THERMOHYDRAULIC PERFORMANCE ANALYSIS OF OFFSET STRIP FIN MICROCHANNEL HEAT EXCHANGERS

Akhilesh V. Bapat\textsuperscript{1} and Satish G. Kandlikar\textsuperscript{2}
Thermal Analysis and Microfluidics Laboratory,
Mechanical Engineering Department,
Rochester Institute of Technology, Rochester, NY 14623.
\textsuperscript{1}avb9689@rit.edu
\textsuperscript{2}sgkeme@rit.edu

ABSTRACT
Recent experimental work on enhanced Silicon microchannels (Colgan et al., 2005, Steinke, 2005) indicates significant improvement in the heat transfer coefficient during single-phase liquid flow. The enhanced microchannel consists of multi-pass arrangements with offset strip fins of short lengths. An analytical treatment employing thermal boundary layer development in the entrance region of each offset fin geometry passage is performed. The analytical results are compared with the experiments and the reasons for the differences are discussed.

1. INTRODUCTION
Thermal management of electronic devices is one of the important aspects for electronics packaging. With the development in microelectronics technology, the need for power dissipation has also increased and as well the need for smaller heat removal equipment.

The heat transfer coefficient \( h \), given by Eq. 1 below, increases with reduced hydraulic diameter \( D_h \). Thus heat transfer is enhanced in microchannels.

\[
h = \frac{Nuk}{D_h}
\]

However the pressure drop also varies inversely with \( D_h \) thus there is increased pressure drop in microchannels. The balance between the heat transfer rate and the pressure drop becomes an important issue in designing the coolant flow passages for the high flux heat removal encountered in microprocessor chip cooling.

\[
\Delta p = \frac{2 f \rho u_m^2 L}{D_h}
\]

The conventional fluid flow theories in microchannel flows for liquids are expected to hold good. Upadhye and Kandlikar (2004) carried out detailed analyses for single-phase flow in microchannels. They analyzed the experimental data available in the literature on microchannels using classical developing flow theory (entrance region effects) and fully developed flow. The present work focuses on the flow of single-phase liquids in microchannel passages. The compressibility effects slip boundary condition, and the rarefied flow concerns do not apply for these single-phase liquid flows in microchannel. Also, no electrokinetic effects are seen at the scales considered in the reported experiments. In the present work, hydrodynamic and thermally developing flows have been assumed.

Figure 1 below shows a set of microchannels with 42\( \mu \)m trenches that are etched to greater than 100\( \mu \)m. Channels such as above enhance the heat transfer considerably with an increase in pressure drop required to pump the fluid through the channels. Tukerman and Pease (1981) first gave the concept of direct chip cooling using single phase heat transfer. They showed that small flow channels are capable of dissipating large amount of energy.
Since then researchers have been trying to develop techniques to enhance heat transfer in microchannels. Stienke and Kandlikar (2004) identified single-phase heat transfer enhancement techniques for use in microchannels and minichannels.

Colgan et al. (2005) tested an offset strip fin Silicon microchannel cooler for a single phase flow. Figure 2 gives the Silicon microchannel used by them. To compensate for the increased frictional losses, shorter flow lengths were used by using multiple entry and exit vias. The Nusselt numbers obtained were as high as 25 and staggered fin arrangements at a pitch of 100\(\mu\)m and fin lengths of 210\(\mu\)m and 250\(\mu\)m. The resulting heat transfer coefficients were as high as 130,000 W/m\(^2\) °C, however with considerably higher friction factors. The Nusselt number and friction factor plots obtained experimentally by Colgan et al. (2005) can be seen in Figs. 3 and 4.

In Fig. 4, the prediction of Nusselt number using laminar theory is also shown by the continuous line. It is seen that the values are under predicted by the laminar flow theory and a different model is required.

Offset strip fin microchannels offer enhancements in heat transfer at the expense of increased pressure drop. However the offset strip fin heat exchangers have been studied extensively in the literature. Some of the literature on offset strip fin heat exchanger is reviewed before analyzing them with the developing flow theory.

