Numerical Investigation of Heat Transfer in Rectangular Microchannels Under H2 Boundary Condition During Developing and Fully Developed Laminar Flow

Study of fluid flow characteristics at microscale is gaining importance with shrinking device sizes. Better understanding of fluid flow and heat transfer in microchannels will have important implications in electronic chip cooling, heat exchangers, MEMS, and microfluidic devices. Due to short lengths employed in microchannels, entrance header effects can be significant and need to be investigated. In this work, three dimensional model of microchannels, with aspect ratios \( a/b \) ranging from 0.1 to 10, are numerically simulated using CFD software tool FLUENT. Heat transfer effects in the entrance region of microchannel are presented by plotting average Nusselt number as a function of nondimensional axial length \( x^* \). The numerical simulations with both circumferential and axial uniform heat flux (H2) boundary conditions are validated for existing data set for four wall heat flux case. Large numerical data sets are generated in this work for rectangular cross-sectional microchannels with heating on three walls, two opposing walls, one wall, and two adjacent walls under H2 boundary condition. This information can provide better understanding and insight into the transport processes in the microchannels. Although the results are seen as relevant in microscale applications, they are applicable to any sized channels. Based on the numerical results obtained for the whole range, generalized correlations for Nusselt numbers as a function of channel aspect ratio are presented for all the cases. The predicted correlations for Nusselt numbers can be very useful resource for the design and optimization of microchannel heat sinks and other microfluidic devices. [DOI: 10.1115/1.4004934]

Keywords: numerical simulation, computational fluid dynamics, microchannel, heat transfer, H2 boundary condition, mathematical formulation

Introduction

With rapid advancements in fabrication of microelectromechanical systems (MEMS), highly compact and effective cooling technologies are required for the dissipation of heat generated by microelectronic devices. Microchannels are the most efficient way of high heat flux removal from small areas. High surface area to volume ratio, smaller volume and mass, and high convective heat transfer coefficient are the features of microchannels that make such high heat flux removal possible. Due to their inherent advantages, heat sinks with small passage dimensions have found applications in automotive heat exchangers, cooling of high power electronic devices, and aerospace heat exchangers.

Use of microchannels as high performance heat sink for cooling of electronic devices was first demonstrated by Tuckerman and Pease [1] in 1981. Fabricating rectangular microchannel heat sinks in silicon wafer, and using water as coolant, they showed that microchannel heat sinks were able to dissipate 790 W/cm² with substrate to coolant temperature difference being kept as 71 °C. Deviations from classical theory of conventional channels has been reported for heat transfer in microchannels by various investigators. In their experiments with rectangular microchannels, Wu and Little [2] found the Nusselt number to be higher than conventional channels. For laminar flow, Choi et al. [3] showed that Nusselt number depends upon Reynolds number unlike conventional channels where Nusselt number was constant for fully developed flow. Adams et al. [4] performed experiments on single phase turbulent flow and found the Nusselt number to be higher than the predicted values for large sized channels. In turbulent regime, Yu et al. [5] found the Nusselt number to be higher than those predicted by Dittus-Boelter equation. In the recent years, Celata et al. [6] and Bucci et al. [7] have also found the Nusselt number to exceed the theoretical predictions for conventional channels.

Many researchers have also found the Nusselt number to lie below the predicted theoretical values for conventional channels. Peng et al. [8] and later Peng and Peterson [9] performed experiments on rectangular microchannels of different dimensions and found the Nusselt number to lie below the predicted values of Dittus-Boelter equation. Based on their experiments, Qu et al. [10] showed the Nusselt number to be lower than the predicted values in the laminar region. Rahman and Gui [11,12] found the Nusselt number to be higher than theory for laminar flow whereas lower in the case of turbulent flow.

There have also been various studies in which heat transfer in microchannels was found to be accurately predicted by macros-scale equations. Lee et al. [13] performed numerical analysis on copper microchannels over a range of Reynolds numbers and found that heat transfer can be predicted using classical theory for conventional sized channels. Conducting experiments on array of microchannels, Harms et al. [14] found the Nusselt number to be similar as the macroscale predictions. Qu and Mudawar [15]...
carried out numerical and experimental work on microchannels in the laminar region and found the heat transfer results to have good agreement with theory. Owaib and Palm [16] performed experiments on microchannels in turbulent region and found the heat transfer results to be accurately predicted by conventional correlations.

