EFFECTS OF ROUGHNESS ON TURBULENT FLOW IN MICROCHANNELS AND MINICHANNELS

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ABSTRACT

The effect of roughness ranging from smooth to 24% relative roughness on laminar flow has been examined in previous works by the authors. It was shown that using a constricted parameter, \( \varepsilon_F \), the laminar results were predicted well in the roughened channels ([1],[2],[3]). For the turbulent regime, Kandlikar et al. [1] proposed a modified Moody diagram by using the same set of constricted parameters, and using the modification of the Colebrook equation. A new roughness parameter \( \varepsilon_F \) was shown to accurately portray the roughness effects encountered in laminar flow. In addition, a thorough look at defining surface roughness was given in Young et al. [4]. In this paper, the experimental study has been extended to cover the effects of different roughness features on pressure drop in turbulent flow and to verify the validity of the new parameter set in representing the resulting roughness effects. The range of relative roughness covered is from smooth to 10.38% relative roughness, with Reynolds numbers up to 15,000. It was found that using the same constricted parameters some unique characteristics were noted for turbulent flow over sawtooth roughness elements.

NOMENCLATURE

\begin{align*}
 p & \quad \text{Pitch of roughness elements} \\
 P & \quad \text{Pressure or perimeter, apparent from eqn.} \\
 Q & \quad \text{Volumetric flow rate} \\
 Ra & \quad \text{Average roughness ASME definition} \\
 Re & \quad \text{Reynolds number} \\
 Re_c & \quad \text{Critical Reynolds number} \\
 Re_{cf} & \quad \text{Constricted Reynolds number} \\
 Re_{ce} & \quad \text{Critical constricted Reynolds number} \\
 R_p & \quad \text{Maximum peak height} \\
 Re_o & \quad \text{Critical Reynolds number for} \frac{\varepsilon}{D_{h,cf}}=0 \\
 RR & \quad \text{Relative Roughness} \\
 RR_{cf} & \quad \text{Relative Roughness base on constricted} \\
 Rv & \quad \text{Roughness valley method} \\
 Sm & \quad \text{Mean spacing of roughness irregularities} \\
 v & \quad \text{Flow velocity} \\
 x & \quad \text{Distance from channel beginning} \\
 z & \quad \text{Profilometer scan heights} \\
 \end{align*}

\begin{align*}
 \alpha & \quad \text{Aspect ratio} \\
 \alpha_{cf} & \quad \text{Aspect ratio based on constricted}\ b_{cf} \\
 \varepsilon & \quad \text{Height of a roughness element} \\
 \varepsilon_F & \quad \varepsilon \text{ based on proposed parameters} \\
 \rho & \quad \text{Density of the fluid} \quad \text{kg/m}^3 \\
 \mu & \quad \text{Dynamic viscosity} \quad \text{Ns/m}^2 \\
 m & \quad \text{Mass flow rate} \quad \text{kg/s}
\end{align*}

INTRODUCTION

Literature Review

The effects of roughness in turbulent flow in microchannels is rarely studied, due to the high pressure drops that this regime requires in small channels. In addition, most processes of interest in microfluids work with laminar flow. Because of this,
few studies ever cover this range and those will be outlined below.

Celata et al. [5] studied the heat transfer on 6 capillary tubes of diameter 130 μm. The roughness element height in these channels was 3.45 μm, leading to a relative roughness of 2.65%. In the laminar regime, the experimental results follow closely with theory, however nearly all the data points collected fall above this line, in agreement with our past work on the laminar regime. In the turbulent regime, the experimental data lie between the Blasius correlation (smooth tubes) and Colebrook (for the roughened parameters) predictions. They found that the Colebrook equation overpredicted the results in the turbulent regime.

Bavier et al. [6] used stainless steel capillary tube ranging from 172 μm to 520 μm with water Reynolds numbers up to 6000. The roughness in these capillaries ranged from 1.49 μm to 2.17 μm. For the larger two capillaries where the turbulent flow regime could be reached, the results matched well with results that were obtained using the Colebrook equation.

Tu and Hrnjak [7] tested RF134a in 5 different rectangular channels with differing aspect ratios. Their relative roughnesses varied from 0.14% to 0.35%. They found excellent agreement with macroscale theory in all regimes of flow. The turbulent data was found to fit the Colebrook equation at all roughnesses tested.

Bavier et al. [8] examined friction factor in channels varying from 7μm to 500μm. Based on plots presented in the work, it appears that their experimental results in the turbulent region show correlate well to macroscale theory.