Sparrow and Liu (1979) studied basic heat transfer and pressure-drop results for laminar airflow through arrays of inline or staggered plate segments from numerical solutions of the fluid flow and energy equations. The heat transfer and pressure drop results incorporating the entrance region was studied for both in-line and staggered configuration and was presented in the form of table.
Table 1. Results for the fully developed regime as a function of entrance length, Sparrow and Liu (1979)

<table>
<thead>
<tr>
<th>(L/2s)/Re</th>
<th>Nu&lt;sub&gt;f&lt;/sub&gt;</th>
<th>fRe</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-line</td>
<td>Stagg.</td>
<td>In-line</td>
</tr>
<tr>
<td>-----------------</td>
<td>---------------</td>
<td>------</td>
</tr>
<tr>
<td>0.0003</td>
<td>24.6</td>
<td>23.5</td>
</tr>
<tr>
<td>0.0005</td>
<td>23.6</td>
<td>22.4</td>
</tr>
<tr>
<td>0.0007</td>
<td>22.8</td>
<td>21.5</td>
</tr>
<tr>
<td>0.0010</td>
<td>21.8</td>
<td>20.6</td>
</tr>
<tr>
<td>0.0014</td>
<td>20.9</td>
<td>19.5</td>
</tr>
<tr>
<td>0.0025</td>
<td>19.1</td>
<td>17.6</td>
</tr>
<tr>
<td>0.0050</td>
<td>17.0</td>
<td>14.9</td>
</tr>
<tr>
<td>inf</td>
<td>7.54</td>
<td>7.54</td>
</tr>
</tbody>
</table>

Joshi and Webb (1987) presented analytical models to predict the heat transfer coefficient and the friction factor of the offset strip fin heat exchanger surface geometry in the laminar and turbulent flow regimes. They also studied the transport of energy and momentum in the boundary layers of the fins because of the oscillating velocities developed from the wakes. Thus the wake distribution was also studied by them to take into considerations the effect of fin length, fin thickness and the fin spacing on the wave flow pattern.

Manglik and Bergles (1995) reanalyzed the empirical f and j data for the rectangular offset strip fin compact heat exchangers. They presented rational design equations for f and j in the form of continuous expressions covering the laminar, transition and the turbulent flow regimes.

3. OBJECTIVES

The present paper has three objectives:

1. Check the validity of the classical developing flow theory with the more sophisticated Joshi and Webb (1987) model for offset strip fin heat exchangers.
2. Apply the same classical developing flow theory for microchannel flows and compare the results with the available experimental data on Silicon Microchannel cooler by Colgan et al. (2005).
3. To indicate further research work needed for offset strip fin microchannel flow.

4. THEORETICAL ANALYSIS-DEVELOPING FLOW

Hydrodynamically developing flow is very important in microchannels. Due to often short lengths, the developing flow could dominate the entire the flow length of the microchannel. This is the assumption on which the available experimental data has been compared to.

The microchannel which is analyzed in this paper has an offset strip fin arrangement, with pitch of 100µm and fin length of 210µm. The channel is 180µm deep and 65µm wide. Because of the shorter fin lengths and the developing flow assumption is valid since the flow is expected to be fully developed after Re/20 diameters (Convective Heat Transfer, Kays and Crawford). The flow with this staggered configuration cannot be fully developed because of the interrupting flow passages. The Silicon microchannel cooler used by Colgan et al. (2005) has an overall nominal channel length of 3000µm. The Reynolds number range used for their experiments was in the range between 26 and 282.

To account for the developing region, the pressure drop equations are presented in terms of apparent friction factor. Apparent friction factor f<sub>app</sub> accounts for the pressure drop due to friction and the developing region effects. It represents an average value of the friction factor over the flow length between the entrance section and the location under consideration. Thus the pressure drop in a channel of hydraulic diameter D<sub>h</sub> over a length x from the entrance is expressed as in Eq. 1.

The non-dimensionalized length x+ is defined as:

\[
x^+ = \frac{x}{D_h Re}
\]

Shah and London (1978) and Kakac et al. (1987) have presented comprehensive summaries of the available literature on various geometries of interest in microfluidics applications. Phillips (1987) reviewed the available information, including that by Curr et al. (1972) and compiled the results for the apparent friction factor in a rectangular duct. Phillips (1987) gives the values of the apparent friction factor in a tabular form.