Deviations in heat transfer predictions by classical theory have been mainly attributed to surface roughness, entrance and exit effects, axial conduction effects, and thermal and flow boundary conditions. Lee et al. [13] showed that flow in microchannels is generally thermally developing with considerable entrance effects by numerically simulating microchannels with inlet and exit manifolds. Applying correct thermal boundary conditions is also very important in determining accurate heat transfer coefficients. Different thermal boundary conditions generally applied in the fluid domain are: H1 (axially constant wall heat flux and circumferentially constant wall temperature), H2 (uniform wall heat flux, axially, and circumferentially), and T (uniform wall temperature, axially, and circumferentially) [17]. Using generalized integral transform techniques, Aparecido and Cotta [18] analytically solved for bulk temperature and Nusselt number for thermally developing laminar flow in rectangular ducts with uniform wall temperature (T) as the boundary condition. Montgomery and Wibulswas [19] performed numerical analyses of thermally developing flow in rectangular microchannels with constant wall temperature and constant wall heat flux (H1) boundary conditions. Lee and Garimella [20] numerically investigated heat transfer in thermally developing flow in microchannels having aspect ratios ranging from 1 to 10 with the H1 thermal boundary condition.

However, there is little information available for the entrance region effects for smooth microchannels under the H2 boundary condition. Previously, Dhariaya et al. [22] presented a data set for three walls and two opposed walls H2 boundary condition in rectangular microchannels for aspect ratios from 0.1 to 10, and the work is being exclusively extended in this current paper.

**Objectives**

In the present work, rectangular microchannels with aspect ratio ranging from 0.1 to 10 are numerically simulated using commercial Computational Fluid Dynamics (CFD) software FLUENT for constant wall heat flux H2 (constant axial and circumferential wall heat flux) boundary condition and different heated wall configurations. The following five different boundary conditions are investigated: (1) uniform heat flux on all four walls, (2) uniform heat flux on three walls, (3) uniform heat flux on two opposing walls, (4) uniform heat flux on one wall, and (5) uniform heat flux on two adjacent walls. Fully developed Nusselt number (NusselH2) results of four wall heating conditions are compared with the published data of Shah and London [17] for validating the numerical model. In order to study the effects of entrance conditions on Nusselt number, microchannels having inlet and outlet headers are studied. Numerical results of this geometry (with headers) are compared with the results for microchannels of the previous case having abrupt entrance (plain microchannels with no headers). In the next step, rectangular microchannels with aspect ratios ranging from 0.1 to 10 are simulated to generate large data sets for Nusselt number with uniform heat flux H2 boundary condition, circumferentially and axially at three walls, two opposed walls, one wall, and two adjacent walls. In addition, generalized correlations are developed to fit the numerical results for predicting the fully developed Nusselt numbers for rectangular microchannels with different heated wall configurations under the H2 boundary condition.

**Mathematical Formulation**

The steady state continuity and energy equations with associated H2 boundary conditions for all the cases were solved using finite-volume approach under the following assumptions:

1. laminar steady state conditions,
2. incompressible fluid flow,
3. constant fluid properties along the length,
4. negligible effects due to viscous dissipation, and
5. negligible radiative effects.

Table 1 shows the physical dimensions of rectangular microchannels used in numerical simulations with aspect ratios varying between 0.1 and 10. The length (L) and height (b) of rectangular microchannels are kept constant for all the computations and width (a) is varied in accordance with change in aspect ratios as shown in Table 1. Figure 1 represents the cross-sectional area of rectangular microchannel with constant heat flux boundary conditions on all the four walls. The aspect ratio and hydraulic diameter for all the different boundary conditions of rectangular microchannels are defined by Eq. (1)

\[
\alpha = \frac{a}{b} \quad \text{and} \quad D_h = \frac{4 \cdot a \cdot b}{2 \cdot (a + b)} \quad (1)
\]

