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Effects of Structured Roughness on Fluid Flow at the Microscale Level

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It has been well established that there are no differences between microscale and macroscale flows of incompressible liquids. However, surface roughness has been known to impact the transport phenomena. This work aims to systematically quantify the effect of structured roughness geometries on friction factor in the laminar and turbulent flows as a precursor to the detailed heat transfer studies on these geometries. Experiments were conducted by varying the pitch (150–400 µm) and height (36–131 µm) of transverse rib roughness structures in rectangular channels such that the pitch-to-height ratio ranged from 2 to 8. The channel width was fixed at 12.70 mm and the length at 152.4 mm, while the channel gap was varied (230–937 µm). Tests were conducted over a Reynolds number range of 5–3400. The results are compared to the existing models, which do not account for specific roughness features such as pitch and height. A theoretical model is developed to predict the effect of roughness pitch and height on pressure drop along the channel length. Validation of the proposed theory is carried out by comparing the predictions with the experimental results. The model and the experimental results provide an understanding of the effect of two-dimensional structured roughness on the frictional losses in fully developed laminar flow.

INTRODUCTION

Surface roughness is known to have an effect on the fluid transport phenomena at the macroscale level as well as at the microscale level. Uniform roughness has been studied extensively in the past and its effects can be predicted using the existing models, such as the constricted flow method, as demonstrated by Young et al. [1]. However, the effect of structured roughness features, such as uniformly spaced ribs of different shapes and sizes, is not well understood. In an effort to identify surface features that are attractive from a heat transfer standpoint, it is essential to identify friction factor characteristics before evaluating their heat transfer performance.

Roughness features smaller than the boundary-layer thickness in internal flows typically have negligible influence in the laminar flow regime, and may be significant in turbulent flow if the feature height is larger than the viscous boundary layer [2]. At the microscale level, channel walls are very closely spaced, and the height of individual asperities may be on the order of the channel dimensions. This large roughness can then lead to greater frictional losses in the laminar regime and earlier transition to turbulence.

Surface roughness is conventionally characterized through the use of average amplitude parameters that are in common use because they are simple to obtain and provide a sufficient description of surface roughness in macroscale applications. These are, however, insufficient in representing structured or periodic roughness. To understand the effect of height, slope, spacing, etc. of roughness on fluid flow, it is useful to consider structured roughness, which allows both control and measurement of those critical aspects of surface geometry.

LITERATURE REVIEW

In recent literature, researchers have reported that friction factors in noncircular geometries begin to differ from established predictions as the channel size decreases to the microscale level. Prior works on liquid flow in microchannels have reported significant increases [3–6] or decreases [7, 8] in friction factors from conventional laminar theory, while others reported negligible deviations [9–13]. In addition to these, other authors...
have reported increases in friction factors for some channel geometries or aspect ratios, but decreases for others [14, 15]. The major reasons for these experimental differences are attributable to errors in measurement of geometric parameters, such as the channel size, and lack of proper accounting for entrance region effects. Many of these authors also highlighted the role of wall roughness in their observed deviations.

The Moody diagram, used to predict frictional losses in pipes, is based on experimental data for uniformly rough surfaces. The original data of Nikuradse [16], for example, was obtained by sifting and resifting sand such that the grain diameters were approximately the same, and then using a lacquer to adhere the sand to the inner surfaces of pipes. Since then, authors have attempted to represent surface roughness, even structured roughness, with an “equivalent sand grain roughness.” Such an approach to roughness characterization is appropriate for uniform roughness, but is questionable for structured roughness because of the inherent differences between uniform roughness and repeated two-dimensional structured roughness.

Menezes et al. [17] roughened steel surfaces with wet and dry emery paper in various patterns in order to correlate existing roughness parameters with the coefficient of friction for each surface. He found that the performance of each plate was independent of the average roughness, but correlated well with the mean slope $\Delta_a$ of the profile. This parameter could potentially be combined with an amplitude or hybrid parameter in order to predict hydraulic performance.

Kandlikar et al. [18] proposed the use of a constricted parameter in estimating pressure drop in roughened microchannels. The constricted parameter $\epsilon_Fp$ is a function of the average roughness $R_s$, but correlated well with the mean slope $\Delta_a$ of the profile. This parameter could potentially be combined with an amplitude or hybrid parameter in order to predict hydraulic performance.

Structured roughness is obtained by adding or removing material from a surface in such a way that a two- or three-dimensional pattern arises. This artificial type of roughness was studied at least as far back as the 1920s with Hopf and Fromm’s sawtooth style two-dimensional roughness [19, 20], followed by Schlichting’s three-dimensional arrays of spheres, cones, and angled roughness [21].

