Effect of Sawtooth Roughness on Pressure Drop and Turbulent Transition in Microchannels

TIMOTHY P. BRACKBILL and SATISH G. KANDLIKAR
Mechanical Engineering Department, Rochester Institute of Technology, Rochester, New York, USA

Roughness elements affect internal flows in different ways. One effect is the transition from laminar to turbulent flow at a lower Reynolds number than the predicted Re = 2300. Initial work at RIT in the subject area was performed by Schmitt and Kandlikar [1] and Kandlikar et al. [2], and this study is an extension of these efforts. The channel used in this study is rectangular, with varying separation between walls that have machined roughness elements. The roughness elements are saw-tooth in structure, with element heights of 107 and 117 µm for two pitches of 405 µm and 815 µm, respectively. The resulting hydraulic diameters and Reynolds numbers based on the constricted flow area range from 424 µm to 2016 µm and 210 to 2400, respectively. Pressure measurements are taken at sixteen locations along the flow length of 88.9 mm to determine the local pressure gradients. The results for friction factors and transition to turbulent flow are obtained and compared with the data reported by Schmitt and Kandlikar [1]. The roughness elements cause an early transition to turbulent flow, and the friction factors in the laminar region are predicted accurately using the hydraulic diameter based on the constricted flow area.

INTRODUCTION

Work in the area of roughness effects on frictional factors in internal flows was pioneered by Colebrook, Nikuradse, and Moody. Their work was limited to relative roughness values of less than 0.05, a value that may be exceeded in microfluidics applications where smaller hydraulic diameters are encountered.

Wu and Little [3] noticed an early transition to turbulent flow in microminiature refrigerators. Their channels were etched on glass and silicon with Dh from 45.46 to 83.07 µm. Wu and Little [4] fabricated microchannels with the same process of Dh around 150 µm, and found unusually high frictional factors. They found that roughness contributed to low critical Reynolds numbers from 400–900.

Schmitt and Kandlikar [1] performed work in this area using roughened rectangular minichannels of hydraulic diameters ranging from 325 µm to 1819 µm with air and water as the working fluids. They found early turbulent transition and irregular pressure drops when compared to conventional values. Also shown was that the laminar friction factor could be calculated by using the constricted hydraulic diameter, Dh,cf. They also found a relationship between the critical Reynolds number and the relative roughness (ε/Dh,cf).

Kandlikar et al. [5] reports laminar-to-turbulent transitions at far lower values than the accepted value of Re = 2300. It was shown, citing work by Schmitt and Kandlikar et al. [1], that increasing relative roughness values resulted in decreasing critical Reynolds numbers. They give the relation governing the transition Reynolds number in Eq. (1). Also resulting from this work is a modified Moody Diagram, based on constricted hydraulic diameter determining the friction factor; this is shown in Figure 1.

0 < ε/Dh,cf ≤ 0.08 Re,cf = 2300 − 18, 750(ε/Dh,cf)

0.08 < ε/Dh,cf ≤ 0.15 Re,cf = 800 − 3, 270(ε/Dh,cf − 0.08) (1)

Recently, Kandlikar et al. [2] proposed new roughness parameters of interest to roughness effects in microfluidics, as shown in Figure 2. First, Rp is proposed as the maximum peak height from the mean line. Next, RSm is defined as the mean separation of profile irregularities. This is also defined in this paper as the pitch of the roughness elements. Finally, FdRa is defined as
the distance of the floor profile ($F_p$) from the mean line. This is the average of roughness heights that low below the mean profile line, or the average of these heights. These values are established to correct for the assumption that different roughness profiles with equal values of $R_a$, average roughness, may have different effects on flows with variations in other profile characteristics. The value of the roughness heights, or $\varepsilon$, is determined by $F_d R_a$ added to $R_p$.

Rawool et al. [6] performed numerical CFD-ACE+ simulation of sawtooth roughness elements, similar in nature to those used in this experiment. In the simulation, serpentine channels were examined. They found that differing pitches of identical triangular roughness elements led to variations in pressure drop and velocity profiles. It was found that pressure drop decreases with an increase in the pitch of the elements. A diagram of this discrepancy can be found in Figure 3.

Kandlikar [7] performed a review of available literature and commented on past and current work in roughness and pressure drop. He concluded that work much older than the 1990s on microscale pressure drop included uncertainties that prevented accurate conclusions from being drawn. It also reaffirmed the effect of surface roughness on friction factor and early turbulent transition, while calling for lower relative roughness experimentation.

