SONIFICATION OF SIMULATED BLACK HOLE MERGER DATA

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Abstract— *The field of Sonification is a subset of auditory display which human beings interface with wherein data are converted to audible signals. Sonification seeks to translate relationships in data into sounds that exploit the auditory perceptual abilities of human listeners such that the data relationships are intelligible. In 2015, the Laser Interferometer Gravitational-Wave Observatory (LIGO) recorded gravitational waves observed from a black hole merger for the first time. In this project we utilize Black Hole merger simulation data from the Center for Computational Research and Gravitation (CCRG) to create novel sonifications that accompany visualizations based upon that same data. Through experimentation, we found that convolving a random Gaussian sine wave buffer frequency modulated by object velocity with the Head-Related Transfer-Function of the appropriate azimuth, multiplied by the inverse square of the object's radius provided the best results. This methodology holds promise for sonification of interstellar phenomena in the domain of public outreach and can further aid discovery in astronomical data.*

Keywords: Sonification, Black Holes, Gravitational Waves, Convolution, Head-Related Transfer-Function

1. Introduction

The field of sonification, for lack of a unified theory, is immensely interdisciplinary and includes knowledge from disparate fields such as computer science, data mining, audio engineering, psychoacoustics, psychology, the cognitive sciences, music, linguistics, and the social sciences [1]. The technique of sonification can be broadly generalized as a process utilizing an information system that has three components: aggregate quantitative (numerical) or qualitative (verbal) data collected from a source or sources, a transmitter or auditory display, and the information receiver e.g. the human listener. There are three broad categorizations of approaches to sonification: (1) Event-based, wherein the auditory display is designed to convey meaning generated out of the aggregated data referring to a specific event post-hoc, such as an alert or notification. (2) Model-based, where a model is a "virtual object with which users can interact... [and] instead of mapping data parameters to sound parameters, the display designer builds a virtual model whose sonic responses to user input are derived from data" [1]. (3) Continuous, which is defined as a dynamic model which mathematically describes the evolution of a system in real-time, where the streams of data are continuous feedback mechanisms for the human listener to use that information to make decisions based on the sonified data, e.g. a sonar device on a ship. We shall focus here on event-based sonifications for application to the black hole merger simulation data-set provided by the Center for Computational Research and Gravitation (CCRG). The motivation for this experiment came from the authors' desire to make a data-driven, yet aesthetically appealing sonification based on black hole merger data using traditional and novel sonification techniques. Our assumptions which contributed to the motivation for this project are as follows: (1) Of the many examples of astronomical data sonifications in the scientific literature, there are none specifically addressing gravitational waves to the authors' knowledge. (2) The sonifications of stellar phenomena that do exist in the literature do not utilize the exact techniques discussed in this paper, combine data sonification with simultaneous visualization, nor employ techniques to achieve binaural audification insofar as the authors are aware. (3) This technique may have potential applications for both public outreach and scientific discovery, leveraging the highly evolved sense of human hearing which has the ability to detect complex, high-dimensional patterns beyond what the human eye is capable of. As Data is commonly modeled using Three and Four-Dimensional techniques via graphical and temporal visualization, sonification techniques such as the one described in this paper hold promise to add additional dimensionality (at least 5D) to the representations of data which scientists can employ. Therefore, there exists the potential for pairing sophisticated sonifications with visualizations that could aid in knowledge discovery and wider audience scope, particularly in the public sector. However, these implications have not been validated experimentally and further research is needed to assess the generalization of the techniques discussed herein. The remaining sections of the paper will include a brief foray into psychoacoustics, some background on black holes and Einstein's General Relativity, a detailed description of our chosen solution path, it's implementation, and finally our conclusions with implications for future research.

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2. Psychoacoustics

Sound is encoded physiologically by the human auditory system and the perceptual dimensions of pitch, timbre, and loudness are all affected by the filtering mechanisms of this auditory system. It is sufficient for our purposes here to say that the sound is transformed and filtered by the outer, middle, and inner ear, all of which have structural sub-components that decompose complex spectra into frequency bands and subsequently recombine them into neural impulses that travel to the brain to reconstruct the auditory perceptual field. It is key to note that the pinna and concha collect and filter sound from the outer ear and transfer it to the inner ear via the auditory canal. These structures are asymmetrical and are utilized in spacial localization of sound-source objects. By the nature of the structures, incoming sound is also spectrally filtered dependent on the relative direction of the incoming sound. The auditory system uses this decomposition as location queues for positioning an object in the environment [2].