For the current configuration with aspect ratio of 0.3611, curve fit is done, the final equation being as given in Kandlikar (2006).

\[
y = \frac{a + cx^{0.5} + dx^1}{1 + bx^{0.5} + fx^{1.5}}
\]

where,

\[
x = x^+ = \frac{x}{D_h Re}
\]

\[
y = f_{app} Re
\]

and a=29.45684, b=2391.784, c=19648.24, d=294692, e=1223378, d=20202.6 which are found from Kandlikar et al. (2006).

For the thermally developing boundary layer, the non dimensional thermal entry length for flow in ducts can be expressed as

\[
x^*=x/Dh Re Pr
\]

For rectangular channels with four-sided heating configuration, Nusselt numbers in the thermally developing region are given in table format from Phillips (1987) work. For three side heating configuration, Phillips (1990) suggested a scheme,

\[
Nu_{x,3} = Nu_{x,4} \frac{Nu_{fd,3}}{Nu_{fd,4}}
\]

Where subscripts x,3 and x,4 refer to location at a distance x in the heated length for the three-sided and four-sided heating cases respectively and fd,3 and fd,4 refer to the fully developed Nusselt number for 3 and 4 sided heating respectively. The Nusselt number for four-sided heating can be given from the curve fit as suggested by Kandlikar et al. (2006).
\[ y = \frac{a + cx + ex^2}{1 + bx + dx^2 + fx^3} \]  

(7)

Where,

\[ x = x^* \]

And \( a=142.0733, \ b=-6.31297, \ c=965.6553, \ d=943.256, \ e=13565.73, \ f=-73.6945 \) as per Kandlikar et al. (2006).

Analysis of flow using these equations for Silicon microchannel cooler has been done in the following sections.

5. COMPARISION: OFFSET STRIP FIN HEAT EXCHANGER GEOMETRY VERSUS DEVELOPING FLOW THEORY

Thermo hydrodynamic performance on the strip fin heat exchanger has been carried out by many researchers in the past. Joshi and Webb (1987) presented analytical models to predict heat transfer coefficient and the friction factor of the offset strip-fin heat exchanger surface geometry with two flow regimes, laminar and turbulent.

Since it is assumed that the flow in microchannels is laminar, in this paper, the treatment by Joshi and Webb (1987) for the offset strip fin exchangers is limited only for the laminar flow. Their model incorporates the entrance length effects only for the fin side heat transfer while it is assumed that the fins do not affect the boundary layer from the top and bottom surfaces. The equations used by them for laminar regimes are stated below.

\[ Nu_s = 0.37 \times 10^6 (l_s^+)^2 - 3692(l_s^+) + 24.2 \]  

(8)

where,

\[ l_s^+ = (l / 2s) / Re_s \]

\[ Re_s = \rho u m 2s / \mu \]

The flow along the fin sides is assumed to be developing while the flow on the bottom and top surfaces is assumed not to be affected by the fin sides and hence fully developed flow is assumed for these two surfaces. Very low Reynolds number flow are not applicable as the friction factor and the Nusselt number values predicted in that case are aberrant and predict very high values. The curve fit made by Joshi and Webb (1987) is based on the data given by Sparrow and Liu (1987). The curve fit is done by using values of \((L' / 2sRe)\) values ranging from 0.0003 - 0.0050. Hence it does not predict values for higher values of the non dimensional length. The above equations are derived from Sparrow and Liu (1979) for aspect ratio of 0. For configurations with aspect ratio other than 0, Joshi and Webb (1987) suggested use of the relation,

\[ \frac{Nu_{EL,a}}{Nu_{EL,0}} = -0.0327 \ln(l_s^+) + 0.5965 \]  

(10)

A particular geometry is selected from the data available from Joshi and Webb (1987). The various parameters are, aspect ratio of 0.246 and hydraulic diameter 13.487 mm. The channel width is 9.347 mm.