OBJECTIVES

The objective of the work is to study the phenomena of transition to turbulent flow at lower Reynolds numbers and the differing friction factor caused by triangular roughness elements of different pitch.

EXPERIMENTAL SETUP

A schematic of the setup is shown in Figure 4. Distilled water is used as the experimental fluid and is stored in a stainless steel reservoir. From the reservoir, it is delivered to a bronze gear pump, Oberdorfer N991RM-FO1, which is driven by a Dayton 5K918C. It then branches through a 1 $\mu$m woven filter (Shelco OSBN-384DUB) or goes through the pass-through back to the reservoir. From here, it is delivered to a bank of three rotameter flow meters, parts Omega FL-5551C. The distilled water then enters the test section, exits the test section, and is released back into the reservoir.

The test section is rectangular in shape, with a fixed width of 12.2 mm (0.48 in) and a variable height, fixed by various
set screws. The test section is 88.9 mm (3.5 in) long and has static pressure taps along the length of the channel. These taps are spaced 6.35 mm (0.25 in) apart for a total of 16 taps along the channel length, and are formed with a #60 drill (diameter = 1.016 mm) in the wall of aluminum. The taps are all connected to a pressure manifold, with a separate valve for each tap. At the end of the pressure manifold is a Honeywell 0-690 kPa (0–100 psi) differential pressure sensor powered by an Electro Industries Laboratory DC power supply. At the inlet and outlet, two K type thermocouples measure the temperature of the water. To control the separation, two Mitutoyo Digimatic Micrometer heads with ±2.54 µm accuracy are used to set the separation, which is then fixed in place with screws.

The pressure sensor is connected to one channel of a LM324 op amp, with the gain set at 67. The design used is a basic non-inverting amplifier circuit. The op amp circuit is powered by an Electro Industries Laboratory DC power supply (at +31VDC). The amplified voltage reading is calibrated to a pressure using an OMEGA DPS-610 pressure calibrator. Pressure readings are then obtained using the voltage recorded by a Craftsman 82040 multimeter. A simple schematic of the setup can be seen in Figure 4.

**UNCERTAINTY**

Due to the similarity of this work compared to the work performed by Schmitt and Kandlikar [1], the uncertainty is estimated to be the same. The same flow meters, micrometer heads, temperature setup, and pressure sensor are used, lending credibility to this assumption. As such, the uncertainty is taken to be the same as that by Schmitt and Kandlikar, or 8.81%. This value uses temperature uncertainty of ±0.1°C, flow rates of ±3%, channel dimensions of ±5 µm, pressure readings of ±0.25%, pressure tap distances of ±5 µm, and pressure tap locations of ±5 µm.

**TEST SECTION SAMPLES**

First, the experimental setup is validated using samples that were ground to be flat and smooth. The values obtained from these tests are expected to match theoretical values for smooth rectangular channels.

Two different sample sets were machined using a ball end mill of an appropriate diameter. The ball end mill was run perpendicular to the test samples to machine grooves at a shallow depth. A schematic with the dimensions of the channel is provided in Figure 5, with height (ε), pitch (p), constricted separation (bcf), and separation (b) marked.

The smooth channel profile as obtained by a stylus profilometer can be seen in Figure 6. The height of roughness from the grinding process is around ε = 2 µm.

For this experiment, two pitches of 405 µm and 815 µm were employed. The profile of the p = 405 µm sample is shown in Figure 7. It can be seen from the examination of the entire roughness test that the height of the roughness elements are ε = 107 µm. The 815 µm pitch sample had a roughness profile that is shown in Figure 8. Every other roughness element on these samples was machined somewhat differently. Only the tops of every other element were removed, rather than the entire element. On these samples, the height of the roughness elements is ε = 117 µm.

The geometries of the samples are summarized in Table 1. The value of Rₐ is calculated from the raw profilometer data.
by averaging the heights. The value of $F_p$ was then obtained with a simple program that ignored all data above $R_a$ and found the average of the rest of the data. $F_p$ is then the distance from the average roughness to the floor profile line.

