# Exponents

## I. Exponents

a. Definition: An *exponent* refers to the number of times a base needs to be multiplied.

b. $x^4 = x \cdot x \cdot x \cdot x$

Note: Since $x$ = base and $4$ = exponent, there are $4$ $x$'s in the product.

c. $\left(\frac{1}{2}\right)^3 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

d. An exponent refers only to that which is immediately to its left.

1. $3^2 = 9$
2. $-3^2 = -9$ Here the exponent refers only to the 3, not the sign.
3. $(-3)^2 = 9$ Here the ( ) are to the left of the exponent so the exponent refers to everything in the ( ).

## II. Laws of Exponents

1. $a^{-m} = \frac{1}{a^m}$
2. $a^n a^m = a^{n+m}$
3. $\frac{a^m}{a^n} = a^{m-n}$
4. $(a^m)^n = a^{m \times n}$
5. $(ab)^m = a^m b^m$
6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
7. $a^{m/n} = \sqrt[n]{a^m}$
8. $b^0 = 1$ for any $b \neq 0$

## III. Examples

1. $a^{-2} = \frac{1}{a^2}$ (Rule #1)

2. $2^3 2^5 = 2^{3+5} = 2^8 = 256$ (Rule #2)

3. $(xy)^2 = x^2 y^2$ (Rule #5)

4. $8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$ (Rule #7)

   Or

   $8^{2/3} = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$

5. $(b^6)^7 = b^{6\times 7} = b^{42}$ (Rule #4)

6. $\left(\frac{2}{m}\right)^2 = \frac{2^2}{m^2} = \frac{4}{m^2}$ (Rule #6)

7. $(3a)^0 = 1$ (Rule #8)

8. $-27^{2/3} = -\left(\sqrt[3]{27}\right)^2 = -3^2 = 9$ (Rule #7)

9. $\left(\frac{2p^{-2}}{3r^3}\right)^{-4} = \frac{2^{-4}(p^{-2})^{-4}}{3^{-4}(r^3)^{-4}} = \frac{2^{-4}p^8}{3^{-4}r^{-12}} = \frac{3^4 p^8 r^{12}}{2^4}$ (Rule #4)

   (Rule #1)

   $= \frac{81p^8 r^{12}}{16}$

Practice Problems:

1. $\left(x^3\right)^7$

2. $\left(5ab^3c^2\right)\left(b^2c\right)$

3. $\left(xy^{-2}\right)^4\left(xy\right)^3$

4. $\dfrac{-36p^3q}{-4p^5}$

5. $\dfrac{4(x+y)^2}{3(x-y)(x+y)}$

6. $\left(25x^8y^6\right)^{1/2}$

7. $81^{-1/2}$

8. $\left(xy^{-4}\right)^{-2}\left(xy^{-1}z\right)$

9. $\dfrac{27a^3b^7}{-3a^5b^2}$

10. $\left(x^{1/2}y^{-2/3}z^{5/6}\right)^6$

11. $y^{2n}\cdot y^{4n+1}$

12. $\left[\left(2a^4b^3\right)^3\right]^2$

13. $\left(b^{2n-1}\right)^n$

14. $\dfrac{a^{5n}}{a^{3n}}$

15. $\left(\dfrac{4^{-2}xy^{-3}}{x^{-3}y}\right)^3\left(\dfrac{8^{-1}x^{-2}y}{x^4y^{-1}}\right)^{-2}$

16. $\dfrac{x^{-3/5}}{x^{1/5}}$

17. $\left(x^8y^2\right)^{1/2}$

18. $a^{-1/4}\left(a^{5/4}-a^{9/4}\right)$

19. $\left(x^{5n}\right)^{2n}$

20. $\left(x^{n/4}y^{n/8}\right)^8$

Answers to Exponents:

1. $x^{21}$
2. $5ab^5c^3$
3. $\frac{x^7}{y^5}$
4. $\frac{9q}{p^2}$
5. $\frac{4(x+y)}{3(x-y)}$
6. $5x^4y^3$
7. $\frac{1}{9}$
8. $\frac{y^7z}{x}$
9. $\frac{-9b^5}{a^2}$
10. $\frac{x^3z^5}{y^4}$
11. $y^{6n+1}$
12. $64a^{24}b^{18}$
13. $b^{2n^2-n}$
14. $a^{2n}$
15. $\frac{x^{24}}{64y^{16}}$
16. $\frac{1}{x^{4/5}}$
17. $x^4y$
18. $a-a^2$
19. $x^{10n^2}$
20. $x^{2n}y^n$