3. Einstein and Gravitational Waves

As the aim of this paper is to describe the procedure by which the authors were able to achieve sonification of black hole merger data, a short discussion of black holes and gravitational waves in relation to Einstein's theory of General Relativity is in order. Black holes are gravitationally massive stellar phenomena whose exact nature are not completely elucidated, but are generally accepted to be defined as regions of space where the gravitational field is so intense that no matter or radiation, including light can escape. There are many compelling theoretical explanations to date with significant research that has been done in the field of astrophysics and related disciplines which will not be discussed here. We instead will focus on an anticipated result of Einstein's General Relativity, which predicted that the merging of two sufficiently gravitationally massive objects would produce disturbances in the fabric (curvature) of space-time across the entire universe that would propagate outward at the speed of light [7]. Einstein predicted these gravitational disturbances would be observable by delicate-enough instruments, and are hence called gravitational waves. The Laser Interferometer Gravitational-Wave Observatory (LIGO), which is the largest laser interferometer ever built, published a paper on February 11th, 2016 announcing that it had detected gravitational waves on September 14, 2015 [3]. In the LIGO paper, they announced the detection of two roughly 30 solar-mass black holes merging about 1.3 billion light-years from Earth, whose gravitational waves propagated all the way to Earth and measured by their interferometer. These gravitational waves are constantly passing the Earth. Even the strongest have an insignificant effect and their sources are generally at extreme distances. For example, the waves given off by the cataclysmic final merger of GW150914 reached Earth after traveling over 1 billion light-years, which changed the length of a 4-kilometer LIGO measuring arm by a ten thousandth of the width of a proton. This is proportionally equivalent to changing the distance to the nearest star outside the Solar System by one hair's width [4]. Such a minuscule effect requires the most sensitive of equipment to detect, and is why they have gone unobserved for so long, and thus why it was crucial in proving Einstein's General Relativity Theory.

For more information regarding LIGO and the MIT/Caltech collaboration, please see references [9] and [10] for links to the MIT and Caltech LIGO websites.

4. Sonification Techniques

Here, we define key mathematical concepts and techniques that will be used in the solution section:

4.1 Convolution

A function derived from two input functions by yielding the integral point-wise multiplication of the two input functions that expresses how the shape of one function modifies the other [8]. This can be thought of as a sliding window function, that multiplies and sums the inputs where both functions overlap and is expressed in general form where the integral goes from negative infinity to positive infinity, however can be any arbitrary time domain window. A function f convolved with a function g is shown below in Equation 1.

$$f * g = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)\,d\tau \tag{1}$$

4.2 Head-Related Transfer-Function

The HRTF is essentially a function of three variables: azimuth, elevation, and frequency. In order to find the sound pressure that an arbitrary source, denoted $x(t)$, will produce at the ear drum, we need the impulse response $h(t)$ from the source to the ear drum. This is the Head-Related Impulse-Response, and its Fourier transform $H(f)$ is the Head-Related Transfer-Function [4]. The HRTF captures the physical cues to source localization via amplitude roll-off for each ear from the Inter-Aural Time Difference (ITD), mimicking what the human auditory system accomplishes with the complex structures of the middle and inner ear. We must also compute HRTFs of both the left and right ear in order to account for the ITD. With this information we can synthesize accurate binaural signals from a monoaural source, giving the impression of spacial location to a sound object when played with headphones or more accurate representation with a complex surround sound-system. Many databases exist online that have taken measurements of the Head-Related Impulse Response (HRIR) and produced HRTF measurements available to the public. The HRTF measurements that were used for this project were derived from the OpenSource MIT KEMAR database. KEMAR stands for Knowles Electronics Manikin for Auditory Research, a manikin that is fitted with highly accurate manufactured human ears and is commonly utilized to take such measurements.

