APPLIED CALCULUS SAMPLE PROBLEMS

A list of sample problems that could be used to assess the two general education student learning outcomes is provided. Included are suggested questions, their mapping to each learning outcome, and the motivation behind that mapping.

Topic: LIMITS

1. **Student Learning Outcome: Perform College-level Mathematical Operations**

   Question: Evaluate the following limit, if it exists.
   \[
   \lim_\limits{x \to -3} \frac{x^2 + 1}{x}
   \]
   Motivation: Evaluate a simple limit.

2. **Student Learning Outcome: Comprehend and Evaluate Mathematical Information**

   Question: Let \(f\) be the function whose graph is
   ![Graph of a function]
   (a) Find \(\lim_\limits{x \to 1} f(x)\) if it exists.
   (b) Does \(\lim_\limits{x \to 3} f(x)\) exist?
   If so, determine the value. If not, explain why the limit does not exist.
   Motivation: Relate the graph of a function to the definition of the limit.

3. **Student Learning Outcome: Perform AND Comprehend**

   Question: Evaluate the following limit, if it exists.
   \[
   \lim_\limits{x \to 2} \frac{x - 2}{x^2 - 4}
   \]
   Motivation:
   - Comprehend: Recognize the indeterminate form.
   - Perform: Be able to factor and cancel to evaluate.
Topic: DIFFERENTIATION

1. Student Learning Outcome: Perform College-level Mathematical Operations
   
   Question: Differentiate the following with respect to $x$.
   
   $$y = e^{2x} + x^3 + 5$$
   
   Motivation: Differentiate a function using the rules and

2. Student Learning Outcome: Comprehend and Evaluate Mathematical Information
   
   Question: Let $f$ be the function whose graph is provided

   ![Graph](image)

   (a) Determine at what $x$-values, if any, the function is not differentiable.
   (b) Determine at what $x$-values, if any, the derivative is zero.
   (c) Is $f'(2)$ positive, negative, or zero?
   (d) Put the following values in order from least to greatest.
       $f'(0), f'(0), f'(2), f'(4)$

   Motivation: Relate the graph of the function to the definition of

3. Student Learning Outcome: Perform AND Comprehend
   
   Question: At what value(s) of $x$, if any, does the graph of the function have a horizontal tangent line?

   $$f(x) = 2x^3 - 3x^2 - 36x + 5$$

   Motivation:
   - Comprehend: Know that the derivative of a function is zero where the graph of the function has a horizontal tangent line.
   - Perform: Differentiate a function and solve an equation for $x$. 
Topic: INTEGRATION

1. **Student Learning Outcome: Perform College-Level Mathematical Operations:**

   Question: Integrate the following.
   \[ \int (5x^3 + 7e^{2x} + 1) \, dx \]
   
   Motivation: Integrate a function using the rules and formulas.

2. **Student Learning Outcome: Comprehend and Evaluate Mathematical Information:**

   Question: Suppose the shaded area is caught between the graph of \( f \) and the \( x \)-axis. Use the figure to set up the definite integral that describes the net area of the shaded region.

   ![Graph](image)

   Motivation: Connect the definite integral with the graph of a region.

**Student Learning Outcome: Perform AND Comprehend**

Question: Find the area of the bounded region enclosed by the graphs of the functions \( y = 4 - x^2 \) and \( y = x \).

Motivation:
- Comprehend: Sketch the region \( R \) and set up the definite integral defining its area.
- Perform: Determine the limits of integration and evaluate the integral to calculate the area.