**EXPERIMENTAL PROCEDURE**

First, the samples to be tested are put in the test apparatus, which is sealed with small quantities of gray putty tape. The constricted height ($b_{ct}$) of the channel is calibrated at zero by placing two gage blocks of equal size between the samples, then zeroing the micrometers when the samples are touching the gage blocks. After removing the gage blocks, the height is set to the desired distance and secured in the set position with screws. The bypass valve in the water flow loop is then opened the entire way to allow excess flow back into the reservoir. The pump is then turned on. The flow through the test channel is set with the bank of flow meters. When the flow stabilizes, the static pressure at each tap is measured successively using a bank of connecting valves appropriately. The flow rate is then changed over the desired range of Reynolds numbers to obtain the flow characteristics at each hydraulic diameter. This is performed for each set of samples, with the flat samples used to validate the experimental setup.

**RESULTS**

The pump used in this experimentation limited the possible Reynolds numbers that could be run. Hydraulic diameters ranged

<table>
<thead>
<tr>
<th>Pitch ($R_{Sm}$)</th>
<th>$\epsilon$</th>
<th>$R_a$</th>
<th>$F_dRa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>µm</td>
<td>µm</td>
<td>µm</td>
<td>Mm</td>
</tr>
<tr>
<td>Smooth</td>
<td>---</td>
<td>0.314</td>
<td>---</td>
</tr>
<tr>
<td>405 µm</td>
<td>405</td>
<td>107</td>
<td>27.427</td>
</tr>
<tr>
<td>815 µm</td>
<td>815</td>
<td>117</td>
<td>24.187</td>
</tr>
</tbody>
</table>

from $D_h = 424$ µm to $D_h = 1.697$ µm. Reynolds numbers from 200 to 3000 were used. Three sample sets were used: smooth, 405 µm aligned sawtooth roughness, and 815 µm aligned sawtooth roughness.

First, the smooth channel samples were tested for validation. The static pressures at each Reynolds number yielded a plot of pressure versus distance from the channel beginning (m). The theoretical pressure drop can be found with Eq. (2):

$$\Delta P = \frac{2fL\rho V^2}{D_h}$$  \hspace{1cm} (2)

Hydraulic diameter is found simply by Eq. (3):

$$D_h = \frac{4\cdot\text{Area}}{\text{Perimeter}} = \frac{4ab}{2(a+b)}$$  \hspace{1cm} (3)

The actual pressure drop versus distance can be seen against the theoretical in Figure 9 for the smooth samples, without readings from the first 0.0254 m. (Note that for the smooth channel testing, the 405 µm pitch sample tests, and some of the 815 µm pitch sample tests, only the taps beginning after 0.0254 m were used to eliminate the entrance region effect.)
Figure 11  Uncorrected, aligned sawtooth roughness: $p = 405 \, \mu m$, $D_h = 1240 \, \mu m$, $b = 653 \, \mu m$, $a = 12.192$.

Figure 12  Corrected $D_{h,cf}$, aligned sawtooth roughness: $\varepsilon/D_{h,cf} = 0.126$, $p = 405 \, \mu m$, $D_{h,cf} = 847 \, \mu m$, $a = 12.192$ mm.

Figure 13  Uncorrected, aligned sawtooth roughness: $p = 815 \, \mu m$, $D_h = 1241 \, \mu m$, $b = 654 \, \mu m$, $a = 12.192$.

Figure 14  Corrected $D_{h,cf}$, aligned sawtooth roughness: $\varepsilon/D_{h,cf} = 0.144$, $p = 815 \, \mu m$, $D_{h,cf} = 847 \, \mu m$, $a = 12.192$ mm, $b_{cf} = 439 \, \mu m$.

Figure 15  Corrected $D_{h,cf}$, aligned sawtooth roughness: $\varepsilon/D_{h,cf} = 0.252$, $p = 405 \, \mu m$, $D_{h,cf} = 424 \, \mu m$, $a = 12.192 \mu m$, $b_{cf} = 216 \, \mu m$.