4.3 Fast-Fourier Transform

The Fourier Transform (Equation 2) is a transform function that converts from the time domain to the frequency domain. This is significant, as convolution becomes simple multiplication in the frequency domain according to the Convolution Theorem [8], thus giving a considerable programmatic performance improvement. The time to compute convoultion in time domain is $\mathscr{O}(n^2)$ whereas convolution in frequency domain is linear, $\mathscr{O}(n)$.

$$\mathscr{R}\left\{\frac{1}{2\pi}\int_0^\infty 2F(\omega)e^{i\omega t}\,d\omega\right\} \tag{2}$$

4.4 Biquadratic Filter

The biquadratic filter shown in Equation 3 and used herein is a recursive linear filter transfer function such that in the Z-domain is the ratio of two quadratic functions:

$$H(s) = \frac{(\frac{s}{Q})}{(s^2 + \frac{s}{Q} + 1)} \tag{3}$$

It is derived from an analog prototype and had been digitized using the Bilinear Transform [5]. In our implementation, the function takes a center-frequency (fc), bandwidth (Q), and the sampling frequency (sf) as arguments and outputs two $3x1$ matrices that will contain the filtered data.
The coefficients used to compute the output are shown below in Equation 4 and Equation 5, while the resulting output matrices B and A are shown in Equation 6, using the coefficients from Equations 4 and 5:

$$\Omega = \frac{2\pi * fc}{fs} \tag{4}$$

$$\alpha = \sin(\Omega) * Q \tag{5}$$

$$B = \begin{bmatrix} \alpha \\ 0 \\ -\alpha \end{bmatrix},\ A = \begin{bmatrix} 1+\alpha \\ -2*\cos(\Omega) \\ 1-\alpha \end{bmatrix} \tag{6}$$

4.5 Fade-in and Fade-out

A fade-in/out is a function that takes a specific sample size vector and monotonically dampens the amplitude of the signal, either from front to back or back to front depending on what orientation is desired. This is applied using the equal power mode rule in Equation 7:

$$sin^2 + cos^2 = 1 \tag{7}$$

4.6 White Gaussian Noise

A simple random Gaussian distribution, which in our case is applied to the generation of the signal buffer, is to be utilized in the convolution with the HRTF data rather than using a a pure tone. The application of random white noise minimizes undesired noise that appears with the application of convoultion with a pure tone. This is due to an overtone that is inadvertently generated by the phase displacement caused by the iterative concatenation of the input buffers.

5. Solution

As the main objects in this project are black holes, being complex and gravitationally massive stellar phenomena, it is imperative that we make certain assumptions that allow for ease of modeling and computation. In this experiment, we treat black holes as point-like sound-source objects which have no behavior other than coordinate displacement (movement), velocity, acceleration, and assume they have constant mass throughout. These assumptions are not realistic of complete data-sets describing actual black hole mergers, but nonetheless allow for simplification of data such that sonification is more readily achievable. Our overall method to achieve sonification can be modeled as a sequential pipe with several general categories that involve organizing and preparing data, object feature modeling, filtering, and convolution as follows:

5.1 Data preparation

We must extract and modify features of the data such that specific key features will be converted into the appropriate form. In order to select the correct HRTF input to convolve with a sound-source impulse, we compute the degree of the azimuth of the object in relationship to the origin. This can be achieved by converting the Cartesian coordinates at each time-step into polar coordinates. We must calculate the hypotenuse of the right triangle formed by the (x, y) coordinates from the Cartesian plane and apply the arctan function plus the quadrant displacement to find the angle θ_t for that time-step. The radius is simply the hypotenuse previously calculated. The overall process is shown sequentially in Illustration 1 and Illustration 2 on the following page.

The Cartesian to polar conversion involves computing the radius r at time t (Equation 8):

$$r_t = \sqrt{x^2 + y^2} \tag{8}$$

The angle θ_i at time t is computed by using *arctan* and adding the quadrant offset (Equation 9), and is dependent on the signs of x and y, where QUADRANT (Equation 10 is a function that takes the (x, y) coordinate as input and by determining the signs of each axis, returns the value in the dictionary QUADRANT appropriate to the input:

$$\theta_t = \arctan(\frac{y_t}{x_t}) + QUADRANT_t \tag{9}$$

$$QUADRANT = \{\text{'I'}: 0^o, \text{'II'}: 90^o, \text{'III'}: 180^o, \text{'IV'}: 270^o\} \tag{10}$$

