
Experiments were conducted to investigate local heat transfer coefficients and flow characteristics of air flow in a 962 μm inner diameter stainless steel microtube (minichannel). The effects of heat loss, axial heat conduction and viscous heating were systematically analyzed. Heat losses during the experiments with gas flow in small diameter tubes vary considerably along the flow length, causing the uncertainties to be very large in the downstream region. Axial heat conduction was found to have a significant effect on heat transfer at low Re. Viscous heating was negligible at low Re, but the effect was found to be significant at higher Re. After accounting for varying heat losses, viscous heating and axial conduction, Nu was found to agree very well with the predictions from conventional heat transfer correlations both in laminar and turbulent flow regions. No early transition to turbulent flow was found in the present study. [DOI: 10.1115/1.4007876]

Keywords: heat transfer coefficient, microtube, heat loss, axial heat conduction, viscous heating, minichannel

1 Introduction

Heat transfer to gas flow in small diameter tubes is important in a number of MEMS devices. To design these microdevices properly, a fundamental understanding of heat transfer and fluid flow characteristics in small diameter channels is essential. Although there is no reason for heat transfer to be affected by diameter at these scales, large deviations from theory are reported in existing literature on experimental data for gas flow in microtubes. The study is aimed at providing quantitative information on the sources for such discrepancies.

Heat transfer research in microchannels was first conducted by Tuckerman and Pease [1] in 1981. They demonstrated that the planar integrated circuit chips can be effectively cooled by a laminar flow of water through microchannels with hydraulic diameters of 86–95 μm. Their work was followed by others who explored the heat transfer and fluid flow characteristics in microchannels. For liquid flow in microchannels, the available literature clearly indicates that friction factors are in good agreement with the conventional (macroscale) correlations [2,3]. A number of studies investigated the heat transfer characteristics of liquid flows in microchannels [3–10]. The earlier studies revealed that the heat transfer coefficients can deviate significantly from the predictions of conventional correlations [7–10] due to experimental uncertainty, while some of the recent studies indicated that the heat transfer can be predicted well by conventional correlations for liquid flow [3–6].

A few studies on gas flow characteristics in microchannels found that due to roughness effects, the friction factor was higher than in conventional correlations [11,12]. Peiyi and Little [11] studied trapezoidal channels with a hydraulic diameter in the range of 45–83 μm, while Tang et al. [12] investigated circular tubes with smooth and rough surfaces. The friction factors in both studies were in good agreement with predictions from conventional correlations for smooth tubes, but were higher than the predictions for rough tubes. Microchannels with hydraulic diameters on the order of 1–10 μm were investigated by a few investigators who found that the friction factors were lower than the predictions from conventional theory due to rarefaction effects [13–18]. Kohl et al. [19] reported that their friction factors were in good agreement with the predictions from the conventional theory for gas flow in 25 μm channel in laminar flow and for 100 μm channel in turbulent flow. A delayed transition to turbulent flow was observed with the transition occurring at a Re of 6000 for smooth tubes. Turner et al. [20] indicated that for hydraulic diameters greater than 4.7 μm and Kn < 0.04, the friction factors agreed well with continuum theory. The friction factors for rough channels with a relative roughness of up to 6%, were also found to be the same in laminar flow.

There are very few studies reported in literature on turbulent flow, due to difficulties encountered by the higher pressure drops in the experimental setup design. Similarly, in the case of gas flow in microtubes, there are very few experimental studies reported in literature for heat transfer due to complexities in heat transfer experiments at microscale [21]. Acosta et al. [22] applied heat and mass transfer analogy and studied mass transfer coefficients in narrow channels by applying the limiting current method. Choi et al. [14] and Wu and Little [23] indicate that the heat transfer coefficients in microchannels were significantly higher than predictions from the conventional correlations, and that the Colburn analogy was not valid. The actual reasons for this discrepancy will be the focus of this work.