Most all previous work on roughness, even in macroscale focuses on the region less than 5% relative roughness. Most fluidic devices fall within this region, however surpassing this limitation is possible and likely as fluidic devices channel size decreases. Kandlikar et al. [1] proposed using a constrained parameter, εp, to model roughness in channels. First, the use of a new roughness element height is proposed by the parameter εp. By changing the base dimension of the channel and recalculating parameters, you obtain a set of constrained parameters. Using this method, they proposed a modified Moody diagram to account for the effect of roughness essentially decreasing the free flow area of a circular channel. They propose replacing the friction term in the Colebrook turbulent friction factor equation with the following relation. In this equation, Dt is the root diameter of the tube

\[ f_{Moody, cf} = f_{Moody} \left[ \frac{D_t - 2 \epsilon_p}{D_t} \right]^5 \]  

When this is used to replot the Moody diagram, all values of relative roughness between 3-5% plateau to a friction factor of f=0.042 for high Reynolds numbers. It is difficult to find a work that tests relative roughness up to and past 3%, so this work aims to test past this region.

OBJECTIVES OF CURRENT WORK

In this work, two sets of samples with tall ribbed roughness elements are tested up to a relative roughness of 17.03% under turbulent flow conditions in rectangular channels. The testing was performed with water around 25°C over a Reynolds number range from Re = 1000 to 15,000 limited by the maximum pressure drop across the pump head.

THEORETICAL CONSIDERATIONS

Roughness height determination

A brief overview of the method of calculation of the proposed peak to floor roughness height, εp, is presented here. The full context can be found in Kandlikar et al. [1] along with Taylor et al. [9]. The following calculations are performed after the profile of a surface is obtained using a stylus profilometer or other similar instrument. These parameters are illustrated graphically in Fig. 1.

The Mean Line is the arithmetic average of all the points from the raw profile, which physically relates to the height of each point on the surface. It is calculated as shown in Eq. 2.

\[ \text{Mean Line} = \frac{1}{N} \sum_{i=1}^{N} Z_i \]  

Rp is the maximum peak height from the mean line, which translates to the highest point in the profile sample minus the mean line. It is calculated by Eq. 3.

\[ R_p = \max \{Z_i - \text{Mean Line}\} \]  

Sm is defined as the mean separation of profile irregularities, or the distance along the surface between peaks. It is defined as the distance between peaks of a roughness irregularity. In this case, on the ribbed sample the rib separation is 405μm.

Fp is defined as the floor profile. It is the arithmetic average of all the points that fall below the mean line value. As such, it is a good descriptor of the baseline of the roughness profile. It can be calculated from Eq. 4.

\[ \text{Fp} = \frac{1}{N} \sum_{i=1}^{N} z_i \]  

FdRa is defined as the distance of the floor profile (Fp) from the mean line. It is found with Eq. 5.

\[ \text{FdRa} = \text{Mean Line} - \text{Fp} \]  

εFP, or the value of the proposed roughness height, is determined by Eq. 6. It is the distance between the floor profile and the peak of the roughness.

\[ \epsilon_{FP} = R_p + \text{FdRa} \]  

Let \( z \in Z \) s.t. all \( z_i = Z_i \) iff \( Z_i < \text{Mean Line} \)
A final measure of roughness is the parameter Ra, or average roughness. This is an ASME defined parameter, mainly used to characterize surface finish.

\( \varepsilon_{FP} \) was conceived because Ra has been shown in previous works ([2],[3]) to not accurately predict the hydraulic effects of roughness. Additionally, a more in depth look ([2]) at the sand grain roughness that so much of our knowledge of roughened hydraulic effects are based on, shows that the use of Ra under predicts the sand grain roughness height by over 50%. However, the use of \( \varepsilon_{FP} \) more accurately predicts this roughness element height.

**Definition of Constricted Parameters**

The derivation of constricted parameters is paramount to determining important predictors for the friction factor in high roughness channels. These constricted parameters are based on the channel dimensions when the area the roughness occupies is removed. They can be found by subtracting the roughness element height from their respective dimension of the channel. First, we have to define the constricted channel height. An ordinary channel has a cross section of height b, and width a. However, with roughness on 2 sides of the channel the parameter \( b_{cf} \) represents the new constricted channel height. These parameters are illustrated with generic ribbed roughness in Fig. 2. Note that the pitch, p, of the ribbed samples is 405μm.

\[
b_{cf} = b - 2\varepsilon_{FP} \quad (7)
\]

\[
A = ab \quad (8)
\]

\[
A_{cf} = ab_{cf} \quad (9)
\]

Perimeter and constricted perimeter follow, substituting the \( b_{cf} \) value.

\[
P = 2a + 2b \quad (10)
\]
\[ P_{cf} = 2a + 2b_{cf} \]  

(11)

Hydraulic diameter is calculated using \( D_h \) and constricted hydraulic diameter are found with the following

\[ D_h = \frac{4A}{P} \]  

(12)

\[ D_{h, cf} = \frac{4A_{cf}}{P_{cf}} \]  

(13)

Now we calculate the Reynolds number. It is given by Eq. 14. The constricted Reynolds number is given in Eq. 15.