As there is no “universal” parameter for structured roughness, many researchers resort to using roughness height values and roughness ratios, such as the relative roughness, $\epsilon/D$, in order to compare the effects of different roughness geometries on fluid flow. Sams [22] experimented with structured roughness in internal flow by threading and cross-threading macroscale pipes ($D \approx 12.7$ mm). He concluded that the conventional “relative roughness” concept is not sufficiently representative of the effects of structured roughness on hydraulic performance.

Numerical simulations have been employed for assessing structured roughness effects in microchannel flow by a number of researchers, either through the use of commercial CFD software or by programming finite difference methods manually. Rawool et al. [23] simulated laminar air flow in microchannels possessing two-dimensional transverse rib roughness, and systematically varied the roughness cross section from triangular to trapezoidal to rectangular, and also varied the rib height and pitch. The authors found that roughness pitch plays a definite role in fluid flow, in that the friction factor increases as roughness elements are brought closer together. Friction factors were also found to be greater for triangular and rectangular geometries, and lower for trapezoidal roughness. These results are in agreement with the numerical simulations of Wang et al. [24] and Sun and Faghri [25], reinforcing the idea that the roughness geometry should be taken into consideration.

Coleman et al. [26] experimentally and numerically assessed the effect of transverse rib roughness pitch-to-height ratios, $\lambda/h$, on turbulent flow and identified a “transitional” roughness, at $\lambda/h \approx 8$, as having the most predominant effect on fluid flow. The authors reported that values of $\lambda/h < 5$ indicate closely spaced ribs, $d$-type roughness, or skimming flow, while $\lambda/h > 5$ indicates isolated roughness elements, $k$-type roughness, or interactive flow. In the extremes where $\lambda/h$ is significantly greater or less than 5, the roughness effect is expected to diminish.

**Objectives**

The objectives of this study are (a) to develop a theoretical model to predict the effect of surface features on the friction factor by systematically varying the roughness pitch and height in the laminar and turbulent regions and (b) to obtain experimental results for friction factor in rough channels to validate the theoretical model.

**THEORETICAL APPROACH**

Figure 1 shows a schematic of the flow in a rough tube with artificial roughness. The channel separation $b$ is significantly less than the length $L$, and the channel aspect ratio $(b/a)$ is small; the flow in this geometry is therefore comparable to flow between two infinite parallel plates. The boundaries for the flow are given by the transverse rib roughness, represented by the functions $f(x)$ and $g(x)$ for the lower and upper walls, respectively. This periodic roughness is described by its pitch $\lambda$, and height $h$.

![Figure 1](image-url)
The lubrication approximation was applied by assuming that the slope of the trajectory of fluid elements is small. That is, velocity \( w \) in the \( z \) direction is significantly less than velocity \( u \) in the \( x \) direction. Figure 2 provides a visual representation of this assumption, which is also referred to as the small slope approximation, as the slope of the boundaries is assumed to be small at every point, such that \( \frac{\partial(x)}{\partial x} \ll 1 \) and \( \frac{\partial(z)}{\partial x} \ll 1 \).

**Lubrication Approximation**

Application of the lubrication approximation makes no change to the continuity equation, but reduces the Navier–Stokes (N-S) equations to the following form:

\[
N - S \begin{cases} 
  x \text{- component: } \frac{\partial P}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2} \\
  y \text{- component: } \frac{\partial P}{\partial y} = \rho g
\end{cases}
\]  

(1)

Integrating the \( x \) component of the simplified N-S equations and applying the no-slip boundary conditions yields the following two-dimensional velocity profile:

\[
u = \frac{1}{2\mu} \frac{\partial P}{\partial x} (z - g(x))(z - f(x))
\]  

(2)

Integration of the continuity equation across the channel separation brings about a volumetric flow rate per unit depth relation, which is used in conjunction with the flow velocity equation to obtain the following differential pressure–flow relation:

\[
\frac{\partial P}{\partial x} = -12\mu \frac{Q}{a} \frac{1}{(g(x) - f(x))^3}
\]  

(3)

Integrating this differential equation along a specified length results in the following equation for pressure drop:

\[
P_L - P_0 = -12\mu \frac{Q}{a} \int_0^L \frac{1}{(g(x) - f(x))^3} \, \partial x
\]  

(4)

Equations for \( g(x) \) and \( f(x) \) are used to evaluate the integral and thereby predict the pressure drop along the length of the rough channel. These two equations are the wall functions that represent structured two-dimensional roughness, so this approach is referred to as the wall function method.

**Roughness Representation**

The form of the periodic roughness chosen for this study was obtained through curve fitting of existing two-dimensionally rough surfaces:

\[
f(x) = h \cos^{p} \left( \frac{\pi x}{\lambda} \right) - \frac{b}{2}
\]  

(5)

where \( h \) is the height of roughness elements, \( \lambda \) is the pitch or peak-to-peak spacing, the cosine power \( p \) is a positive even number, which controls the slope of the peaks, and \( b \) is the root channel separation measured from the valley-to-valley of the opposing surfaces. The opposing wall \( g(x) \) is the negative of Eq. (5) (Figure 1). To evaluate the effect of alignment of roughness peaks on fluid flow, a phase-shift variable may be added into the cosine function of one of the walls.