From the reviewed literature, it can be summarized that for liquid flow in microchannels there are no differences between experimental data and the theoretical predictions. With channel diameters greater than 15 μm, friction factors agree well with conventional correlations. Similarly, available data for channel diameters greater than 0.1 mm shows that heat transfer coefficients are also in good agreement with predictions from conventional correlations. For gas flow in microchannels with hydraulic diameters smaller than about 7 μm at atmospheric pressure, friction factors are found to be lower than the conventional theory due to rarefaction effects. Yu et al. [24] reported that their friction factor data of water and gas flows in 52–102 μm channels were similar and lower than predictions. However, Judy et al. [2] indicated that in this diameter range, the data for water were in good agreement.

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with conventional correlations. Since the friction factor data of N\textsubscript{2} gas by Yu et al. [24] were the same as that of water, microscale effects were certainly not expected for liquid flow in microchannels within that diameter range [3,4,6,25]. Some other effects, such as heat loss, axial conduction and viscous dissipation, may not be important at macroscale and/or in incompressible flow. However, these effects may become important at microscale. Heat loss is another important consideration in heat transfer at microscale, and not accounting for it correctly is believed to be one of the major reasons for the discrepancies found in experimental data reported in literature. Axial conduction was indicated to have an important effect; although it was not quantitatively assessed and corrected in comparisons presented by previous investigators. Regarding viscous dissipation, accurate formulations are available for laminar flow, although there are no correlations available for turbulent flow.

With the above shortcomings in the available literature in mind, the present study is undertaken with the following objectives:

(i) Experimentally investigate the heat transfer coefficient of compressible air flow in 962 \( \mu \)m diameter smooth tube.
(ii) Carefully evaluate the effects of heat loss, axial conduction and viscous dissipation and compare the results with the theoretical predictions in laminar and turbulent regions.

2 Experimental Setup

Figure 1(a) shows a schematic of the present experimental setup. The test facility was an open system with the outlet exposed to atmosphere. A high-pressure air bottle and a pressure regulator capable of regulating pressures up to 20 MPa were used to supply air in the tube. Mass and volume flow meters with flow rate ranges of 100, 10, and 1 standard liter per minute, and accurate to within 0.8% reading were used to measure the flow rates. Pressure drop of fluid flowing through the tube was measured by two different working range pressure sensors to increase the accuracy. Pressure sensors were calibrated by a pressure calibrator with an accuracy of 52 Pa. The inlet air temperature was measured by resistance temperature detectors (RTD) with an accuracy of 0.1 °C. Fine wire 36 gage E type thermocouples were attached on the tube to measure the outside wall temperature. Thermocouples were calibrated to be within an uncertainty of 0.1 °C. A high current dc power supply with 0–100 A and 0–20 V was used to supply power to the stainless steel tube. The test section was enclosed in a vacuum chamber to reduce the natural convection heat losses. Flow meters, thermocouples, a pressure transducer and a power supply were interfaced with a LabView program for data acquisition. The inlet pressure was measured by a pressure sensor mounted near the inlet as shown in Fig. 1(b). The outlet was at atmospheric pressure as the flow was discharged into the atmosphere after exiting the test section.

A schematic drawing of the test tube assembly enclosed in a vacuum chamber is shown in Fig. 2. The two ends of the test tube were bonded to low thermal conductivity Garolite extensions with epoxy. One end of the test section was connected to a high-pressure compression tube fitting for fluid inlet while the other side was exhausted to the atmosphere. Two copper connectors were soldered to the two sides of the stainless steel test tube for power supply wires following the same design as reported by Kandlikar et al. [26]. The test section was heated directly by a dc power supply. Three E-type thermocouples were attached on the surface of the tube at uniform spacing to measure the surface temperatures. All thermocouple wires and electric wires inside the vacuum chamber were connected to data acquisition system and power supply through vacuum feed-through national pipe thread fittings. The tube cross-section image taken by an optical microscope and the internal tube surface image taken by a laser confocal scanning microscope are shown in Fig. 3. The average tube diameter is 962.0 \( \mu \)m which is calculated from the tube flow area of several individual cross-section images. The surface roughness was determined from the images to be Ra = 0.8 \( \mu \)m.

