Complex Fractions

I. Fractions

a. Definition: A complex fraction is a fraction where either the numerator, denominator, or both contain fractions.

b. Examples:

1. $\dfrac{\frac{a}{b}}{c}$

2. $\dfrac{\frac{1}{x}+4}{3}$

3. $\dfrac{\frac{a}{b}+\frac{c}{d}}{\frac{x}{y}-4}$

II. To Simplify A Complex Fraction

a. Find the least common denominator of all fractions within the fraction.

b. Multiply the numerator and denominator of the complex fraction by the LCD found in part a.

c. Simplify the resulting fraction.

III. Mixed Numbers & Improper Fractions

a. $\dfrac{\frac{3}{x}}{2}$

$\frac{3}{x}$ is the only fraction within the complex fraction. The LCD is x.

$$\frac{3}{x}\times\frac{x}{1}$$

$$2\times\frac{x}{1}$$

Multiply the top and bottom by x.
Simplify as the last step.

Answer: $\dfrac{3}{2x}$

b. $\dfrac{\frac{3}{x}}{\frac{2}{y}}$

$\frac{3}{x}$ and $\frac{2}{y}$ are the only fractions within the complex fraction. The LCD is xy.

$$\frac{\frac{3}{x}\times\frac{xy}{1}}{\frac{2}{y}\times\frac{xy}{1}}$$

Multiply the top and bottom by xy. Simplify as the last step.

Answer: $\dfrac{3y}{2x}$

c. $\dfrac{\frac{3}{x}+1}{\frac{2}{y}+\frac{3}{4}}$

$\frac{3}{x}$, $\frac{2}{y}$ and $\frac{3}{4}$ are the only fractions within the complex fraction. The LCD is $4xy$.

$$\frac{\left(\frac{3}{x}+1\right)4xy}{\left(\frac{2}{y}+\frac{3}{4}\right)4xy}$$

Multiply <u>every</u> term in the top and bottom by $4xy$. Simplify as the last step.

Answer: $\dfrac{12y+4xy}{8x+3xy}$

Practice Problems:

1. $\dfrac{1/a}{1/a^2}$

2. $\dfrac{1/a}{1/b}$

3. $\dfrac{1/x}{y}$

4. $\dfrac{\frac{1}{x}+\frac{1}{y}}{xy}$

5. $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{y}+\frac{1}{z}}$

6. $\dfrac{\frac{1}{x+2}-\frac{1}{x}}{2}$

7. $\dfrac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x^2y^2}}$

8. $\dfrac{\frac{1}{x-3}+\frac{1}{x^2-2x-3}}{\frac{1}{x+1}}$

9. $\dfrac{\frac{1}{a}+\frac{1}{b}}{a^2-b^2}$

10. $\dfrac{2-\frac{1}{x}}{2x-1}$

11. $\dfrac{\frac{1}{3}-\frac{1}{a}}{\frac{1}{b}}$

12. $\dfrac{\frac{1}{a+b}-\frac{1}{a-b}}{\frac{2b}{a^2-b^2}}$

13. $\dfrac{\frac{1}{x+y}-\frac{1}{y}}{\frac{1}{x^2-y^2}}$

14. $\dfrac{\frac{1}{2x}-\frac{1}{6y}}{\frac{1}{3y}+\frac{1}{4x}}$

Answers to Complex Fractions:

1. a

2. $\frac{b}{a}$

3. $\frac{1}{xy}$

4. $\frac{x+y}{x^2y^2}$

5. $\frac{yz+xz}{xz+xy}$

6. $\frac{-1}{x(x+2)}$

7. y^2-x^2

8. $\frac{x+2}{x-3}$

9. $\frac{1}{ab(a-b)}$

10. $\frac{1}{x}$

11. $\frac{ab-3b}{3a}$

12. -1

13. $\frac{xy-x^2}{y}$

14. $\frac{6y-2x}{4x+3y}$