In prior work, Kandlikar et al. [18] proposed a method for assessing a surface of significant random roughness or structured roughness. This roughness factor, referred to as the constricted flow parameter, is a function of the average parameters typically obtained through surface analysis:

\[
\varepsilon_{FP} = R_p + F_p = (\text{Max} - R_a) + F_p
\]  

(6)

The floor profile, \( F_p \), is defined as the average of all points below the mean line, \( R_a \).

The constricted parameter was used to develop a constricted channel separation \( b_{cf} \) for use in internal flow calculations, calculated as:

\[
b_{cf} = b - 2\varepsilon_{FP}
\]  

(7)

where \( b \) is the root separation, measured valley to valley. By this definition, any amplitude parameter could be used to “constrict” the channel separation. As discussed previously, amplitude parameters and the concept of “relative roughness” do not account for the two-dimensional roughness elements [22].

In previous studies [1, 27, 28], the constricted parameters were successfully used to fit experimental data for uniform surface roughness to laminar theory for smooth ducts. These parameters were able to predict the friction factors reasonably well, but the model was unable to account for the effect of variation in pitch. The new model incorporates these effects through the wall function profiles.

**Application of Theory**

In order to test the wall function method (Eq. (4)), and to compare it with conventional laminar theory, the pressure-loss form of the Darcy–Weisbach equation was used to calculate experimental laminar friction factors:

\[
f = \frac{\Delta P}{L} \frac{D_h}{2p} \left( \frac{Q}{A} \right)^2
\]  

(8)

where \( D_h \) is the hydraulic diameter, calculated as four times the cross-sectional area divided by the wetted perimeter.
For comparison with conventional laminar theory, laminar friction factors in smooth, rectangular ducts were evaluated from the following correlation by Kakac et al. [29]:

\[
f = \frac{24}{\text{Re}} \left(1 - 1.3553\alpha + 1.9467\alpha^2 - 1.7012\alpha^3 + 0.9564\alpha^4 - 0.2537\alpha^5\right)
\]

where the aspect ratio was calculated as the channel width divided by channel height, and the Reynolds number was calculated as:

\[
\text{Re} = \frac{\rho Q D_h}{\mu A}
\]

**EXPERIMENTAL APPROACH**

Experiments were performed with a series of structured roughness test pieces to systematically evaluate the effects of roughness pitch and height on friction factor. Roughness elements were designed and manufactured matrix in stainless steel via wire electrical discharge machining (EDM). Multiple sample lengths of each surface were scanned on a laser confocal microscope for profile evaluation, with a total of 10 scans averaged for each roughness type. A sample three-dimensional (3-D) image from the \(\lambda/h = 3\) surface is shown in Figure 3 along with the roughness profile.

The profile equation (Eq. (5)) was fitted to the two-dimensional surface data for each roughness test piece. Figure 4 shows a sample of the result from curve fitting the \(\lambda/h = 3\) surface shown in Figure 3. Measured roughness geometry is summarized in Table 1.

The results of curve fitting and the study of roughness parameters showed that the cosine power for the curve fits of three of the five surfaces differed from the designed values, and no standard roughness parameter was able to represent the cosine power. The “average step height,” noted as \(R_{\text{step}}\), unique to the laser confocal microscope, provided an ideal fit to the peak height of each surface and compared well with the designed height values. In addition, the mean spacing of peaks, \(R_{\text{SM}}\), not only provided an excellent fit to the pitch of the surface data, but was also within less than 1 \(\mu\)m of the designed pitch for every profile. These two parameters were used for the experimental and theoretical calculations, and are shown in Table 1 with \(\lambda = R_{\text{SM}}\) and \(h = R_{\text{step}}\). The values for cosine power, \(p\), were obtained through curve fitting of the measured profiles, by the least sum of squares method.

**Channel Assembly**

The test fixture, shown in Figure 5, consists of a base block and inlet and outlet blocks. The base was drilled with 15 pressure tap holes for differential pressure transducers. The outlet block was fixed rigidly to the base and aligned with the circular hole in the gauge block. The inlet block slid along a slot in the base, while a custom clamping device was used to provide axial compression to seal the inlet and outlet blocks.