3 Data Reduction

In the present study, the friction factor and Nusselt number were calculated to obtain the fluid flow and heat transfer performance data for air flow in a microtube. The pressure drop and mass flow rate in the microtube were measured to calculate the friction factor. Neglecting the entrance region effect as the tube diameter to length ratio is quite large (343), the friction factor is calculated from the following equation:

\[
f = \frac{\Delta P_t}{\frac{D_h \rho_{avg} \text{L}}{2G^2}}
\]  

where \( \Delta P_t \) is the pressure drop due to friction and \( \rho_{avg} \) is the average of inlet and outlet fluid densities, and \( D_h, L, \text{and} \ G \) are the hydraulic diameter, tube length and mass flux, respectively. In compressible flow, the measured total pressure drop \( \Delta P_{\text{tot}} \) is the sum of friction pressure drop \( \Delta P_f \) and acceleration pressure drop \( \Delta P_a \):

\[
\Delta P_{\text{tot}} = \Delta P_f + \Delta P_a
\]

The acceleration pressure drop \( \Delta P_a \) is given by

\[
\Delta P_a = \frac{\rho \text{G}^2}{2}
\]
while in turbulent flow with \( \text{Re} \) higher than 2300 the viscous heating is given by

\[
q_{\text{viscous}} = \frac{2}{7} \pi \mu \nu L
\]  

(9)

The input heat to the fluid \( q_{\text{in}} \) is calculated by the following equation:

\[
q_{\text{in}} = q_{\text{mea}} + q_{\text{axial}} + q_{\text{viscous}} - q_{\text{loss}}
\]  

(10)

where \( q_{\text{mea}} \) is the measured total heat input by the DC power supply and \( q_{\text{loss}} \) is estimated by a separate experiment as described in the next section.

4 Results

4.1 Heat Loss Estimation. Since the heat input needed during an actual test is quite small, on the order of a few Watts, heat losses constitute a large portion of the heat supplied in the present study. Therefore, it is essential to reduce the heat losses from the system. This was accomplished by enclosing the test section in a vacuum chamber. Heat losses are determined by heating the tube without any fluid flowing through the test section. The tube was heated directly by the power supply and the applied power was quantified as the heat loss corresponding to the difference between the local wall surface and the ambient temperature.

Figure 4 shows \( \Delta T \) (difference between the wall and the ambient temperature) versus \( q_{\text{loss}} \) for different levels of vacuum in the chamber. It is seen from Fig. 4 that \( q_{\text{loss}} \) increases with increasing \( \Delta T \). As expected, \( q_{\text{loss}} \) decreases with a decrease in the vacuum chamber pressure. At \( \Delta T = 40^\circ \text{C} \), the heat loss values at pressures of 760 Torr (atmospheric pressure) and 0.008 Torr are 1.7 W and 0.7 W, respectively. The heat loss values at pressures of 0.012 Torr and 0.008 Torr are the same, which indicates that the heat loss at the lower pressures is due to radiation. Further reduction in pressure did not lower the heat losses.

Heat loss is a function of wall temperature as shown in Fig. 4, while the wall temperature during a test with fluid flow is a function of the distance from the entrance. It was therefore essential to calculate the local heat loss separately near each wall thermocouple location. To achieve this, heat loss \( q_{\text{loss}} \) versus the temperature difference \( \Delta T \) at three different locations at distance \( L_x \) from the entrance was recorded as shown in Fig. 5. The heat loss estimates in the actual heat transfer experiments were then calculated using this plot. As shown in the figure there is no significant difference in the heat loss estimation at these locations. The heat loss is thus a function of the local temperature alone for all three locations.