For numerical simulations of the case with uniform heat flux on three walls of microchannels, wall 1 in Fig. 1 is kept adiabatic. Similarly, for the two opposed wall and one wall heating boundary conditions, walls 3 and 4 and walls 2, 3, and 4 are assumed adiabatic walls, respectively. Moreover, for the two adjacent wall heat flux H2 boundary condition, walls 2 and 3 are given constant heat flux circumferentially and axially, whereas walls 1 and 4 are kept adiabatic. A finite-volume approach is employed to investigate the thermally developing flow regime in microchannels. The local and average Nusselt numbers are calculated numerically as a function of nondimensional axial distance and channel aspect ratio. The heat transfer coefficient and Nusselt number for rectangular microchannels can be calculated using

\[
h = \frac{q''}{T_{w,avg} - T_{f,avg}} \quad \text{and} \quad Nusselt = \frac{D_h \cdot h(x)}{k_f} \quad (2)
\]

In Eq. (2), the fluid average temperature along the length of the microchannel is calculated using the energy balance equation as follows:

**Table 1 Channel dimensions used for numerical simulations for each cases of rectangular microchannel with varying aspect ratios of 0.1 to 10**

<table>
<thead>
<tr>
<th>(\alpha) (a/b)</th>
<th>a ((\mu m))</th>
<th>b ((\mu m))</th>
<th>L (mm)</th>
<th>(D_h) ((\mu m))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>150</td>
<td>1500</td>
<td>100</td>
<td>272.7</td>
</tr>
<tr>
<td>0.25</td>
<td>150</td>
<td>600</td>
<td>100</td>
<td>240.0</td>
</tr>
<tr>
<td>0.33</td>
<td>150</td>
<td>450</td>
<td>100</td>
<td>225.1</td>
</tr>
<tr>
<td>0.50</td>
<td>150</td>
<td>300</td>
<td>100</td>
<td>200.0</td>
</tr>
<tr>
<td>0.60</td>
<td>150</td>
<td>250</td>
<td>100</td>
<td>187.5</td>
</tr>
<tr>
<td>0.75</td>
<td>150</td>
<td>200</td>
<td>100</td>
<td>171.4</td>
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<td>150</td>
<td>100</td>
<td>150.0</td>
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<td>150</td>
<td>30</td>
<td>100</td>
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<tr>
<td>10</td>
<td>150</td>
<td>15</td>
<td>100</td>
<td>27.3</td>
</tr>
</tbody>
</table>

![Fig. 1 Cross-sectional area of rectangular microchannel with uniform heat flux on all four walls](image_url)
\[ T_{f,a} = \left( \frac{q'' \cdot P_h \cdot x}{m \cdot C_p} \right) + T_{f,in} \]  

(3)

The temperature distribution around the rectangular channel walls is not uniform under \( H_2 \) boundary condition. The temperature is the highest at the corners of the wall whereas it is the lowest at the center-line on the walls subjected to \( H_2 \) boundary condition. Representations of wall temperature profile in a fully developed region for a rectangular channel having an aspect ratio of 0.5 (width—300 \( \mu \text{m} \) and height—150 \( \mu \text{m} \)) are shown in Figs. 2–4. It can be seen that the wall temperature is not constant circumferentially along the rectangular channel. Therefore, heat transfer estimation depends upon the location of wall temperature measured along the circumference. Figures 3 and 4 show a variation of temperature along the width of 300 \( \mu \text{m} \) and height—150 \( \mu \text{m} \), respectively in a fully developed region. In order to estimate heat transfer coefficient accurately, several points are taken along the temperature profile and an average wall temperature is estimated. The average wall temperature, obtained from the computational results, is calculated using temperature at different nodes on each heated walls along the length of the microchannel. Initially, several points are computed along the heated walls to estimate the average wall temperature. Thereafter, to simplify the calculations, five nodes are selected on the heated wall, which would able to predict the average wall temperature with maximum error of 0.06%.

Figure 2 shows the representation of the five-node method used to average the wall temperature profile for wall 1. Similarly, using the five nodes on the wall, the average temperature is determined for each heated wall. As discussed above, for each wall of the rectangular microchannel under constant wall heat flux, temperature peak is observed at corners and hence the temperature values at each corner nodes are halved while calculating the average temperature for each wall (as represented by Eq. (4) for wall 1)

\[ T_1 = \left( \frac{[(0.5 \ast T_{N2}) + T_{N2} + T_{N3} + T_{N4} + (0.5 \ast T_{N5})]}{4} \right) \]  

(4)

where, subscripts \( N1, N2, \ldots, N5 \) represents the different nodes which are used to calculate the average wall temperature for different heating configurations.