\[ Re = \frac{4m}{\mu P} \]  

(14)

\[ Re_{cf} = \frac{4m}{\mu P_{cf}} \]  

(15)

**EXPERIMENTAL VERIFICATION**

**Experimental Setup**

For this experimentation, a similar setup to those previously reported ([2],[3]) is used. A pump capable of at most 5.5 lpm at 8.5 bar differential pressure is used. Pressurized water passes through a filter and flow meter, which cause a significant pressure drop and dampens any flow fluctuations. The flowmeter is capable of 500 mL/min to 5000 mL/min with a stated 1% FS accuracy. Careful calibration however shows errors of at the very worst 1.2% error of the reading. The water then enters the test section. A differential pressure sensor is connected between the two to obtain the pressure gradient.

**Testing**

A schematic of the test section can be seen in Fig. 5. The test section consists of two samples that are machined from aluminum. These samples form the channels longer two sides and the short sides are formed by one wall containing pressure taps and the other of a sealing gasket. One of the two samples of each sample set is secured in place to remain immobile, and the other sample is placed in the movable ground steel holder opposite to the first. The samples are brought together with the set screws onto two precision gage blocks of the same known width and the micrometer heads zeroed on them. One gage block is placed between the samples at each end of the channel, to ensure that the channel is parallel. This gage block is placed between the samples at each end of the channel, to ensure that the channel is parallel. This gage block width is then known to be the \( b_{cf} \) separation of the channel. The separation of the samples is then set to the desired value with the micrometer heads and held in place with set screws. The channel is then sealed. The pump is turned on and controlled by the computer to maintain the flow rate. The DAQ system also acquires data. While the experiment is running, the pressure taps are monitored to make sure they are outside the developing
region and operating properly. The tests that were run with this setup are summarized in Fig. 4, along with other relevant parameters for testing.

**Samples**

The sample blanks are machined to near-dimensions, and then are precision ground to exact dimensions. The parallelism and flatness is then verified to ensure proper channel geometry. The smooth channel samples were then lapped first with 5μm lapping compound and then 1μm lapping compound to create a mirrored smooth surface.

The 405μm samples have patterned ridges using a small ball end mill in a CNC mill. A small ball end mill is run in perpendicular cuts across the ground sample blanks, forming ridges on the metal. This is done along the entire length of the sample, resulting in a repeating ridge profile to the roughness. Due to the nature of the machining process, the height of the elements varies slightly with a periodic nature. Every other element is approximately 20μm shorter than the rest. The 1008μm samples are machined with the same technique, but a more accurate CNC mill, so the shape is more as intended.

The profile of these surfaces is then taken with a stylus profilometer, and the roughness element height is determined using ε_{FP}. The results of 8 profilometer scans are averaged and a value of ε_{FP} is found for both the smooth and roughened samples. For easy visualization of the samples, a 3D surface is composed using a digital microscope and also an interferometer for one sample. This 3D surface along with a profile scan can be seen in Fig. 3.

The roughness parameter Ra has often been used in studies to represent the height of the roughness elements. We've found that this neither is a good basis for representing sand grain roughness, nor allows for accurate prediction of the hydraulic effects of roughness elements. Roughness element height for the samples can be seen in in Fig. 4. This figure also shows an overview of the test separations that are run. For a more in depth look at the parameterization of different machined surfaces using this method, refer to Young et al [4].

**Uncertainty**

This analysis is based on being able to find the uncertainty of each measurement in the experiment. The calibration of each sensor is used to determine uncertainties. The points used for the linear calibration are used to find the error between measured and the calibration value. For each sensor, 30 points are checked, and the maximum value of error of the 30 is recorded. The average of these maximum errors is used for the error of the pressure sensors. This approach yields extremely conservative reading error values, of 0.998% for pressure sensors and around 2.2% for the flow sensors.

The result of a performing uncertainty analysis on this data is that at worst, the friction factor in the fully turbulent regime (Re >5000) has an uncertainty from 3.2% to 11.18% for a separation of 600μm and 200μm respectively. Uncertainty in the Reynolds number is dependent on only the flow meters, and is 1.36%.