![Figure 4](http://heattransfer.asmedigitalcollection.asme.org/ on 04/15/2014 Terms of Use: http://asme.org/terms)
In an actual experiment, since the wall temperature increases along the tube length, the heat losses also increase accordingly. To account for the heat losses during a test at a given location, the varying heat loss needs to be integrated along the tube length.

To accomplish this, the test section was divided into three zones along the tube length, and the heat losses were calculated for each zone using the following equation:

\[ q_{\text{loss},n} = \left[ q_{\text{loss},\Delta T_{n-1}} + q_{\text{loss},\Delta T_{n}} \right] \frac{(L_{h,n} - L_{h,n-1})/2}{L_{h,\text{total}}} \]  

(11)

where \( q_{\text{loss},\Delta T_{n-1}} \) is the heat loss calculated from the local temperature difference \( \Delta T_{n-1} \) at location \( n-1 \).

The heat input to the fluid in the zone located between \( L_{h,n-1} \) and \( L_{h,n} \) is given by

\[ q_{\text{in},n} = q_{\text{in},n-1} + q_{\text{mea}} \frac{(L_{h,n} - L_{h,n-1})}{L_{h,\text{total}}} - q_{\text{loss},n} \]  

(12)

Figure 6 shows \( q_{\text{loss}}/q_{\text{in}} \) as a function of Re at three different locations along the test section length. It is observed that \( q_{\text{loss}}/q_{\text{in}} \) increases with increasing distance \( L_{h} \). This is because of the higher local wall temperatures that result in higher heat losses at downstream locations. The \( q_{\text{loss}}/q_{\text{in}} \) decreases with an increase in Re, as the heat losses become a smaller fraction of the heat carried away by the fluid.

4.2 Uncertainty Analysis. To assess the accuracy of the measurement, an uncertainty analysis was performed. The calculated parameters such as Nu, Re, G and f are generally denoted as \( \delta y \). They are functions of variables \( x_1, x_2, \ldots, x_n \). The uncertainty in a parameter is determined as described below

\[ y = f(x_1, x_2, \ldots, x_n), \]

\[ \delta y = \left[ \left( \frac{\partial y}{\partial x_1} \delta x_1 \right)^2 + \ldots + \left( \frac{\partial y}{\partial x_n} \delta x_n \right)^2 \right]^{1/2} \]  

(13)

where \( \delta x_1, \delta x_2, \ldots, \delta x_n \) are the uncertainties in the independent variables.

The uncertainties in the experimental parameters are listed in Table 1. The uncertainty in diameter was calculated using 14 sets of diameter measurements taken at different locations in the tube sample. A laser confocal scanning microscope was used to measure the internal surface roughness. From these measurements, the average tube diameter was 962 \( \mu m \). The uncertainties in the experimental parameters are listed in Table 1. The uncertainty in diameter was calculated using 14 sets of diameter measurements taken at different locations in the tube sample. A laser confocal scanning microscope was used to measure the internal surface roughness. From these measurements, the average tube diameter was 962 \( \mu m \). The uncertainties in the experimental parameters are listed in Table 1. The uncertainty in diameter was calculated using 14 sets of diameter measurements taken at different locations in the tube sample. A laser confocal scanning microscope was used to measure the internal surface roughness. From these measurements, the average tube diameter was 962 \( \mu m \). The uncertainties in the experimental parameters are listed in Table 1. The uncertainty in diameter was calculated using 14 sets of diameter measurements taken at different locations in the tube sample. A laser confocal scanning microscope was used to measure the internal surface roughness. From these measurements, the average tube diameter was 962 \( \mu m \). The uncertainties in the experimental parameters are listed in Table 1. The uncertainty in diameter was calculated using 14 sets of diameter measurements taken at different locations in the tube sample. A laser confocal scanning microscope was used to measure the internal surface roughness. From these measurements, the average tube diameter was 962 \( \mu m \). The uncertainties in the experimental parameters are listed in Table 1. The uncertainty in diameter was calculated using 14 sets of diameter measurements taken at different locations in the tube sample. A laser confocal scanning microscope was used to measure the internal surface roughness. From these measurements, the average tube diameter was 962 \( \mu m \). The uncertainties in the experimental parameters are listed in Table 1. The uncertainty in diameter was calculated using 14 sets of diameter measurements taken at different locations in the tube sample. A laser confocal scanning microscope was used to measure the internal surface roughness. From these measurements, the average tube diameter was 962 \( \mu m \).
similar values for each flow rate by controlling the input power. This is done mainly to eliminate the effect of variations in heat losses, which depend on the local wall temperatures. The heat input and heat flux values are therefore higher at higher Re. In laminar flow, the heat transfer coefficient is constant and independent of Re, which causes \((T_{w,x} - T_{f,x})\) to increase with heat flux and Re. The low heat flux and \((T_{w,x} - T_{f,x})\) values in the low Re case thus lead to higher uncertainty in Nu.