The average wall temperature is defined differently based on different wall boundary conditions. Following are the equations used to estimate average wall temperature as a function of temperature variation along each heated wall and channel aspect ratio:

For four wall uniform heat flux boundary condition

\[ T_{w,avg} = \frac{T_w \cdot x + T_b}{1 + x} \]  

(5)

where \( T_w = \frac{T_1 + T_2}{2} \) and \( T_b = \frac{T_3 + T_4}{2} \)

For three-wall uniform heat flux boundary condition

\[ T_{w,avg} = \frac{T_w \cdot 2x + T_b}{1 + 2x} \]  

(6)

where \( T_w = \frac{T_1 + T_2}{2} \) and \( T_b = T_3 \)

For two opposed wall uniform heat flux boundary condition

\[ T_{w,avg} = \frac{T_1 + T_2}{2} \]  

(7)

For one wall uniform heat flux boundary condition

\[ T_{w,avg} = T_1 \]  

(8)

For two adjacent wall uniform heat flux boundary condition

\[ T_{w,avg} = \frac{(T_2x + T_3)}{(1 + x)} \]  

(9)

Model Description

The fluid flow and heat transfer effects in rectangular microchannels with varying aspect ratio from 0.1 to 10 are investigated using commercial CFD software, FLUENT. GAMBIT is used as the presolver software for designing geometric models, grid generation, and boundary definition. Water enters the rectangular microchannels with a fully developed velocity profile at an inlet temperature of 300 K. The numerical simulation is initially performed for rectangular microchannels with four side constant wall heat flux boundary condition. This case is tested for different aspect ratios of 0.1, 0.25, 0.33, 0.5, and 1, and the results are verified.
by the scheme proposed by Shah and London [17] which are the only published data available for NuH2 for varying aspect ratios are for four wall heating conditions. Following the validation, data sets under uniform heat flux (H2) boundary condition for three walls and two walls. One wall and two adjacent walls are numerically generated. The rectangular channels used in Shah and London’s investigation have abrupt entrance type as shown in Fig. 5, and in general, the channels always have some type of headers for practical applications. Hence, new geometric models are designed with same rectangular microchannel dimensions as earlier along with a smooth entrance type (with inlet and outlet headers) as shown in Fig. 6. This comparison helps us to understand the effect of entrance types on heat transfer characteristics in entrance region.

The presolver GAMBIT is used for grid generation in microchannels. Figures 5 and 6 also show the meshed geometry of rectangular channel having aspect ratio of 1 with abrupt entrance type and smooth entrance type, respectively. Hexwedge cooper scheme is used for generating mesh in rectangular geometries. The mesh spacing is kept as low as possible to justify the conformity of obtained results. Finer meshes are created to predict the fluid flow and heat transfer effects more accurately. Each of the rectangular geometries possess approximately $2 \times 10^6$ grid elements and the processing time for each simulation is noted to be around 7–8 h (Intel Core 2 Duo processor). Grid independence test is also carried out in order to ensure the best mesh spacing for the geometrical model.

The numerical simulations are performed using commercial CFD software package FLUENT. Reynolds number is kept constant as 100 for all cases in order to confirm laminar flow through rectangular microchannels. Pressure-based solver is used for simulating to achieve steady state analysis. The semi-implicit method for pressure-linked equations (SIMPLE) algorithm is used for introducing pressure into the continuity equation. The energy equation is activated during the analysis to predict heat transfer effects in microchannels and check the effect of entrance type on heat transfer. The flow momentum and energy equations are solved with a first-order upwind scheme. The simulations are performed for a convergence criterion of $10^{-6}$.

**Results and Discussions**

Initially, the simulations are performed for H2 boundary conditions on all four walls for different aspect ratios from 0.1 to 1. The results of fully developed Nusselt number are then compared with the values reported by Shah and London [17]. Further simulations are performed on all the possible cases of wall boundary conditions; four wall, three wall, two opposed wall, one wall, and two adjacent wall for different aspect ratios for two different cases of abrupt and smooth entrance types. The results from these simulations are used to predict the effect of entrance type on thermally developing entrance region in rectangular microchannels. Large data sets are generated for thermally developing Nusselt number for three wall, two wall, one wall, and two adjacent wall uniform heat flux boundary condition (H2) for whole range of aspect ratio varying from 0.1 to 10. The aspect ratio is defined as $a/b$ for all the numerical simulations under H2 boundary condition with different heated channel wall configurations.