All interfaces between the microchannel assembly and the test fixture were sealed with an adhesive-backed silicon gasket. Precision gauge blocks were used to set the channel separation and align the roughness peaks. The hole through the gauge block allowed for a smooth transition from circular tubing to the rectangular channel, thereby alleviating cavitation issues identified in prior versions of the test set. A matching pair of test pieces was assembled with a matching pair of gauge blocks to form a rectangular channel of constant cross section, with channel widths ranging from approximately 100 \(\mu\)m to 1 mm.
Table 1  Measured roughness geometry and parameters

<table>
<thead>
<tr>
<th>Pitch, λ (µm)</th>
<th>Height, h (µm)</th>
<th>Power, p</th>
<th>Pitch-to-height ratio, λ/h</th>
<th>Constricted parameter, εP (µm)</th>
<th>RMS roughness, Rq (µm)</th>
<th>Arithmetic mean, Ra (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>249.5</td>
<td>131.0</td>
<td>4</td>
<td>2</td>
<td>85.5</td>
<td>29.8</td>
<td>25.4</td>
</tr>
<tr>
<td>149.8</td>
<td>49.6</td>
<td>4</td>
<td>3</td>
<td>27.7</td>
<td>9.9</td>
<td>8.6</td>
</tr>
<tr>
<td>250.2</td>
<td>49.4</td>
<td>12</td>
<td>5</td>
<td>28.3</td>
<td>9.7</td>
<td>8.5</td>
</tr>
<tr>
<td>400.4</td>
<td>49.6</td>
<td>26</td>
<td>8</td>
<td>33.8</td>
<td>10.8</td>
<td>9.0</td>
</tr>
<tr>
<td>250.6</td>
<td>35.6</td>
<td>8</td>
<td>7</td>
<td>17.5</td>
<td>5.8</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Channel height was fixed at 12.7 mm and length at 152.4 mm. Channel separation values were chosen to meet the criteria $b/L \ll 1$ and $\alpha \ll 1$.

**Test Loop**

Testing was performed using degassed distilled water in a closed test loop, shown in Figure 6. A bank of flowmeters was employed to measure separate ranges of flow rate, with sufficient overlap between them. An array of 15 pressure transducers, denoted by the P in Figure 6, spaced 6.53 mm apart, measured gauge pressure along the channel length. Thermocouples, labeled T, situated at the inlet and outlet measured bulk fluid temperature for interpolation of fluid properties. The micropump was controlled through a custom LabVIEW program that also recorded data from all sensors.

The bias error inherent in each sensor and measurement device was combined with the error associated with repeatability to provide a total uncertainty for the friction factor and Reynolds number for laminar and turbulent flow, which can be found in Table 2. Bias error was reported by the manufacturer of the flowmeters as 3% and for the pressure transducers as 1%. For the calipers used in measuring channel length and height, bias error was 0.2%, while the laser confocal microscope used for roughness and channel separation measurement had a bias error of 0.001%. Precision error is associated with the standard deviation of steady-state measurements, and in the laminar regime was found to be 1.25% for flow rate.

Because the bias error for the laser confocal microscope is approximately zero, and the standard deviation for all roughness measurements was small, there is effectively zero difference between uncertainty calculated with and without surface roughness. As laminar flow is the aim of this work, the percent errors of interest are $Re_{lam} = 3.3\%$ and $f_{lam} = 6.6\%$. In all plots of friction factor versus Reynolds number, the size of data points is approximately equal to half the experimental uncertainty of the friction factors. For purpose of visual clarity, error bars are omitted and percent difference is noted in the tables.

**Validation of Test Set With Smooth Channels**

Preliminary experiments were conducted for validation with smooth channels. Table 3 summarizes the measured smooth

---

**Table 2  Experimental uncertainty**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bias (%)</th>
<th>Precision (%)</th>
<th>Total uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re_{lam}$</td>
<td>3.007</td>
<td>1.249</td>
<td>3.256</td>
</tr>
<tr>
<td>$Re_{turb}$</td>
<td>3.007</td>
<td>1.796</td>
<td>3.503</td>
</tr>
<tr>
<td>$f_{lam}$</td>
<td>6.116</td>
<td>2.495</td>
<td>6.605</td>
</tr>
<tr>
<td>$f_{turb}$</td>
<td>6.116</td>
<td>3.593</td>
<td>7.093</td>
</tr>
</tbody>
</table>
Table 3  Measured channel geometry—Smooth surfaces

<table>
<thead>
<tr>
<th>Separation, b (µm)</th>
<th>Hydraulic diameter, Dh (µm)</th>
<th>Relative roughness, εFP/Dh (%)</th>
<th>Percent difference from laminar theory (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>101.8</td>
<td>202</td>
<td>0.6%</td>
<td>−4.1%</td>
</tr>
<tr>
<td>377.5</td>
<td>733</td>
<td>0.2%</td>
<td>0.4%</td>
</tr>
<tr>
<td>548.4</td>
<td>1051</td>
<td>0.1%</td>
<td>0.1%</td>
</tr>
<tr>
<td>751.0</td>
<td>1418</td>
<td>0.1%</td>
<td>1.2%</td>
</tr>
</tbody>
</table>

channel geometry that coincides with the data shown in Figure 7, as well as the percent error between friction factors obtained from experimental data and laminar theory. A negative percent difference indicates experimental friction factors are greater than the conventional laminar theory prediction.