The uncertainty in Nu is also a function of heating location distance \(L_h\) and increases proportionately. For large \(L_h\), the wall temperature and heat losses are high, which increases the uncertainty in the derived fluid temperature and Nu. At the location \(L_h = 25\) mm, the Nu uncertainty is less than 23%. At \(L_h = 125\) mm and \(L_h = 229\) mm, the Nu uncertainty is quite high and the experimental data are not reliable. However, in the turbulent flow regime, the Nu uncertainty at the three different locations was less than 20%.

### 4.3 Friction Factor

Figure 9 shows the comparison of friction factor data derived from Eqs. (1) and (4). There is no significant difference observed in the data derived by the two different equations indicating little influence from rarefaction effects. In the laminar flow, the data are in good agreement with Hagen–Poiseuille flow values, while in the turbulent region, the deviation of experimental data from Blasius and Filonenco equations is less than 10% and is considered as a good agreement with the conventional correlations. Comparing the slopes of the f-Re plots, it is concluded that for Re less than 2000 the flow is laminar and that earlier transition from laminar to turbulent flow is not observed. In the laminar flow, the friction factor is slightly higher than predicted at higher values of Re. This is due to higher entrance region effects.

### 4.4 Heat Transfer Coefficient

Figure 10 shows Nu as a function of Re for the 962 \(\mu\)m diameter tube at \(L_h = 25\) mm from the entrance. It is seen that in turbulent flow, the data can be predicted very well by the Petukhov–Gnielinski correlation. In laminar flow, Nu agreed well with the theoretical laminar fully developed flow value of 4.36. In higher Re range of laminar flow, Nu was slightly higher than the laminar fully developed flow value. This was due to the flow being in the developing region. The entrance length for the tube diameter of 962 \(\mu\)m corresponding to Re \(= 2000\) is about 67 mm, which is significantly higher than the heating length \(L_h = 25\) mm. The transition flow was observed at Re in the range 2000–3000.

It should be noted that the effects of heat loss, and axial conduction and viscous heating were accounted for in the data plotted in Fig. 10. Without considering these factors, the data were not well correlated with the conventional correlations.

### 4.5 Effect of Viscous Heating

Viscous heating effects are usually negligible in macroscale systems or with liquid flow in both microscale and macroscale systems due to the relatively low velocities employed. During gas flow is microscale systems, however, the small hydraulic diameter results in a high velocity, and consequently the thermal energy generated due to viscous heating results in additional heat input to the fluid. Viscous heating...
increases the fluid temperature and the calculated heat transfer is altered accordingly.

To accurately evaluate the experimental heat transfer coefficient, it is essential to estimate the viscous heating effect. In laminar flow, viscous heating can be derived by considering the laminar fully developed flow velocity profile and is also available in literature [29]. However, there are no equations available in literature to account for viscous heating effects in turbulent flow. In the present study, viscous heating in turbulent flow was estimated from the proposed model using Eq. (9).