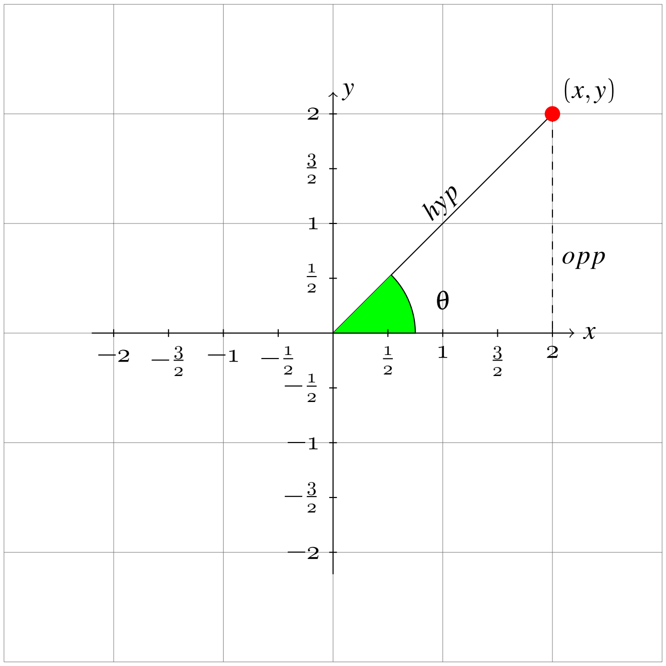

Illustration 1: Conversion to polar involves computing the angle theta (Equation 9) by taking the arctan of the y divided by x Cartesian coordinates and the radius, which is the hypotenuse of the right triangle shown above (Equation 8):

After the conversion from Cartesian to Polar, the θ values are put through an artificial filter function to create discrete output theta values in increments of 5^o to match the incremental values for the MIT KEMAR HRTF set (Equation 11):

$$mod(\theta) = \begin{cases} \lceil \frac{\theta}{5} \rceil + 1 & \text{if } \theta \bmod 5 > 2 \\ \lfloor \frac{\theta}{5} \rfloor + 1 & \text{if } otherwise \end{cases} \tag{11}$$

This function returns a natural number between 1 and 72, inclusive, to use with the MIT KEMAR dataset, which has 72 HRTFs with elevation of zero for azimuth increment of 5^o. The each of the 72 HRTF examples from the database is a 512 sample recorded sound taken from the KEMAR dummy head such that when convolved with a sound-source signal, it will give the effect of originating from the particular azimuth (direction) of the HRTF vector. Then we compute the velocity at each time t to create a multiplier for the frequency such that if an object has greater velocity, it will have a higher frequency, and otherwise have a lower frequency. This can be accomplished by taking the Cartesian straight line distance (SLD) between the object $\Delta(x,y)$ (Equation 12), and dividing by Δt (Equation 13):

$$\Delta(x,y)_{t_0:t_1} = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} \tag{12}$$