The laminar theory line in Figure 7 and all subsequent plots was calculated using Eq. (9). Due to the small change in aspect ratio across test cases, there is a difference in laminar theory friction factor of about 6% when switching between the smallest and largest channel separations. All experimental data for smooth channels fall well within experimental error of laminar theory for flow in smooth channels.

These results indicate that the experimental test set is within the anticipated uncertainty of 6.6% for the friction factor in the laminar regime. The greatest deviation from the smooth channel correlation occurred for the 102 µm channel separation, which is not unexpected, as the literature indicates friction factors increase as channel size decreases.

RESULTS AND DISCUSSION

Experimental data for rough channels is presented and compared with the smooth channel correlation (Eq. (9)) and the wall function method (Eq. (4)). The Reynolds number was varied from approximately 50 to 4000, although the entire range was not possible for every test case.

The five rough surfaces were tested for multiple channel separations, at a minimum of 5 trials per test case, in order to ensure repeatability of the experimental setup. Figures 8–12 summarize the experimental data based on roughness type in order of increasing effect on fluid flow; λ/h = 7, 8, 5, 3, and 2, respectively. Only the minimum and maximum gap values for b are plotted to avoid overcrowding the plot. The intermediate values of b follow the same trend. It can be seen that λ/h has a marked effect on the agreement between the theoretical prediction and the experimental data. For λ/h = 2, shown in Figure 12, the agreement is quite good, but as λ/h increases to 3, the agreement deteriorates as seen in Figure 11. For λ/h = 5, the discrepancy is found to be quite large, as seen in Figure 10,
but again improves for higher $\lambda/h$ values of 7 and 8 shown in Figures 8 and 9, respectively.

All experimental friction factors were found to be greater than conventional laminar theory, and the percent difference increased as channel separation decreased, with the difference becoming more pronounced as $\lambda/h$ decreased. Similarly, experimental friction factors were found to be greater than the wall function method’s predictions in all cases but for the largest channel separations for $\lambda/h = 8$ and 2.

Table 4 details the measured channel geometry for each test condition, as well as the relative roughness values. Surface geometry is identified by the pitch-to-height ratio in the left-hand column, listed in order of increasing apparent effect on fluid flow. The percent difference between experimental data and predictions by conventional laminar theory and the wall function method are also presented in Table 4. The cases for which percent difference is within experimental uncertainty are highlighted in gray.

From Table 4, it can be seen that in the constant-roughness pitch cases, with $\lambda/h = 7, 5,$ and 2, the roughness height effect on flow is evident. In the event that the relative roughness is similar for two surfaces, the friction factors are greater when roughness height is greater. The wall function method predicted the lowest $\lambda/h$ surface well for all separations, but for the next tallest roughness was unable to correlate with any of the data. However, excellent agreement was found for the smaller channel separations, or lowest aspect ratios, for the $\lambda/h = 2$ surface, while error increased for the larger channel separations. In all cases, the transition to turbulence occurred at lower Reynolds numbers as the channel size decreased.

The three cases for constant roughness height, $\lambda/h = 8, 5,$ and 3, where $h = 50 \mu m$, all resulted in similar values of relative roughness (both $h/D_h$ and $\varepsilon_f/D_h$) for the channel sizes tested, yet the experimental friction factors differed for each surface. The $\lambda/h = 5$ and $\lambda/h = 3$ surfaces exhibited similar friction factors, while the $\lambda/h = 8$ surface exhibited lower effect on fluid flow of these three. The same trend can be seen in the percent errors between experimental friction factors and the wall function method’s predictions.

The Poiseuille number, $Po$, is useful for comparison across an array of data sets where the Reynolds numbers are not consistent throughout. This parameter is equivalent to the friction factor multiplied by the Reynolds number and is a constant value for laminar flow through smooth channels, $Po = 23$. Figure 13 is a plot of the Poiseuille numbers obtained from the experimental results and from the wall function method as $\lambda/h$ varies. The channel aspect ratio $\alpha = 0.03$ for each test case in this plot ($b \approx 400 \mu m$). As was seen in previous plots, experimental friction factors decrease with increasing pitch-to-height ratio. Figure 13 shows the wall function method’s ability to predict friction factors in rough channel flow, given...
Table 4  Test matrix and results summary