**Validation of Numerical Model.** The first objective is to validate the results of numerical simulation with available published data of Shah and London [17] for uniform heat flux boundary condition (H2) on all the four walls of rectangular microchannel. Table 2 shows the fully developed Nusselt number found from numerical simulation results for varying aspect ratios of 0.1, 0.25, 0.33, 0.50, and 1 with H2 boundary condition. It can be seen from the Table 2 that fully developed Nusselt number found from numerical simulation results are in good agreement with the fully developed Nusselt numbers from Shah and London [17]. For rectangular channels, the values of fully developed Nusselt numbers depend mainly on two factors: channels aspect ratio and wall thermal boundary conditions. In general, the most commonly used thermal boundary conditions in literature are T, H1, and H2. Figure 7 shows the comparison of results obtained for rectangular ducts for fully developed Nusselt numbers for different thermal boundary conditions: $N_uT$ (constant wall temperature), $N_uH1$ (constant circumferential wall temperature), and $N_uH2$ (constant wall heat flux, circumferentially, and axially). The plot shows that the numerical model used to predict NuH2 for four walls with varying aspect ratio in current work is in good agreement to published data by Shah and London [17]. Moreover, it can be observed that H1 and T thermal boundary conditions tend to follow exponential graph with increase in aspect ratios, whereas there is a significant change in thermal behavior under H2 boundary condition. For rectangular ducts with varying aspect ratios, the H2 boundary condition follows a linear trend with very little change in values of fully developed Nusselt number.

**Effects of Entrance Type on Nusselt Number.** Figures 8 and 9 are the plots showing variation in Nusselt number along the length of the microchannel having uniform heat flux on four walls and three walls, respectively. These cases are plotted only for the first 10 mm length of rectangular microchannel in order to see the effect of entrance headers on Nusselt number. Results for microchannels with and without abrupt entrance are shown for two different aspect ratios of 0.1 and 0.5. From these plots, it can be seen that the entrance type does not have any significant impact on the Nusselt number in the entrance region of thermally developing flow. Similar results are observed for two wall and one wall H2 boundary conditions as well.

<table>
<thead>
<tr>
<th>Aspect ratio, a/b</th>
<th>From Ref. [17]</th>
<th>Current numerical work</th>
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</thead>
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<tr>
<td>0.1</td>
<td>2.950</td>
<td>2.924</td>
</tr>
<tr>
<td>0.25</td>
<td>2.940</td>
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<tr>
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<td>3.301</td>
</tr>
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</table>

Fig. 5 Schematic and meshing of geometric model with abrupt entrance type

Fig. 6 Schematic and meshing of geometric model with smooth entrance type

Table 2 Comparison of $N_uH2,fd$ of numerical work with proposed values by Shah and London [17]
Results for H2 Boundary Condition. The next step in this work is to generate large data set for Nusselt number in thermally developing flow with H2 boundary condition. Figures 10–12 are the plots showing Nusselt number along the nondimensional length of a microchannel with uniform heat flux on three walls, two walls, and one wall, respectively. As expected, the graphs show that the Nusselt number is very high at the beginning of the entrance region of microchannel and thereafter decreases exponentially to the fully developed Nusselt number.

The fully developed Nusselt numbers obtained for four walls and three walls H2 boundary condition seem to have an increasing trend with an increase in the channel aspect ratio. A similar trend is observed in the plots of Shah and London [17] for the four wall cases. This trend is also similar to that observed by investigators for the three wall case for H1 and T boundary conditions with only difference in the values of the fully developed Nusselt number [23].

Moreover, for the three walls H2 boundary condition, the values of fully developed Nusselt number converge and are approximately around 3 for all aspect ratios from 0.1 to 10. Also, the trend of Nusselt number along the nondimensional length of microchannels for the three wall H2 condition is similar to that
observed for the four wall cases. The difference in the results of fully developed laminar flow Nusselt number in the three wall uniform heat flux H2 boundary condition as compared to H1 and T thermal boundary conditions are only due to the fact that the longer side (i.e., wall 1) is kept as adiabatic in current work. In fact, all the cases with varying wall boundary conditions tend to follow the same pattern with respect to the dimensionless thermal length. The values of Nusselt number did not seem to vary much for the whole range of channel aspect ratio for the three wall H2 boundary condition as seen in Fig. 10 compared with other cases subjected to H2 boundary condition.