Figure 16  Corrected $D_{h,cf}$, aligned sawtooth roughness: $\varepsilon/D_{h,cf} = 0.177$, $p = 405 \, \mu m$, $D_{h,cf} = 605 \, \mu m$, $a = 12.192 \mu m$, $b_{cf} = 289 \, \mu m$. 
For each trial with the smooth channels, the theoretical laminar Fanning friction factor was found using Eq. (4), as given by Kakac et al. [8]:

\[
f = \frac{24}{\text{Re}} (1 - 1.3553\alpha + 1.9467\alpha^2 - 1.7012\alpha^3 + 0.9564\alpha^4 - 0.2537\alpha^5)
\]

Aspect ratio (\(\alpha\)) can be defined as in Eq. (5):

\[
\alpha = \frac{b}{a}
\]

Theoretical turbulent Darcy friction factor is given by Miller [9], and is divided by four to give the theoretical turbulent Fanning friction factor, as is shown in Eq. (6):

\[
f = \frac{0.25 \left[ \log \left( \frac{\varepsilon}{3.7} + \frac{5.74}{\text{Re}^{0.85}} \right) \right]^{-2}}{4}
\]

Finally, the pressure drop given in Eq. (1) is solved for the Fanning friction factor to get the experimental friction factor, as is shown in Eq. (7):

\[
f = \frac{D_h \Delta P}{2L \rho V^2}
\]

Once the friction factor is computed for each Reynolds number, a chart is made of the friction factor versus Reynolds number. An example from the smooth validation tests is shown in Figure 10. It can be seen that the test setup is validated by data collected using the smooth samples. Also visible here is the transition to turbulent flow around the conventional value of \(\text{Re} = 2300\).

Kandlikar et al. [2] proposed the use of constricted hydraulic diameter to predict the laminar frictional factor using the standard formulas presented above and substituting \(b_{cf}\) for \(b\) and \(D_{h,cf}\) for \(D_h\). To find the constricted hydraulic diameter, Eq. (8) is used. Using this value of \(D_{h,cf}\) instead of \(D_h\) the constricted Reynolds number, \(\text{Re}_{cf}\), can also be found.

\[
D_{h,cf} = \frac{4ab_{cf}}{2(a + b_{cf})} = \frac{4a(b - 2\varepsilon)}{2(a + b - \varepsilon)}
\]

Next, the samples with aligned 405 \(\mu\)m roughness are placed in the test apparatus. The tests are run as before. Again, friction factor is plotted against Reynolds number, as shown in Figure 11. It can be seen from this figure that there is a relatively high amount of error between the experimental friction factor and the theoretical values obtained using just \(D_h\). When the data is analyzed using the constricted hydraulic diameter proposed by Kandlikar et al. (2005) [2], a much better fit is obtained for the data, as seen from Figure 12. A transition to turbulence can be seen easily around \(\text{Re} = 800\) with the departure of the data from the trends shown.

Next, the \(p = 815\ \mu\)m samples were placed in the apparatus. The data follows the same trends present in the \(p = 405\ \mu\)m samples. Figures 13 and 14 show the uncorrected and corrected data for one of the gap settings. This same trend was observed for all hydraulic diameters tested. The results of some of the other trials are shown in Figures 15–18 for constricted flow hydraulic diameters of 424 \(\mu\)m, 605 \(\mu\)m, and 1697 \(\mu\)m. Further testing

<table>
<thead>
<tr>
<th>Sample</th>
<th>(D_{h, \mu m})</th>
<th>(D_{h,cf, \mu m})</th>
<th>(\varepsilon/D_{h,cf})</th>
<th>(Re_{cf})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth</td>
<td>424</td>
<td>421</td>
<td>0.00475</td>
<td>2350</td>
</tr>
<tr>
<td>Smooth</td>
<td>565</td>
<td>557</td>
<td>0.00359</td>
<td>2400</td>
</tr>
<tr>
<td>Smooth</td>
<td>847</td>
<td>840</td>
<td>0.00238</td>
<td>N/A</td>
</tr>
<tr>
<td>Smooth</td>
<td>1697</td>
<td>1690</td>
<td>0.00118</td>
<td>N/A</td>
</tr>
<tr>
<td>405</td>
<td>831</td>
<td>424</td>
<td>0.252</td>
<td>410</td>
</tr>
<tr>
<td>405</td>
<td>1005</td>
<td>605</td>
<td>0.177</td>
<td>610</td>
</tr>
<tr>
<td>405</td>
<td>1240</td>
<td>847</td>
<td>0.126</td>
<td>790</td>
</tr>
<tr>
<td>405</td>
<td>2016</td>
<td>1697</td>
<td>0.063</td>
<td>820</td>
</tr>
<tr>
<td>815</td>
<td>868</td>
<td>424</td>
<td>0.276</td>
<td>210</td>
</tr>
<tr>
<td>815*</td>
<td>1003</td>
<td>605</td>
<td>0.193</td>
<td>350</td>
</tr>
<tr>
<td>815*</td>
<td>1276</td>
<td>847</td>
<td>0.144</td>
<td>380</td>
</tr>
<tr>
<td>815</td>
<td>1991</td>
<td>1589</td>
<td>0.074</td>
<td>960</td>
</tr>
</tbody>
</table>

*means run with full manifold.
in the lower Reynolds number region with better experimental uncertainty is suggested.