Figure 11 shows the Nu obtained by including viscous heating from Eqs. (8) and (9) normalized by the theoretical Nusselt number, Nu_th, as a function of Re for different heating lengths. Equation (8) represents the equation for viscous heating in laminar flow, while Eq. (9) represents viscous heating in turbulent flow. At the location L_h = 25 mm, there was no significant difference for the derived Nu by the two equations. This was due to the short heating length and the lower magnitude of viscous heating. However, for all other lengths, using the laminar flow equation, Eq. (8), in the turbulent flow region at higher Re results in significantly higher values of Nu due to overestimation of the viscous effects. Using the turbulent flow equation, Eq. (9), in the turbulent region results in Nu/Nu_th to be close to 1 for all lengths.

The results shown in Fig. 11 indicate that the derived model in the present study is applicable to estimate viscous heating in turbulent flow. Equation (9) was derived based on a 1/7 power law profile as shown in Appendix C. Although the power exponent is not constant but depends on the Reynolds number, the simple power law equation is used in the present work for ease of calculations. Further refinements in this area are recommended.

4.6 Effect of Heat Loss. The heat losses constitute a large fraction of the heat supplied and significantly affect the experimental uncertainty. Figure 12 shows the comparison of derived Nu data with and without considering heat losses. In turbulent flow, the Nu values evaluated with and without considering heat losses are very close, implying that heat loss effects are negligible. However, in laminar flow, the data without considering heat losses are significantly different from the predictions from the conventional theory. The heat loss effect becomes more prominent in the laminar flow region. Without considering heat losses, the heat input to the fluid was overestimated, along with the heat flux, as illustrated in Fig. 7. Similarly, the local fluid temperature was also overestimated, which caused (T_w,x - T_s,x) to be lower. From Eq. (5) it is seen that heat transfer coefficient is significantly overestimated at higher q^* and T_s,x values.

\[ \frac{Nu_{th}}{Nu_{th}} = \frac{1}{1 + \frac{k_w A_{th}}{k_f A_f (Re Pr)^{1/3}}} \]  

4.7 Effect of Axial Conduction. The axial conduction effect is not negligible in gas flow due to the low heat input and low fluid heat capacity. This effect becomes more significant for smaller diameter tubes, higher thermal conductivity tube materials, and lower Re-Pr product according to the model presented by Lin and Kandlikar [28]. Figure 13 compares the data without correcting for the axial heat conduction effects, and by including them as described by model [28] and Eq. (14).
effects are not accounted properly. For low Re, the Nu is significantly lower than the conventional laminar fully developed flow theory. It is however, in good agreement with the prediction proposed by the Lin and Kandlikar [28] model. The temperature variation of the microtube at different conditions are listed in Table 2. It shows the magnitude of temperatures gradients along the length, as well as the higher temperatures encountered in the downstream region for different flow conditions. The effect of axial heat conduction in the diameter range of the present study is therefore not negligible, especially for low Re.

5 Conclusions

Heat transfer and friction factor of air flow in a smooth 304SS 962 μm inner diameter tube was investigated in this study. The effects of heat loss, axial conduction and viscous heating were analyzed in detail. An uncertainty analysis was performed to evaluate the uncertainty in the data as a function measurement location along the flow length and flow rate. Based on the experimental data the following conclusions were drawn:

1. Heat losses are a function of local wall temperature. Since the wall temperature variation along the tube length increases at lower Re for a given power input, the heat losses are higher in the downstream section.
2. For laminar flow at low Re, the wall temperatures in the downstream region are higher resulting in large heat losses and larger uncertainty, making data in this region unreliable. The experimental Nu uncertainty increases with increasing distance (downstream), and with decreasing Re.
3. Viscous heating effects increase with Re and are negligible for high Re. A new equation is developed for viscous heating in the turbulent region.
4. The effects of axial heat conduction at low Re are negligible. The model of Lin and Kandlikar [28] is able to predict this effect well.
5. The heat transfer coefficient and friction factor for air flow in a 962 μm tube can be predicted very well by conventional correlations after accounting for the variable heat losses along the length of the test section, axial conduction and viscous heating effects.

