{ if } \phi \geq 1.5d \]
\[ H = 0 \text{ if } \phi \leq -1.5d \]
\[ H = 0.5 + \phi/(3d) \]
\[ + \sin[2\pi\phi/(3d)]/(2\pi) \text{ if } |\phi| \leq 1.5d \]  \hspace{1cm} (9)

where \( h \) is the grid spacing

Since the vapor is assumed to remain at saturation temperature, the thermal conductivity is given by –
\[ k = k_i H^{-1} \]  \hspace{1cm} (10)

Level set equation is solved as -
\[ \phi_t + (\dot{u} + \dot{u}_{evp}) \nabla \phi = 0 \]  \hspace{1cm} (11)

After every time step, the level-set function \( \phi \) is reinitialized as –
\[ \phi_t = S(\phi_0)(1-|\nabla \phi|)u_0 \]  \hspace{1cm} (12)
\[ \phi(x,0) = \phi_t(x) \]

S is the sign function which is calculated as -
\[ S(\phi_0) = \frac{\phi_0}{\sqrt{\phi_0^2 + d^2}} \]  \hspace{1cm} (13)

Computational Domain

Figure 2 shows the computational domain. The total domain is 2.97x0.99 non-dimensional units in size. Cartesian coordinates are used with uniform grid.

The domain consists of two parallel plates with an opening in the top wall for the liquid to enter. The bottom wall moves in the positive x-direction. The two other sides are outlets. The number of computational cells in the domain are 240x80 i.e. 80 grids are used per 0.99\( l_0 \).

The liquid and vapor properties are taken at 373K.

Scaling Factors

The distance and velocities are non-dimensionalized with the length scale \( l_0 \) and the velocity scale \( u_0 \). For all calculations we assumed \( l_0=1 \) mm.

In the case with no wall velocity, \( u_0 \) is defined as –
\[ u_0 = \sqrt{g l_0} \]  \hspace{1cm} (14)

In the other cases, \( u_0 \) is the wall velocity and is calculated from the specified Reynolds number.

The non-dimensional temperature is defined as –
\[ T^* = \frac{T - T_{sat}}{T_{sat}} \]  \hspace{1cm} (15)

Initial Conditions

The liquid column is initially placed with center at \( x^* = 0.99 \) and a thickness of 0.99\( l_0 \). Saturated vapor fills rest of the domain. Thus the initial width of the liquid column is same as the distance between the plates. For the case with no wall velocity, the liquid column is placed at the center of the domain.

All initial velocities inside the domain are set to zero. The liquid and vapor temperatures are set to the saturation temperature (\( T^* = 0 \)). The wall temperature is set to the specified superheat (\( T^* = 1 \)). The wall contact angle is specified as 50° at the liquid base, at both ends. The wall superheat is kept fixed at 10K for all cases.

Boundary Conditions

At the bottom wall (\( y^* = 0 \)) -
\[ v^* = 0; \]
\[ u^* = u_{wall}^*; \]
\[ T^* = 1; \phi^* = -\cos \varphi; \]

where \( \varphi \) is the contact angle

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At the top of the domain \((\gamma^* = 0.99)\) :-
\[
\begin{align*}
v^* &= -0.0025 u_{\text{wall}} \quad \text{if } \phi > 0; \\
v^* &= 0 \quad \text{if } \phi \leq 0; \\
u^* &= 0; T^* = 0; \phi_y = 0;
\end{align*}
\]
At the outlets \((x^* = 0 \text{ and } x^* = 2.97)\) :-
\[
\begin{align*}
u_x^* &= 0; \nu_y^* = 0; T_x^* = 0; \phi_x = 0;
\end{align*}
\]

We have assumed constant wall temperature at the bottom wall. It has been shown in [6] that it is a reasonable assumption for a surface with high thermal diffusivity such as copper.

**RESULTS**

Figure 3 shows the velocity and temperature field in the liquid meniscus for the case with no wall velocity at 200 ms. The meniscus takes up a steady hyperbolic profile after some initial fluctuations. The thermal boundary layer builds up gradually from the wall inside the liquid. Non-dimensional temperature contours are plotted at an interval of 0.1. Intense evaporation takes place at the meniscus base near the contact line region. The crowding of isotherms at the contact line region indicates high wall heat transfer. Large velocity vectors can be seen near the meniscus base in the vapor region indicating high rate of evaporation. The velocity vectors present in the liquid inside the meniscus are much smaller compared to those in the vapor and hence cannot be seen as prominently as the latter.

Convergence check is carried out with various grid sizes to demonstrate grid independence. To optimize computation costs we have chosen 80 grids for our calculations. Figure 4 shows the variation of wall heat transfer for different grid sizes for the above case with no wall velocity. The wall heat transfer is averaged over the entire bottom wall of the domain. It is very high initially as the saturated liquid is in contact with the superheated wall.