The transition to turbulent flow is observed to occur at lower Reynolds number with increasing roughness. A summary of the critical Reynolds numbers from this experiment is given in Table 2.

A comparison of the two different pitches on the friction factor is shown in Figure 19. In both cases, the agreement with the theoretical laminar flow friction factors based on constricted flow is seen to be reasonably good, considering the uncertainty bars shown in the figure. However, the two cases show a distinct difference in the transition Reynolds number and in the region following departure from the laminar flow theory. The numerical results of Rawool et al. [6] also indicate the effect of pitch on the friction factor. Further experimental data is needed in this region to draw any further conclusions.

Finally, a critical Reynolds value is determined from the theoretical laminar curve at the location where the flow diverges from the theoretical prediction. This is then plotted in Figure 20 against the relative roughness, using the constricted diameter ($\varepsilon/D_{h,cf}$). Also shown in Figure 20 is the theoretical critical Reynolds number curve given by Eq. (1), as proposed by Kandlikar et al. [5]. There appears to be good correlation of the findings of this paper and previously published findings. It appears that the Kandlikar correlation holds past the $\varepsilon/D_{h,cf} = 0.2$ restriction that was set before. Further modifications may be suggested after a wider set of data become available.

**CONCLUSIONS**

The effect of sawtooth roughness profile on the frictional pressure drop in laminar flow in rectangular minichannels and microchannels has been investigated experimentally. The roughness element heights of 107 $\mu$m and 117 $\mu$m were employed with respective pitches of 405 $\mu$m and 815 $\mu$m. Water was used as the working fluid over the Reynolds number range of 200 to 2400 based on the constricted flow diameter. Theoretical friction factors using the hydraulic diameter calculated from the base dimensions fall considerably below the experimental values. The friction factors calculated with the constricted flow diameter $D_{h,cf}$, as suggested by Kandlikar et al. [2], provided good agreements with the experimental data.

The roughness elements caused early transition from laminar to turbulent flow. The critical Reynolds number decreased with increasing relative roughness. The transition Reynolds number was well correlated with the transition criterion proposed by Kandlikar et al. [5]. However, an effect of the pitch is seen on the transition Reynolds number. Further refinement in this criterion is recommended after a large number of data points become available for different fluids and different roughness structures.

**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>channel base, mm</td>
</tr>
<tr>
<td>b</td>
<td>channel height, $\mu$m</td>
</tr>
<tr>
<td>b$_{cf}$</td>
<td>constricted channel height, $\mu$m</td>
</tr>
</tbody>
</table>
Greek Symbols

α aspect ratio
ε roughness element height, µm
ρ density of the water, kg/m³

REFERENCES


Tim Brackbill is currently a mechanical engineering undergraduate student at the Rochester Institute of Technology. He is looking to perform his master’s degree on surface roughness effects in microchannels at RIT.

Satish Kandlikar is the Gleason Professor of Mechanical Engineering at RIT. He received his Ph.D. from the Indian Institute of Technology in Bombay in 1975 and has been a faculty member there before coming to RIT in 1980. His current work focuses on the heat transfer and fluid flow phenomena in microchannels and minichannels. He is involved in advanced single-phase and two-phase heat exchangers incorporating smooth, rough, and enhanced microchannels. He has published more than 130 journal and conference papers. He is a fellow member of ASME and has been the organizer of the three international conferences on microchannels and minichannels sponsored by ASME. He is a recipient of the Eisenhart Outstanding Teaching award, IBM Faculty award, ASME Best Paper Award, and *Journal of Heat Transfer* Best Reviewer Award. He is the founder of the ASME Heat Transfer chapter in Rochester and founder and first chairman of the E-cubed fair (http://www.e3fair.org), a science and engineering fair for middle school students in celebration of the Engineers’ Week. He is the Heat and History Editor for *Heat Transfer Engineering* and an associate editor for *Journal of Heat Transfer* and *Journal of Nanofluidics and Microfluidics*.