$$v = \frac{\Delta(x,y)}{\Delta t} \tag{13}$$

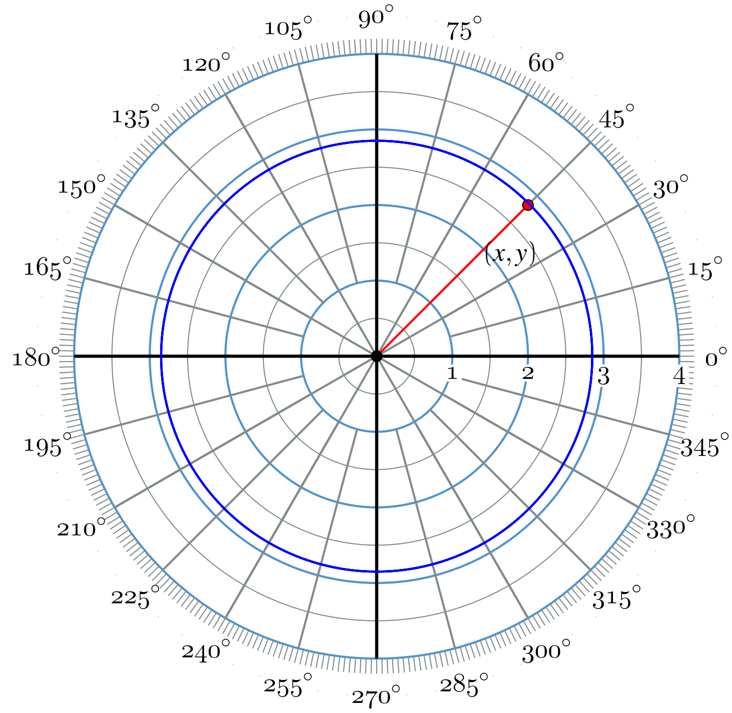

Illustration 2: The Polar graph (above) is representative of the translated values from Cartesian via the process shown in illustration 1 (across)

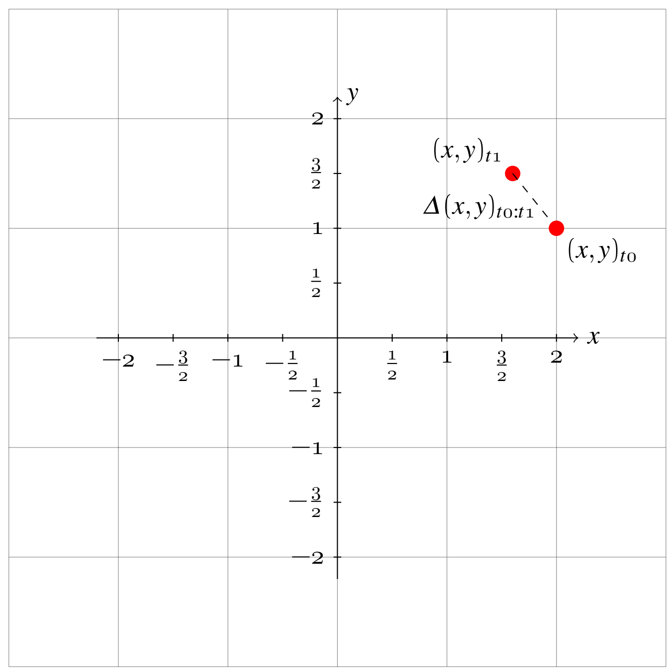

Illustration 3: The above graph illustrates the distance covered by a stellar object in one time-step, from which we can calculate the velocity using Equations 12 and 13

We then aggregate the theta (Equation 14), radius (Equation 15), and velocity (Equation 16) values into separate vectors that will make iteratively applying convolution simpler, as shown below:

$$\Theta = [\theta_0, \theta_1, ..., \theta_n] \tag{14}$$

$$\mathfrak{R} = [r_0, r_1, ..., r_n] \tag{15}$$

$$\mathscr{V} = [v_0, v_1, ..., v_n] \tag{16}$$

5.2 Non-linear factoring

An intermediate step that must be taken is to look at the velocity data and make aesthetic choices about how to extract the meaning we wish to emphasize. The goal here is to factor the data into a range that is effective for modulating the pitch of a particular frame such that it maintains continuity for the listener's ear to follow while remaining true to the relationships in the data. However, as the data is non-linear it does not lend well to being modeled functionally and led, in our case, to make specific choices about where to stretch and shrink subsections of the velocity data to fit our needs. The overall result was to factor the data such that the range between 0.15 and 3, effectively creating multipliers to be applied to the sound-source buffer vector whose initial frequency is relative to the black hole it represents.

5.3 Object Differentiation

In order to allow for the ear the differentiate the three objects in our data-set, we set different base frequencies for each black hole to be modulated by velocity. Here we chose frequencies that stayed in a specific range when multiplied by the velocity factor such that the resulting pitch would be both perceptible and comfortable for the human ear (between 20-10,000 Hertz) while allowing sufficient differentiation (164, 417, and 639Hz). These values are arbitrarily chosen with no mathematical basis for selection, and research into pitch relationships between objects may prove to be fruitful.

