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Turán's problem and Ramsey numbers for trees. (English summary)

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Let T_n^1 and T_n^2 be the two n-vertex trees which have maximum degree $\Delta(T_n^i) = n-3$ for $i \in \{1,2\}$. First, the authors obtain explicit formulas for $\mathrm{ex}(p, T_n^i)$, where $\mathrm{ex}(p, G)$ is the maximum number of edges in any graph on p vertices not containing graph G. Second, these formulas are used to obtain some exact values of two-color Ramsey numbers $r(T_m, T_n^i)$ for several cases where T_m is an m-vertex tree, $m \le n$, and $\Delta(T_m) \le m-3$.

These results on Ramsey numbers extend the existing evidence confirming a well-known 1976 conjecture by S. A. Burr and P. Erdős [Utilitas Math. **9** (1976), 247–258; MR0429622 (55 #2633)] that $r(T_n, T_n)$ is at most $2n-2$ for even n and $2n-3$ for odd n. They also confirm the off-diagonal version of this conjecture by R. J. Faudree, R. H. Schelp and M. Simonovits [Ars Combin. **29** (1990), A, 97–106; MR1412810 (97f:05132)] that $r(T_m, T_n) \le m+n-2$ holds for all trees. The proofs in the current paper are long but elementary. The results extend those of a previous paper by the first two authors [J. Comb. Number Theory **3** (2011), no. 1, 51–69; MR2908182] and another one by the first author [Bull. Aust. Math. Soc. **86** (2012), no. 1, 164–176; MR2960237].

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