The wall heat transfer decreases as the thermal boundary layer thickens inside the liquid with time. Calculations are carried out for 64, 80 and 96 grids. After initial fluctuations die around 50 ms, the results for all three grids show very little difference. It confirms that the calculations with 80 grids provide sufficient resolution to capture details of high heat transfer region near the wall.

In Fig. 5 we plot the meniscus base width as a function of time for the case of Reynolds number of 125. Initially we started with a base width of 0.99 mm. The plot shows initial fluctuation of the meniscus base width till 50 ms. The base width increases from the initial condition and becomes steady at around 1.35 mm which gives us an indication that steady state has been reached.

Figure 6 shows the velocity field in the meniscus for the above case of Re = 125. The figure shown is at 200 ms when a steady state has been reached and there is no appreciable change in the shape of the meniscus. The wall...
velocity has caused the meniscus to get skewed in one direction. The streamlines in the liquid show a circulating flow inside the meniscus. Evaporation takes place at the liquid-vapor interface indicated by the jets of vapor leaving the interface. However, this evaporation is comparatively much more intense at the receding contact line region. The reason for this can be explained by analyzing the temperature field inside the meniscus.

The thermal boundary layer inside the meniscus is shown in Fig. 7. It shows more superheated liquid near the receding liquid-vapor interface compared to the advancing one. The liquid is dragged in the positive x-direction near the wall due to the wall movement and gets heated up due to heat transfer from the wall. The circulation of liquid (shown in Fig. 6) causes the hotter liquid to rise up near the receding contact line, increasing evaporation along the receding liquid-vapor interface. The cooler saturated liquid comes back down near the advancing contact line suppressing evaporation at the interface in that region.

Figure 8 compares the heat transfer from the wall at the meniscus base for all the cases. The cases are one with no wall velocity and two others with Re equal to 125 and 250. The Nusselt number, averaged over the meniscus base, is plotted against the surface velocity. We note that the case with no wall velocity produces the least heat transfer. As the surface velocity is imparted, the liquid circulates near the meniscus base, which in turn increases the heat transfer. The highest heat transfer is obtained for the highest surface velocity.

Figure 9 shows the local heat transfer variation along the meniscus base at 200 ms for the case with Re = 125. The solid line shows the results obtained from the numerical calculations. The highest heat transfer is seen to occur at the advancing contact region. This is due to transient conduction as cooler saturated liquid comes down near the heated wall. The heat transfer steadily decreases from that end till it reaches the receding contact line. There is a jump
in wall heat transfer at the receding contact line. This is where the distance between the liquid-vapor interface and the wall becomes small causing a steep temperature gradient at the wall.

The heat transfer near the advancing edge of the meniscus can be represented by a transient heat conduction model. The heater surface comes in contact with the liquid, which can be assumed to be a semi-infinite medium during the initial contact with the heater surface. In the present modeling, a constant heater wall temperature is assumed. The instantaneous heat flux following the initial contact with a semi-infinite medium (assuming water at saturation temperature) under the constant surface temperature case is given by –

\[ q^* = \frac{k(T_w - T_{sat})}{\sqrt{\pi a t}} \]  \hspace{1cm} (19)

The heat transfer coefficient \( h \) is calculated as –

\[ h = \frac{k}{\sqrt{\pi a t}} \]  \hspace{1cm} (20)

The instantaneous value of the Nusselt number near the advancing edge of the meniscus is calculated using Eqs. (21) and (22).

\[ Nu = \frac{h l_0}{k} = \frac{l_0}{\sqrt{\pi a t}} \]  \hspace{1cm} (21)

\[ t = \frac{x}{u_{wall}} \]  \hspace{1cm} (22)

The distance along the meniscus base \( x \) is calculated starting from the location of the advancing contact region.

The results are plotted on Fig. 9 as a dashed line along with the numerical results discussed earlier. It is seen that the numerical results and the results from the transient conduction model are in excellent agreement at the advancing contact region. This confirms that transient conduction is indeed the primary mechanism of heat transfer from the wall at the leading edge of the meniscus base. However, with increase in distance from the leading edge, the calculated heat transfer is found to be less than the analytical results. It is due to the fact that the circulation of liquid inside the meniscus causes a mixing effect. This mixing increases the average temperature of the liquid which in turn reduces the wall heat transfer.

CONCLUSIONS

1. A steady evaporating meniscus on a moving heated wall is numerically simulated in two dimensions and the velocity and temperature fields are obtained.
2. Circulation of liquid is observed inside the meniscus on the moving wall.
3. Evaporation is found to be predominant on the receding liquid-vapor interface compared to the advancing interface.
4. The wall heat transfer is found to increase with increase in wall velocity for the same wall superheat.
5. The local wall heat transfer varies significantly along the meniscus base. The advancing contact region shows the highest heat transfer due to transient conduction in the liquid from the heated wall.

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