Considering the above geometry the Nusselt number of fully developed flow for the top and bottom surface is given by the equation,

\[ Nu_e = 7.45 - 16.9\alpha + 22.1\alpha^2 - 9.75\alpha^3 \]  

(11)

Equations (8) and (11) are added to give the overall Nusselt number as per Joshi and Webb (1987).

6. COMPARISON WITH SILICON MICROCHANNEL COOLER

A particular geometry specified in the earlier sections is selected from Colgan et al. (2005) and the developing flow theory is applied to that configuration. To predict the Nusselt number Eq. 5 was used.

The apparent friction factors obtained from the experiments are inclusive of the entry and exit entry losses. These are considered while predicting the friction factors using the developing flow theory by adding them. Eq. 4 was used to predict the apparent friction drop for the Silicon microchannel cooler.

The plots for the Nusselt number and apparent friction factor are given in figs. 7 and 8 which are compared with the experimental data obtained by Colgan et al. (2005).

7. RESULTS AND DISCUSSION

The theoretical method used by Joshi and Webb (1987) is based on the work of Sparrow and Liu (1979) for air. The developing flow theory presented in section 4 above is based on the work of Shah and London (1978), Kakac et al. (1987), Phillips (1987) and Curr et al. (1972). The heat transfer and pressure drop predictions from these two models are compared for air flow for the geometries used by Joshi and Webb.

For the water cooled silicon micro coolers, the model by Joshi and Webb (1987) could not be applied as it was specific to the air flow. The developing flow theory is directly compared with the experimental data obtained by Colgan et al. (2005).

First the predicted values for the offset strip fin heat exchanger are compared with Joshi and Webb’s model, which was reported to closely fit their experimental results within 20 percent. Figures 5 and 6 show these plots.
In Fig. 5, the solid line indicates the values predicted using the developing flow theory while the dashed line indicated those obtained by using equations given by Joshi and Webb (1987). It is seen that the developing flow theory under-predicts the Nusselt number for the offset strip fin heat exchanger. The wake effect from the preceding fin is the most probable reason for the higher heat transfer as compared to the developing flow model, which ignores this effect.

In Fig. 6, the friction factor values predicted by the developing flow theory are plotted in solid lines while the dashed line indicates the experimental values obtained by Joshi and Webb (1987) on the offset strip fin heat exchanger. In this case also the values predicted by the developing flow theory are under predicted. The correlations used by Joshi and Webb to predict the friction factor were for the offset strip fin configuration. They formed an analytical solution on the assumption that there is developing flow on the fin sides while fully developed flow takes place on the top and bottom surfaces. The detailed analysis can be found their paper.

The comparison with the Silicon microchannel cooler is now made with the developing flow theory. Colgan et al. (2005) in his experiments with offset strip fin Silicon microchannel obtained Nusselt number as high as 25. The developing flow is tested one of the configurations which were tested by him.

Figure 7 gives the Nusselt number plot against the non-dimensional thermal length. The solid line gives the values of the Nusselt number predicted when the developing flow equations are applied to the Colgan et al. geometry. The dashed line indicates the values obtained experimentally by Colgan et al. (2005). It is observed that the values predicted in the Silicon microchannel cooler case are less than the actual experimental data. Also, the slope in the Colgan et al’s data is higher than the model predictions. The main reason for this difference is believed to be the cross-flow caused in the Colgan et al. geometry. This may also lead to an early transition to turbulent flow. It was shown by Kandlikar et al. (2005) that the transition to turbulence in rough microchannels can occur at Reynolds numbers as low as 300, depending on the roughness geometry. Further work in this area is needed.

In Fig. 8 below, apparent friction factor predictions by the developing flow theory are compared with the experimental data obtained by Colgan et al. (2005). The continuous line indicated the analytical prediction while the dashed line indicates the experimental values.
It is observed that the developing flow theory predicted values are considerably below the experimental values. The transition to turbulent flow is clearly seen at $ReD/s/L$ of around 2.