<table>
<thead>
<tr>
<th>Pitch-to-height ratio, ( \lambda/h )</th>
<th>Separation, ( b ) (mm)</th>
<th>Hydraulic diameter, ( D_h ) (µm)</th>
<th>Relative roughness, ( h/D_h )</th>
<th>Height-to-separation ratio ( h/b )</th>
<th>Percent difference from laminar theory</th>
<th>Percent difference from wall function method</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>231</td>
<td>455</td>
<td>8%</td>
<td>0.15</td>
<td>26%</td>
<td>4%</td>
</tr>
<tr>
<td></td>
<td>439</td>
<td>849</td>
<td>4%</td>
<td>0.08</td>
<td>17%</td>
<td>5%</td>
</tr>
<tr>
<td></td>
<td>572</td>
<td>1094</td>
<td>3%</td>
<td>0.06</td>
<td>9%</td>
<td>2%</td>
</tr>
<tr>
<td>8</td>
<td>258</td>
<td>505</td>
<td>10%</td>
<td>0.19</td>
<td>55%</td>
<td>17%</td>
</tr>
<tr>
<td></td>
<td>444</td>
<td>858</td>
<td>6%</td>
<td>0.11</td>
<td>26%</td>
<td>8%</td>
</tr>
<tr>
<td></td>
<td>593</td>
<td>1134</td>
<td>4%</td>
<td>0.08</td>
<td>19%</td>
<td>2%</td>
</tr>
<tr>
<td></td>
<td>791</td>
<td>1489</td>
<td>3%</td>
<td>0.06</td>
<td>5%</td>
<td>4%</td>
</tr>
<tr>
<td>5</td>
<td>231</td>
<td>453</td>
<td>11%</td>
<td>0.21</td>
<td>142%</td>
<td>88%</td>
</tr>
<tr>
<td></td>
<td>414</td>
<td>802</td>
<td>6%</td>
<td>0.12</td>
<td>59%</td>
<td>29%</td>
</tr>
<tr>
<td></td>
<td>577</td>
<td>1104</td>
<td>4%</td>
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<td>18%</td>
</tr>
<tr>
<td></td>
<td>780</td>
<td>1470</td>
<td>3%</td>
<td>0.06</td>
<td>23%</td>
<td>13%</td>
</tr>
<tr>
<td>3</td>
<td>230</td>
<td>452</td>
<td>11%</td>
<td>0.22</td>
<td>125%</td>
<td>23%</td>
</tr>
<tr>
<td></td>
<td>414</td>
<td>801</td>
<td>6%</td>
<td>0.12</td>
<td>65%</td>
<td>27%</td>
</tr>
<tr>
<td></td>
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<td>1058</td>
<td>5%</td>
<td>0.09</td>
<td>31%</td>
<td>3%</td>
</tr>
<tr>
<td></td>
<td>751</td>
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<td>3%</td>
<td>0.07</td>
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<td>6%</td>
</tr>
<tr>
<td>2</td>
<td>377</td>
<td>732</td>
<td>18%</td>
<td>0.35</td>
<td>266%</td>
<td>1%</td>
</tr>
<tr>
<td></td>
<td>536</td>
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<td>13%</td>
<td>0.24</td>
<td>150%</td>
<td>2%</td>
</tr>
<tr>
<td></td>
<td>755</td>
<td>1425</td>
<td>9%</td>
<td>0.17</td>
<td>103%</td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>937</td>
<td>1745</td>
<td>8%</td>
<td>0.14</td>
<td>71%</td>
<td>8%</td>
</tr>
</tbody>
</table>

the flow rate, fluid temperature, and channel and roughness geometry.

To further illustrate the variation of experimental friction factors in microchannels possessing structured roughness, Figure 14 shows a plot of the Poiseuille numbers versus channel separation with curve fits to highlight the correlations. For decreasing roughness pitch-to-height ratio (taller or more closely spaced roughness elements), and decreasing channel separation (narrower channel, where rough walls are closer together), Poiseuille numbers from these experiments are seen to increase. As channel separation increases, \( P_o \) for each rough case converges towards \( P_{o,\text{smooth}} \). Table 5 provides the channel geometry and Poiseuille numbers for each test case.

Table 5  Comparison between experimental and theoretical (wall function) Poiseuille numbers