5.4 Memory Allocation

The final step to pre-process our data is to structure and allocate internal variables for storing the results of the sonification techniques to apply to the data discussed in the next section (5.5). We initialize a single sample buffer whose length is allocated to the size of one frame from the visualized data-set and is then populated with white Gaussian (random) noise. Generally, we need to find how many samples at a given sample rate will be required for each time-step in the data-set. Take the given sample rate, say 48,000 Hertz and divide by the total number of frames, giving the *samples per frame* for one second of time. We then use this measurement (SPF) to multiply by the duration of the visualization with which we wish to pair to match the total duration. In this case (Equation 17), the visualization we wish to pair is 42 seconds, yielding 48,000 Hz divided by 1012 frames multiplied by 42 seconds, and thus have the full sample size of the output vector at 2,016,000 samples. The generalized formula is displayed in Equation 18:

$$2{,}016{,}000 = \frac{48{,}000}{1012} \times 42 \tag{17}$$

$$TotalOutputSamples = \frac{SampleRate}{TotalFrames} \times FullDuration \tag{18}$$

5.5 Convolution Algorithm

We will allocate and iteratively store the results of convolution (Equation 1) with the Gaussian sine wave buffer with the frequency and amplitude modulation based on the velocity and the inverse square of the radius, respectively. In order to reduce the amount of unwanted noise from the system, we applied two main types of filtering, biquadratic linear filtering (Equation 3) and fade-ins/fade-outs (Equation 7) for each frame iteration. These output vectors are allocated to the full sample size based upon duration, frame rate, and samples per frame. In our case, this amounted to just over 2 million samples at 48kHz. The convolution must be done for both the left and right ear signals in order to combine both channels to achieve the binaural effect (Equations 19 and 20). Before applying the convolution, the Fast-Fourier Transform (Equation 2) is used to move from time-domain to frequency-domain such that applying convolution becomes simple multiplication and achieves the run-time performance improvement mentioned earlier. The sound-source signal is frequency modulated by the velocity at the current time-step, t, and is convolved with the respective left and right ear HRTF functions based on the azimuth of the object at time t. We then use the Inverse Fast-Fourier Transform (IFFT) to return to time-domain after the application of convolution. We then apply amplitude modulation based on the inverse square of the radius in order to create the effect of relative distance from the origin, or observer. The convolution process in full is shown in Equations 19 and 20, where the star operator $(*)$ is defined as convolution:

$$LES_t \leftarrow \sin(\frac{2\pi}{sr} \cdot \mathscr{V}_i) * HRTF_{Left}(\theta_i) \cdot \frac{1}{(\mathfrak{R}_i)^2} \tag{19}$$

$$RES_t \leftarrow \sin(\frac{2\pi}{sr} \cdot \mathscr{V}_i) * HRTF_{Right}(\theta_i) \cdot \frac{1}{(\mathfrak{R}_i)^2} \tag{20}$$

Key:

$$sr = \textit{sample rate}$$
$$\mathscr{V}_i = \textit{Velocity at current frame}$$
$$\mathfrak{R}_i = \textit{Radius at current frame}$$
$$HRTF_{L/R}(\theta_i) = \textit{HRTF data for azimuth at current frame}$$
$$LES_t = \textit{ Left Ear Signal at time } t$$
$$RES_t = \textit{ Right Ear Signal at time } t$$

The full process consisting of convolution, amplitude, and frequency modulation is applied iteratively for each frame, e.g. programmatically in a *for* loop in the MATLAB script found in the GitHub link at the end of this paper.

Algorithm 1 *For a more concrete illustration we provide pseudocode with references to line numbers (below):*

1: **procedure** SONIFY
2: $f \leftarrow$ *frequency*
3: $ft \leftarrow$ *filter size*
4: $fLen \leftarrow$ *Reduced convolved buffer length*
5: $sr \leftarrow$ *sample rate*
6: $tf \leftarrow$ *total frames*
7: $d \leftarrow$ *duration (seconds)*
8: $TotalSamples \leftarrow \frac{sr}{tf} * d$ # (Eq. 17)
9: $outputL \leftarrow$ zeros(1 to $TotalSamples$)
10: $outputR \leftarrow$ zeros(1 to $TotalSamples$)
11: $i \leftarrow 1$
12:
13: **for** j $\leftarrow$ 1 to $|\Theta|$ **do**
14: $f \leftarrow f * \mathscr{V}_j$ # Frequency modulation
15: *[B,A]* $\leftarrow$ biquadbpf(f, Q, *samplerate*) # (Eq. 4,5)
16:
17: # LEFT CHANNEL
18: $bufferL \leftarrow \sin(random(1 \text{ to } |bufferL|))$
19: $bufferL \leftarrow$ filter(B, A, $bufferL$) # (Eq. 6)
20: $bufferL \leftarrow$ fft($bufferL$) # To Freq domain
21: $temp \leftarrow$ convolve($bufferL$, HRTF$\{\theta_j\}$) # (Eq. 18)
22: $temp \leftarrow$ ifft($temp$) # To Time domain
23: $temp \leftarrow temp * (\frac{1}{\Re_j^2})$ # (Eq. 18)
24: $temp \leftarrow$ fadeout(ft, fadein(ft, $temp$)) # (Eq. 7)
25: $outputL_{i:i+fLen} \leftarrow temp$
26:
27: $i \leftarrow i + fLen - ft$ # Update output index position