As seen in Figs. 11 and 12, the fully developed Nusselt numbers for both cases have a decreasing trend with an increase in the aspect ratio for the two walls and one wall H2-constant wall heat flux boundary condition. This trend is similar to the results observed for H1 and T boundary conditions for the two walls and one wall transferring heat [23]. Similar trend is observed for the case of two adjacent heated walls of a rectangular microchannel.
In order to study the heat transfer effects in the entrance region of microchannels, Nusselt number is plotted as a function of non-dimensional length $x^*$ for all the cases. Figures 10–12 shows result for rectangular microchannels for the entire range of aspect ratios investigated under the H2 boundary condition for one or more walls transferring heat. From these figures, it can be seen that a larger channel aspect ratio will have a larger dimensionless thermal entrance length $x_a$ and its value decreases with simultaneous decrease in aspect ratio. The results show that the value of Nusselt number starts high and decreases rapidly along the length of the microchannel.

Generated Numerical Data Set for Fully Developed Laminar Flow Nusselt Number Under H2 Boundary Condition. Table 3 represent the values of fully developed laminar flow Nusselt number found from the results of numerical simulations with different heated wall configurations for rectangular microchannels for the
entire range of aspect ratios. Data sets are generated for constant wall heat flux, both circumferentially and axially, by performing CFD computations for wide range of cases. The numerical values for fully developed Nusselt number for all the possible cases can prove to be a good tool to provide an estimation of heat transfer coefficient used for the design and optimization of microchannel heat sinks and other major microfluidic devices. Figure 13 represents the approximation of fully developed Nusselt number with one or more walls transferring heat. Based on the predicted results, NuH2 approximation was performed to estimate the correlation with respect to channel aspect ratio, within 0.8%. The correlations were curve-fitted to the results simulated with a correlation coefficient (R2 \approx 1) by the following:

For four walls H2 boundary condition (0.1 \leq \alpha \leq 10),

\[ \text{Nu}_{H2, \text{fd}} = \left( 5.0523 \times 10^{-5} \right) \alpha^6 - (1.5144 \times 10^{-4}) \alpha^5 + (1.6317 \times 10^{-3}) \alpha^4 - (6.6889 \times 10^{-2}) \alpha^3 - (1.8511 \times 10^{-2}) \alpha^2 + (0.8680) \alpha + 2.2808 \]

(11)

For three walls H2 boundary condition (0.1 \leq \alpha \leq 10),

\[ \text{Nu}_{H2, \text{fd}} = \left[ (8.8270 \times 10^{-5}) \alpha^6 - (3.1693 \times 10^{-3}) \alpha^5 + (4.5800 \times 10^{-2}) \alpha^4 - (0.3447) \alpha^3 + (1.4805) \alpha^2 - (3.8842) \alpha + 7.1743 \right] \]

(12)

For two walls H2 boundary condition (0.1 \leq \alpha \leq 10),

\[ \text{Nu}_{H2, \text{fd}} = \left[ (0.3816) \alpha + 2.886 \right] \]

(10)

For one wall H2 boundary condition (0.1 \leq \alpha \leq 10),

\[ \text{Nu}_{H2, \text{fd}} = \left[ (0.3816) \alpha + 2.886 \right] \]

(10)

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**Figure 13** Rectangular ducts: NuH2,fd for fully developed laminar flow and for one or more walls transferring heat under uniform heat flux H2 boundary condition

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**Table 3** Rectangular ducts: NuH2,fd for fully developed laminar flow and for one or more walls transferring heat under uniform heat flux H2 boundary condition