<table>
<thead>
<tr>
<th>Surface</th>
<th>Channel separation, µm, ( b )</th>
<th>Hydraulic diameter, µm, ( D_h )</th>
<th>Relative roughness, ( h/D_h )</th>
<th>Experimental Poiseuille number, ( P_{o,\text{exp}} )</th>
<th>Wall function Poiseuille number, ( P_{o,\text{wall}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth</td>
<td>102</td>
<td>202</td>
<td>0.6%</td>
<td>23</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>378</td>
<td>733</td>
<td>0.2%</td>
<td>23</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>548</td>
<td>1051</td>
<td>0.1%</td>
<td>23</td>
<td>23</td>
</tr>
<tr>
<td>( \lambda/h = 7 )</td>
<td>231</td>
<td>455</td>
<td>3.8%</td>
<td>30</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>439</td>
<td>849</td>
<td>2.1%</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>572</td>
<td>1094</td>
<td>1.6%</td>
<td>25</td>
<td>24</td>
</tr>
<tr>
<td>( \lambda/h = 8 )</td>
<td>258</td>
<td>505</td>
<td>6.7%</td>
<td>36</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>444</td>
<td>858</td>
<td>3.9%</td>
<td>29</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>593</td>
<td>1134</td>
<td>3.0%</td>
<td>27</td>
<td>26</td>
</tr>
<tr>
<td>( \lambda/h = 5 )</td>
<td>231</td>
<td>453</td>
<td>6.2%</td>
<td>58</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>414</td>
<td>802</td>
<td>3.5%</td>
<td>37</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>577</td>
<td>1104</td>
<td>2.6%</td>
<td>31</td>
<td>27</td>
</tr>
<tr>
<td>( \lambda/h = 3 )</td>
<td>230</td>
<td>452</td>
<td>6.1%</td>
<td>55</td>
<td>47</td>
</tr>
<tr>
<td></td>
<td>414</td>
<td>801</td>
<td>3.5%</td>
<td>38</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>552</td>
<td>1058</td>
<td>2.6%</td>
<td>30</td>
<td>29</td>
</tr>
<tr>
<td>( \lambda/h = 2 )</td>
<td>377</td>
<td>732</td>
<td>11.7%</td>
<td>84</td>
<td>84</td>
</tr>
<tr>
<td></td>
<td>536</td>
<td>1029</td>
<td>8.3%</td>
<td>57</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>755</td>
<td>1425</td>
<td>6.0%</td>
<td>45</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>937</td>
<td>1745</td>
<td>4.9%</td>
<td>37</td>
<td>31</td>
</tr>
</tbody>
</table>

Figure 13  Poiseuille Number versus roughness pitch-to-height ratio. Channel separation is the same for all data points. (Color figure provided online.)

Figure 14  Poiseuille number versus channel separation with curve fits. (Color figure provided online.)

heat transfer engineering vol. 33 no. 6 2012
seen for the larger channel separations, and the two smallest \( \lambda/h \) surfaces, with better agreement for the larger separations of the \( \lambda/h = 3 \) surface, and better agreement for the smaller separations of the \( \lambda/h = 2 \) surface. The \( \lambda/h = 5 \) surface exhibited the poorest agreement with the wall function method’s predictions for all channel separations tested. It is believed that the fluid recirculation between the roughness elements for \( \lambda/h = 5 \) interacts with the main flow in the core and introduces additional losses. For lower and higher \( \lambda/h \) values, the recirculation effects seem to be contained between the roughness elements. The results are in agreement with the earlier results on the effects of flow instability due to recirculation at certain \( \lambda/h \) ratios and Reynolds numbers on pressure drop as indicated by earlier investigators [23, 30, 31].

**CONCLUSIONS**

Laminar incompressible fluid flow in rectangular channels possessing two-dimensional roughness was investigated from a theoretical standpoint, resulting in a model for pressure versus flow that allowed for the direct input of surface roughness geometry. This model is referred to as the wall function method since it directly incorporates the functions for the rough side walls. Structured roughness of a sinusoidal type was evaluated in order to control and vary the pitch and height. Surfaces possessing this structured roughness were generated for experimental validation of the wall function method.

- From measurements of the structured roughness, the spatial parameter \( R_{SM} \) was found to be in excellent agreement with the designed roughness pitch, while the average step height, \( R_{step} \), a nonstandard parameter, was found to be in good agreement with the designed peak height.
- A more universal roughness parameter is required for representing the peak height, as an alternative to \( R_{step} \). A method for designing and predicting the cosine power, which controls the peak slope, is also needed.
- As channel aspect ratio decreases, or as the hydraulic diameter decreases and relative roughness increases for a given roughness profile, the experimental friction factor increases, with the increase becoming more pronounced for taller and more closely spaced roughness.
- Relative roughness, \( \varepsilon_{\text{fr}}/D_h \), provides a general indication of the effect roughness will have on fluid flow, but is unable to represent the variations in roughness peak height.
- Results from surfaces with the same 250 \( \mu \)m pitch but varying height showed that an increase in roughness peak height causes a significantly greater frictional loss.
- No systematic pattern was observed for the cases of constant roughness height, \( h = 50 \mu \)m, and varying pitch, aside from the intermediate surface, \( \lambda/h = 5 \), consistently exhibiting the largest friction factors of the three surfaces.
- The surface for which \( \lambda/h \) was greatest (\( \lambda/h = 8 \)) was within experimental error of the wall function method for the two larger channel cases. The next largest pitch-to-height ratio was the “short” profile, \( \lambda/h = 7 \), which was predicted best out of all surfaces by the wall function method. The intermediate surface, for which \( \lambda/h = 5 \), resulted in the poorest comparison with the wall function method. The surface of \( \lambda/h = 3 \) saw friction factors comparable to those of the \( \lambda/h = 5 \) surface, though consistently lower, and the wall function method predicted the largest two channel separations for this surface well within experimental uncertainty. The final and “tallest” surface roughness, \( \lambda/h = 2 \), had the most pronounced effect on fluid flow, and the wall function method predicted the two smallest channel separations for this surface exceptionally well.
- The higher friction factors observed at \( \lambda/h = 5 \) are believed to be due to the instability introduced by the recirculation zone between the roughness elements.
- Overall, the limits of the wall function method were tested and it was shown that it is possible to predict frictional losses in a significantly rough microchannel without the complex computations of the fluid in the near wall region. Further exploration and refinement of this method are required.

