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On the multicoloring of the edges of unicyclic graphs.
(Russian. English, Russian summaries)
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For an edge-weighted multigraph $G = (V, E)$, let $w(e)$ denote a non-negative integer weight of edge e. A (proper) multicoloring c of the edges E assigns an interval $I = [a, b]$ to each edge, $c(e) = I$, such that $w(e) = b - a$ and the intervals assigned to adjacent edges do not intersect in inner points. The edge multichromatic number $\mu'(G)$ is the minimum number of (integer) colors in any such multicoloring of G. The classical coloring of edges (or vertices) each with a single color k is a special case of the multicolorings considered in this paper, when all used intervals have the form $[k-1, k]$. Note that $\mu'(G) \geq \Delta(G)$, where $\Delta(G)$ is the maximum weighted degree $d(v)$ of vertices in G, $d(v) = \sum_{\{e|v\in e\in E\}} w(e)$, and in the case when $w(e) = 1$ for all e we have $\mu'(G) = \chi'(G)$, where $\chi'(G)$ is the chromatic index of G. One can also see that finding such multicolorings c generalizes the well-studied open shop problem.

The author conjectures that $\mu'(G) \leq \lfloor 3\Delta(G)/2 \rfloor$ for any edge-weighted multigraph G, which generalizes a classical theorem by C. E. Shannon from 1949 [J. Math. Physics **28** (1949), 148–151; MR0030203 (10,728g)] for coloring unweighted multigraphs. Some examples showing that the upper bound in this conjecture cannot be smaller are presented. The main result of the paper states that $\mu'(G) \leq \lfloor 3\Delta(G)/2 \rfloor$ for all edge-weighted unicyclic graphs G.

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