Allocation of resource variables to be used in the main *for* loop occurs on lines 2-11, where we: declare the identifying frequency of the current object (Line 2), the number of samples to filter (Line 3), how many samples will remain from the convolution operation with the HRTF in the buffer (Line 4), sample rate (Line 5), total number of frames in the visualization (Line 6), duration of the visualization (Line 7), the total number of samples needed for output (Line 8, Equation 18), output vectors (Lines 9-10), and an index variable (Line 11). In the body of the for loop (Lines 13-27), the frequency is scaled by the velocity of the current time-step (Line 14), and we create a biquadratic filter matrix to use for filtering (Line 15). The buffer gets random Gaussian noise (Line 18), and then filter the buffer with the filter function created earlier (Line 15). Fast Fourier Transform is applied (Line 20), and then convolved (Line 21) before conversion back to time-domain with the Inverse Fast Fourier Transform (Line 22). The amplitude modulation is applied (Line 23), along with the fade-in/fade-out for the current frame (Line 24) before being inserted into the output vector (Line 25). The index range of the output vector is updated for the next iteration of the for loop (Line 27). Note: The # symbol denotes comments, to the right of which indicate corresponding equations.

5.6 Output

After the execution of the main *for* loop for each black hole, we condense the three pairs of binaural signals contained in the output vectors for each black hole by applying scalar addition, thus reducing the six channels to two which are then converted into WAV format in MATLAB and exported to disk. Shown in Equation 21, the two channels from the i^{th} black hole are stored as one variable in MATLAB. Each black hole will have two channels, and when linearly added (Equation 22) will then be exported using the *audiowrite()* function (Equation 23).

$$y_i = [outputL',\ outputR'] \tag{21}$$

$$outputVector = y_1 + \ldots + y_n \tag{22}$$

$$audiowrite(fileName.wav,\ outputVector,\ sampleRate) \tag{23}$$

6. Future Work

A natural expansion of this technique will include Z-axis elevations to provide higher dimensional reconstruction and finer resolution of the data with improvement to the technique. Next steps would naturally include experimental validation of dimensional information gain based on the technique, application of rigorous testing methodology for generalization to n-objects and exploration of frequency initialization for n-objects. From there, it will be of interest to apply the technique or some modification of it to objects that have volume or fields, such as nebulae, and then potentially generalize to objects with completely different features or attributes than the ones used herein.

7. Conclusion

Insofar as this technique has produced a novel sonification of an interstellar event of interest, there is certainly more to be desired as to the clarity and resolution of the data reproduction, particularly in domain of object differentiation and recognition, such that the technique may have more potential in aiding knowledge discovery in astronomical data. It is our hope that use and modification of this technique may yield robust algorithmic and computational methodologies for sonifying astronomical data in novel ways that add to the toolkit scientists can employ to aid knowledge discovery in modeling astronomical data using the highly evolved, complex pattern recognition system of the human ear.

8. Appendix

All code used for this project can be found on GitHub at:

`http://github.com/ifrit98/LIGO_Sonification`

This repository includes all data files, graphical figures, Python 3 files for formatting and converting data, MATLAB scrips which implement filtering and convolution, documentation, and a copy of the MIT KEMAR database at zero elevation used in this project.

A link to the visualization made by Dr. Bischof paired with the sonification can be viewed at the link below:

`https://www.youtube.com/watch?v=8sxf9IhrnDs`

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