**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>area, surface or cross-sectional, as noted in the text (m²)</td>
</tr>
<tr>
<td>( a )</td>
<td>rectangular channel height, or longer of the two dimensions (m)</td>
</tr>
<tr>
<td>( b )</td>
<td>rectangular channel separation, or smaller channel dimension (m)</td>
</tr>
<tr>
<td>( b_{cf} )</td>
<td>constricted channel separation (m)</td>
</tr>
<tr>
<td>( D )</td>
<td>pipe diameter (m)</td>
</tr>
<tr>
<td>( D_h )</td>
<td>hydraulic diameter, defined as 4A/P (m)</td>
</tr>
<tr>
<td>( F_p )</td>
<td>average floor profile (m)</td>
</tr>
<tr>
<td>( f )</td>
<td>friction factor</td>
</tr>
<tr>
<td>( f(x) )</td>
<td>lower wall function</td>
</tr>
</tbody>
</table>
\( \rho \) density (kg/m\(^3\))

\( g \) gravity (m/s\(^2\))

\( g(x) \) upper wall function

\( h \) roughness height (m)

\( L \) length (m)

\( \text{Max} \) maximum roughness height (m)

\( P \) pressure (Pa) or wetted perimeter (m), as noted in text

\( P_L \) pressure at exit (Pa)

\( P_0 \) pressure at inlet (Pa)

\( \text{Po} \) Poiseuille number

\( \rho \) power

\( Q \) volumetric flow rate (m\(^3\)/s)

\( R_a \) average roughness (m)

\( R_p \) maximum peak height (m)

\( R_{\text{q}} \) root-mean-square (RMS) roughness (m)

\( R_{\text{step}} \) average step height, unique to the laser confocal microscope (m)

\( R_{\text{SM}} \) mean spacing of peaks (m)

\( Re \) Reynolds number

\( u \) \( x \) component of velocity (m/s)

\( w \) \( z \) component of velocity (m/s)

\( x \) Cartesian coordinate, flow direction

\( y, z \) Cartesian coordinates, normal to flow direction

\( \alpha \) channel aspect ratio; smaller dimension divided by larger

\( \Delta \alpha \) mean slope of roughness profile

\( \Delta P \) pressure drop (Pa)

\( \varepsilon_{FP} \) roughness height parameter, for constricted flow model (m)

\( \lambda \) roughness pitch (m)

\( \mu \) dynamic viscosity (kg/m\( \cdot \)s)

\( \rho \) density (kg/m\(^3\))

Greek Symbols

\( \alpha \) channel aspect ratio; smaller dimension divided by larger

\( \Delta \alpha \) mean slope of roughness profile

\( \Delta P \) pressure drop (Pa)

\( \varepsilon_{FP} \) roughness height parameter, for constricted flow model (m)

\( \lambda \) roughness pitch (m)

\( \mu \) dynamic viscosity (kg/m\( \cdot \)s)

\( \rho \) density (kg/m\(^3\))

Subscripts

\( \text{lam} \) laminar flow

\( \text{smooth} \) smooth surface

\( \text{turb} \) turbulent flow

REFERENCES


**Rebecca Wagner** received her B.S. and M.S. degrees in mechanical engineering from the Rochester Institute of Technology, in 2008 and 2010, respectively. She is currently working at IBM in the Semiconductor Research and Development Center.

**Satish Kandlikar** is the Gleason Professor of Mechanical Engineering at the Rochester Institute of Technology (RIT). He received his Ph.D. degree from the Indian Institute of Technology in Bombay in 1975 and was a faculty member there before coming to RIT in 1980. He has worked extensively in the area of flow boiling heat transfer and CHF phenomena at microscale, single-phase flow in microchannels, high-heat-flux chip cooling, and water management in PEM fuel cells. He has published more than 200 journal and conference papers. He is a fellow of the ASME and associate editor of a number of journals. He is the executive editor of *Heat Exchanger Design Handbook* published by Begell House. He received the RIT’s Eisenhart Outstanding Teaching Award in 1997 and Trustees Outstanding Scholarship Award in 2006. He received the 2008 Rochester Engineer of the Year award from Rochester Engineering Society. Currently he is working on DOE- and GM-sponsored projects on fuel cell water management under freezing conditions, and an NSF-sponsored project on roughness effect on fluid flow and heat transfer at microscale.