<table>
<thead>
<tr>
<th>Aspect ratio, ( \alpha (a/b) )</th>
<th>4-wall BC</th>
<th>3-wall BC</th>
<th>2-opposite wall BC</th>
<th>1-wall BC</th>
<th>2-adjacent wall BC</th>
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<tr>
<td>0.10</td>
<td>2.924</td>
<td>2.463</td>
<td>6.803</td>
<td>5.036</td>
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<td>4.625</td>
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</table>
For two adjacent walls H2 boundary condition (0.1 ≤ z ≤ 1),
\[ \text{N}u_{H2,2adjwalls} = \left[ (1.6980 \times 10^{-4})z^6 - (5.7050 \times 10^{-3})z^5 + (7.5552 \times 10^{-2})z^4 - (0.5054)z^3 + (1.8448)z^2 - (3.8559)z + 5.2720 \right] \]

All the above correlations for fully developed Nusselt number have a very small average residual and an excellent correlation coefficient. The correlations are generalized using a statistical analysis tool, and regression analysis is performed with an extremely low residual tolerance over a large number of numerically simulated values. Figure 13 clearly depicts the trend followed by each boundary condition with varying microchannel aspect ratios.

Conclusions
The numerical simulations are carried out with uniform wall heat flux boundary condition for five different cases; four walls, three walls, two walls, two opposing walls, one wall, and two adjacent walls heating. The numerical model for four wall H2 boundary condition is validated with published data by Shah and London [17] and the results obtained for microchannels are in good agreement with their findings. The effect of entrance type on the heat transfer in the thermally developing entrance region is studied. The result predicts significant effect of entrance type on Nusselt numbers for all the cases studied. Large data sets are generated for wide range of aspect ratios varying from 0.1 to 10 for rectangular microchannels for all cases with uniform heat flux wall configurations. The numerical data obtained for four wall H2 boundary conditions follow a linear trend as compared to an exponential trend observed under H1 and T boundary conditions. Moreover, the values of fully developed laminar flow Nusselt numbers for H1 and T thermal boundary condition show an exponentially decreasing trend with an increase in the aspect ratio; whereas under H2 boundary condition, it follows a linear increasing trend. In general, the heat transfer results for four wall H2 boundary condition did not vary much with an increase in aspect ratio, contrary to that observed by Schmidt [23] for H1 and T boundary condition. On the other hand, fully developed Nusselt number values for three walls, two walls, one wall, and two adjacent walls H2 boundary condition in rectangular microchannels for aspect ratios ranging from 0.1 to 10 followed the same trend as compared to H1 and T boundary conditions. Higher channel aspect ratios predicted larger values of nondimensional thermal entrance length, \( x_{n,fd} \). A comprehensive numerical data set is generated for heat transfer aspects in rectangular microchannels under more practical H2 boundary condition. In addition, generalized correlations are proposed for all the possible heated wall configurations to estimate fully developed H2 laminar flow Nusselt number value as a function of channel aspect ratio in rectangular microchannels to represent the numerical results within 0.8% maximum deviation.

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Nomenclature
- \( a \) = height of rectangular microchannel (\( \mu m \))
- \( b \) = width of rectangular microchannel (\( \mu m \))
- \( L \) = length of rectangular microchannel (mm)
- \( D_h \) = hydraulic diameter (\( \mu m \))
- \( m \) = mass flow rate (kg/s)
- \( A_s \) = cross-sectional area of microchannel (m²)
- \( A_h \) = area of heated walls (m²)
- \( P \) = perimeter of microchannel (m)
- \( P_h \) = perimeter of heated walls (m)
- \( T \) = temperature (K)
- \( Q \) = heat transfer rate (W)
- \( q^* \) = total heat flux (W/m²)
- \( C_p \) = specific heat of fluid (J/kg-K)
- \( k \) = thermal conductivity of fluid (W/m-K)
- \( h \) = heat transfer coefficient (W/m²-K)
- \( Re \) = Reynolds number
- \( Nu \) = Nusselt number
- \( x_{th} \) = thermal entrance length (m)
- \( x^* \) = nondimensional axial thermal length (x/(DhRePr))

Subscripts
- \( w \) = wall
- \( f \) = fluid
- \( H2 \) = uniform heat flux boundary condition
- \( fd \) = fully developed flow
- \( \text{avg} \) = average
- \( x \) = local
- \( m \) = mean

Greek
- \( \alpha \) = channel aspect ratio (refer to Fig. 1)
- \( \rho \) = density of water (kg/m³)
- \( \mu \) = dynamic viscosity (N-s/m²)

References


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