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Nomenclature

\[ A = \text{area, m}^2 \]
\[ A_f = \text{cross-sectional area of fluid flow in channel, m}^2 \]
\[ A_{hs} = \text{cross-sectional area of the channel wall, m}^2 \]
\[ D_h = \text{hydraulic diameter, m} \]
\[ f = \text{fanning friction factor, dimensionless} \]
\[ G = \text{mass flux, kg/m}^2 \text{s} \]

Table 2  Wall and fluid temperature variation along test section

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<thead>
<tr>
<th>( Re )</th>
<th>( L_h = 25 \text{ mm} )</th>
<th>( T_w )</th>
<th>( L_h = 125 \text{ mm} )</th>
<th>( T_w )</th>
<th>( L_h = 229 \text{ mm} )</th>
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<td>62.6</td>
</tr>
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</table>

\[ h = \text{heat transfer coefficient, W/m}^2 \text{C} \]
\[ k_l = \text{thermal conductivity of liquid, W/m C} \]
\[ k_w = \text{thermal conductivity of tube, W/m C} \]
\[ Kn = \text{Knudsen number, dimensionless} \]
\[ L = \text{tube length, m} \]
\[ L_{h,x} = \text{local heating length, m} \]
\[ Nu = \text{Nusselt number, dimensionless} \]
\[ Nu_{th} = \text{theoretical Nusselt number, dimensionless} \]
\[ P_l = \text{pressure in the inlet region, Pa} \]
\[ P_s = \text{pressure in the outlet region, Pa} \]
\[ \Delta P = \text{acceleration pressure drop, Pa} \]
\[ \Delta P_f = \text{friction pressure drop, Pa} \]
\[ q_{axial} = \text{axial conduction heat, W} \]
\[ q_{in} = \text{input heat to fluid, W} \]
\[ q_{loss} = \text{heat loss, W} \]
\[ q_{mea} = \text{measured heat from power supply, W} \]
\[ q_{viscous} = \text{viscous heat, W} \]
\[ q' = \text{heat flux, W/m}^2 \]
\[ Re = \text{Reynolds number, dimensionless} \]
\[ T_{w,x} = \text{local wall temperature, °C} \]
\[ T_{f,x} = \text{local fluid temperature, °C} \]
\[ \mu_m = \text{mean fluid velocity, m/s} \]

Greek Symbols

\[ \mu = \text{viscosity, Ns/m}^2 \]
\[ \rho_1 = \text{fluid density in tube inlet region, kg/m}^3 \]
\[ \rho_2 = \text{fluid density in tube outlet region, kg/m}^3 \]
\[ \rho_{avg} = \text{averaged fluid density, kg/m}^3 \]

Appendix A: Internal Tube Wall Temperature Calculation

From energy equation, for one-dimensional heat conduction with heat generation

\[ \frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) + \frac{q}{k_w} = 0 \]  \hspace{1cm} (A1)

where \( k_w \) is the conductivity of micro tube and \( q \) is the internal heat source generated inside the micro tube by the electric power defined as

\[ q = L_{h,x} \pi (r_o^2 - r_i^2) \]  \hspace{1cm} (A2)

With the adiabatic condition at the outer wall, the inside temperature is given by

\[ T_w = T_{w,0} + \frac{q}{4k_w} (r_o^2 - r_i^2) - \frac{q}{2k_w} r_o^2 L_n \left( \frac{r_o}{r_i} \right) \]  \hspace{1cm} (A3)
Appendix B: Calculation of Local Fluid Temperature