The reasons for the discrepancy are not clear. It is seen that the developing flow theory under predicts the heat transfer and the friction factor for the heat exchangers studied by Joshi and Webb (1987), just like it under predicts with Colgan et al. (2005) data for microchannel geometry using offset-strip fins. The three possibilities are: 1) The uncertainty in the experimental data of Colgan, 2) Additional effects due to specific inlet and outlet configuration, and 3) transition to turbulent flow in the microchannels due to cross-flow structure at the manifolds. The discrepancies in the experimental and analytical results are quite large, and may not be explained by the uncertainty in Colgan et al.’s data as their experiments were conducted under carefully controlled conditions. The manifold effects and the turbulent transition need to be further evaluated.

7. FUTURE RESEARCH DIRECTIONS

The developing flow theory under predicts the values of Nusselt number and friction factor for flow through offset strip fin microchannels. Since offset strip fin microchannel enhances the heat transfer considerably, more detailed understanding of the flow mechanism is required. An analytical solution should be found to predict the Nusselt number and friction factor accurately for the flow through offset strip fin microchannels. Flow visualization on flow through these channels will give a better idea for the high heat transfer rates. More experimental data with water as the test fluid is needed to validate the enhanced performance of the offset strip finned microchannels.

8. CONCLUSIONS

In this paper, the developing flow theory is first applied to the offset strip fin heat exchangers analyzed by Joshi and Webb (1987). The same theory is then applied to the microchannel geometry with strip-fins studied by Colgan et al. (2005) and the heat transfer and friction factor are compared. Following conclusions are derived from this study.

1. The Nusselt number and friction factor values were underpredicted using the developing flow theory as compared to those obtained by Joshi and Webb (1987) for their offset strip fin heat exchanger. Their analytical equations for the offset strip fin heat exchangers using the laminar and turbulent models were able to predict the experimental data for air
2. The same developing flow theory was then applied to water flow in microchannels with offset strip fins studied by Colgan et al. (2005). The results were underpredicted for the Nusselt number and friction factor. The experimental data from Colgan et al. (2005) for this geometry indicate that the Nusselt number and the friction factor are much higher in their enhanced microchannel configuration.
3. The Silicon microchannel cooler studied by Colgan et al. (2005) has a similar geometry to the offset strip fin heat exchanger studied by Joshi and Webb (1987). However, Joshi and Webb model cannot be applied to the Silicon microchannels as their model is based on Sparrow and Liu’s (1979) experimental data which used air as the test fluid. Colgan et al. has used water as test fluid for his experiments on Silicon microchannels.
4. The reasons for the discrepancy are believed to be the inlet and outlet header configurations in the Colgan et al.’s design, which may be giving rise to early transition to turbulent flow in their heat exchangers.

9. NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_h$</td>
<td>Hydraulic Diameter.</td>
</tr>
<tr>
<td>$f$</td>
<td>Single-phase friction factor, dimensionless.</td>
</tr>
<tr>
<td>$f_{app}$</td>
<td>Apparent friction factor accounting for developing flow</td>
</tr>
<tr>
<td>$f_s$</td>
<td>Friction factor for zero aspect ratio.</td>
</tr>
<tr>
<td>$h$</td>
<td>Heat transfer coefficient.</td>
</tr>
<tr>
<td>$l$</td>
<td>Fin length.</td>
</tr>
<tr>
<td>$l' = (l/2s)/Re_s$</td>
<td></td>
</tr>
<tr>
<td>$Nu$</td>
<td>Nusselt number.</td>
</tr>
<tr>
<td>$Nu_s$</td>
<td>Nusselt number for zero aspect ratio.</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number.</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number.</td>
</tr>
<tr>
<td>$Re_s$</td>
<td>$Re = \rho u(2s)/\mu$</td>
</tr>
<tr>
<td>$u$</td>
<td>Flow velocity.</td>
</tr>
<tr>
<td>$u_m$</td>
<td>Mean flow velocity.</td>
</tr>
<tr>
<td>$x$</td>
<td>Length.</td>
</tr>
<tr>
<td>$x^*$</td>
<td>Hydraulic non dimensional length, $x^* = x/ReD_h$</td>
</tr>
<tr>
<td>$x'$</td>
<td>Thermal non dimensional length, $x' = x/RePrD_h$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Aspect ratio.</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity.</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density.</td>
</tr>
</tbody>
</table>

10. REFERENCES