**RESULTS**

**Data Reduction**

Friction factor is found from from the following equation. To find the constricted friction factor, a substitution of the constricted parameters suffices.

\[
f_{\text{experimental}} = \frac{P_2 - P_1}{\rho D_h A^2} \frac{\dot{m}^2}{x}
\]

\[
f_{\text{experimental,cf}} = \frac{P_2 - P_1 \rho D_{h,cf} A_{cf}^2}{\dot{m}^2 x}
\]

(16)
To compare this experimentally obtained friction factor to theory, the following rectangular duct correlation by Kakac, et al [10] is used for the laminar regime (Eqn. 17). The aspect ratio \( \alpha \) is defined by Eq. 18. Again, the constricted aspect ratio, \( \alpha_{cf} \), is defined with the constricted channel height in Eq. 19.

\[
f = \frac{24}{Re} \left( 1 - 1.3553 \alpha + 1.9467 \alpha^2 - 1.7012 \alpha^3 + \ldots \right)
\]

\[
0.9564 \alpha^4 - 0.2537 \alpha^5
\]

\[
\alpha = \frac{b}{a}
\]

\[
\alpha_{cf} = \frac{b_{cf}}{a}
\]

The theoretical turbulent friction factor is calculated using the following equation from Colebrook.

\[
\frac{1}{f^{0.5}} = -2 \log \left( \frac{\epsilon/D_h}{3.7} + \frac{2.51}{Re f^{0.5}} \right)
\]

To modify Eqn. 20 for a rectangular duct, \( D_l \) is used instead of \( D_h \) in Eqn. 20 as prescribed in Kakac, et al. [10]. \( D_l \) is defined in the following equation.

\[
\frac{D_l}{D_h} = \frac{2}{3} + \frac{11}{24} \alpha(2 - \alpha)
\]

**Smooth Channel Verification**

The smooth samples are tested at separations from 200 \( \mu \)m to 1000 \( \mu \)m. This results in hydraulic diameters from 394 \( \mu \)m to 1849 \( \mu \)m. This range of separations effectively covers those that will be encountered with the roughened samples. The results of this verification agree well with theory. At the largest separation of 1000\( \mu \)m, it can be seen that the uncertainty in the data is much higher. This is a result of a low pressure gradient along the region of interest, and as such, there is a lot of uncertainty in this set at low Reynolds numbers. As the Reynolds number increases, and the pressure gradient does also, the data becomes more certain. The smooth channel results can be seen in Fig. 6. After applying the rectangular turbulent modification given in Eqn. 21 it can be seen that the results...
match quite well. What is also apparent is that the differences in aspect ratio between the channels has little influence on this modification, since \( a \gg b_c \). With the test setup verified, we move on to the roughened samples.

**Rough Channel Results**

The roughened channels represent data sets that are obtained for the first time in larger roughness channels under both laminar and turbulent flow conditions. The lowest relative roughness tested is 3.97%. The roughened channels are tested the same way as the smooth channels.

In Fig. 7, the results from the 405 \( \mu \text{m} \) are plotted. In Fig. 7(a) a single data set is plotted, along with a previous prediction for rougher channels by Kandlikar. It can be seen that this theory far under predicts what is observed from testing. All the 405 \( \mu \text{m} \) roughened samples are plotted in Fig. 7(b), and again if the theory was plotted it would under predict the friction factor that is observed.

Interestingly, for the 405 \( \mu \text{m} \) samples in the turbulent regime, all relative roughness values appeared to converge to a single line for friction factor. Note that the data does not yet form a horizontal line at the highest Reynolds numbers tested, because that does not occur until Reynolds numbers greater than 100,000. Kandlikar *et al.* [1] predict convergence to a Darcy friction factor of 0.042 in circular tubes. With the limitation of Reynolds number on the current data set, not much can be said about the accuracy of this prediction. Limitations in the capabilities of some of the sensors and the pump in the system prevent tests from being run to further Reynolds numbers, however this is currently being fixed so the tests can be extended further into the turbulent regime.

In Fig. 8, both of the pitched samples are plotted. It appears that the ratio of the height of the roughness element over the pitch of the elements (\( p/\varepsilon_f \)) appears to play an important role in the determination of which friction factor line the samples will converge to. When looking at this parameter, the 405 \( \mu \text{m} \) samples correspond to a \( p/\varepsilon_f \) ratio of 4.1, and the 1008 \( \mu \text{m} \) samples correspond to a \( p/\varepsilon_f \) ratio of 19.2. Intuitively, as this ratio approaches infinity, the turbulent data should become closer to smooth channel results because the elements are becoming more sparse. Further experimentation with other pitched samples will be required for more detailed correlations on this phenomena.

**CONCLUSIONS**

Rough channels were tested for relative roughness values from 3.97\% to 17.03\%, representing roughness that have not been looked at extensively in literature. When tested to a Reynolds numbers up to 15,000, the constricted data appear to indicate that the friction factor converges to a single line if plotted on the Moody diagram, based on the ratio of pitch to roughness element height. This extremely rough nature of the samples in this testing provides data that is far outside of what has been tested before, and is indeed new ground. Kandlikar *et al.* [1] predict a plateau of the turbulent friction lines when plotting the modified Moody diagram with the constricted parameters. The data from this experimentation seem to show that this could indeed be the case, however the sawtooth roughness used in this experimentation seems to indicate a different value for the plateau. Further work is needed to properly characterize this effect in turbulent flow, and the effect of modifying the ratio of pitch to roughness element height.