The inlet velocity $u_1$ can be determined from the correlation as follows:

$$ u_1 = \frac{4n\bar{m}}{\rho_1 \pi d^2} \quad (B1) $$

where $\bar{m}$, $\rho_1$ are mass flow rate and inlet fluid density, respectively. The inlet speed of sound, $a_1$, is expressed as $a_1 = \sqrt{RT_1}$, where $\gamma$, $R$, and $T_1$ are ratio of specific heats, gas constant and inlet fluid temperature. Hence, inlet flow Mach number can be determined $M_1 = (u_1/a_1)$, and

$$ \frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2 \quad (B2) $$

where $T_{01}$ is the temperature which the fluid element achieves when $u = 0$, defined as total pressure. Hence, $T_{01}$ is obtained as

$$ T_{01} = T_1 \left(1 + \frac{\gamma - 1}{2} M_1^2 \right) \quad (B3) $$

From the energy balance

$$ T_{02} = \frac{\dot{q}}{c_p} + T_{01} \quad (B4) $$

where $\dot{q}$ is heat added per unit mass defined as $\dot{q} = (\dot{q}_{in}/\bar{m})$. Therefore

$$ \frac{T_{01}}{T_0} = \frac{\gamma + 1}{1 + \gamma M_1^2} \left[2 + (\gamma - 1) M_1^2 \right] \quad (B5) $$

$$ \frac{T_{02}}{T_0} = \frac{T_{02}}{T_{01}} \frac{T_{01}}{T_0} \quad (B6) $$

From the compressible flow table, local Ma $M_2$ can be determined. The local fluid temperature $T_2$ can be determined from the following equation where $T_2^*/T_1$ can be determined from $M_1$:

$$ T_2 = T_2 \left(1 + \frac{\gamma + 1}{1 + \gamma M_2^2} \right)^2 \quad (B7) $$

$$ T_2 = \frac{T_{02} T_{01}}{T_{01} T_{0}} \left(1 + \frac{\gamma + 1}{1 + \gamma M_2^2} \right)^2 \quad (B8) $$

Appendix C: Viscous Dissipation

$$ \Phi = 2\mu \left\{ \left( \frac{\partial v}{\partial r} \right)^2 + \left[ \frac{1}{r} \left( \frac{\partial \theta}{\partial \theta} + v_r \right) \right]^2 \left( \frac{\partial w}{\partial z} \right)^2 + \frac{1}{2} \left( \frac{\partial v}{\partial z} \right)^2 + \frac{1}{2} \left( \frac{\partial w}{\partial z} \right)^2 \right\} \quad (C1) $$

Assume steady and uniform flow, $v_\theta \approx 0$, $\partial/\partial \theta \approx 0$, $\partial w / \partial z \approx 0$, $\partial/\partial z \approx 0$

$$ \Phi = 2\mu \left( \frac{\partial v}{\partial r} \right)^2 \quad (C2) $$

In laminar flow

$$ u(r) = 2u_1 \left[1 - \left( \frac{r}{L} \right)^2 \right] \quad (C3) $$

$$ \frac{\partial u}{\partial r} = -4u_1 \frac{r}{L} \quad (C4) $$

$$ \Phi = \int_0^R \int_0^{R/2} \left( \frac{1}{6} \mu \int_0^{R/2} \left( \frac{1}{R} \right)^{-\gamma} \right) 2\pi r dr dx \quad (C5) $$

$$ \Phi = 16\pi \mu u L \quad (C6) $$

In turbulent flow

$$ \Phi = \int_0^R \int_0^{R/2} \left( \frac{1}{6} \mu \int_0^{R/2} \left( \frac{1}{R} \right)^{-\gamma} \right) 2\pi r dr dx \quad (C7) $$

$$ \Phi = \frac{2}{3} \pi \mu u^2 L \quad (C10) $$

References


