Logarithms

I. Logarithms

a. Definition: A *logarithm* is the exponent required to produce a given number.

b. The *General Form* is: $y = \log_a x$. It can be rewritten as $a^y = x$.

   1. Ex) $\log_2 8 = 3$ In exponential form: $2^3 = 8$

c. $\log_a x$ is only defined if a and x are both positive, and $a \neq 1$.

d. The *Common Logarithm* has a base of 10. In general usage, the subscript 10 is omitted when dealing with common logs.

   1. Ex) $\log 10 = 1$ In exponential form: $10^1 = 10$

e. A *Natural Logarithm* is a logarithm where the base is the number "e" (where $e = 2.72828$). $\log_e x$ is replaced with $\ln x$. Natural logarithms play an important role in theoretical mathematics and the natural sciences.

II. Properties of Logs:

Identity	Example
Log of a Product: $\log xy = \log x + \log y$	$\log_2 24 = \log_2 6 + \log_2 4$
Log of a Quotient: $\log \frac{x}{y} = \log x - \log y$	$\log_2 \frac{10}{3} = \log_2 10 - \log_2 3$
Log with Exponent: $\log x^a = a \log x$ $\log \sqrt[a]{x} = \log x^{1/a} = \frac{1}{a} \log x$	$\log_2 (8^3) = 3 \log_2 8$ $\log \sqrt[4]{32} = \log (32)^{1/4} = \frac{1}{4} \log 32$
Identities: $\log_a a = 1$ $\log_a 1 = 0$ $\ln e = 1$ $\ln 1 = 0$	$\log_3 3 = 1$ $\log_2 1 = 0$
Negative Exponents: $\log_a \frac{1}{x} = -\log_a x$	$\log_3 \frac{1}{9} = -\log_3 9$
Change in base (from base a to base 10): $\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$	$\log_2 5 = \frac{\log 5}{\log 2} = \frac{\ln 5}{\ln 2}$

III. Solving Equations Using Logarithms

1. $4^x = 7$

$\ln 4^x = \ln 7$	Take the log of both sides. You can use the common logarithm or natural log, but in practice the natural log (or ln) is used more.
$x \ln 4 = \ln 7$	Use the exponent property to rewrite, then divide to solve for x.
$x = \dfrac{\ln 7}{\ln 4}$ or $\log_4 7$	

2. $6(10^x) = 11^x$

$\ln[6(10^x)] = \ln 11^x$	Take the log of both sides.
$\ln 6 + \ln 10^x = \ln 11^x$	
$\ln 6 + x \ln 10 = x \ln 11$	Use exponent property.
$\ln 6 = x \ln 11 - x \ln 10$	Get x terms on one side.
$\ln 6 = x(\ln 11 - \ln 10)$	Factor out the x terms.
$x = \dfrac{\ln 6}{\ln 11 - \ln 10}$	Divide and solve for x.

3. $4e^x = 7$

$e^x = \dfrac{7}{4}$	Divide by 4 to get e^x by itself.
$\ln e^x = \ln \dfrac{7}{4}$	Take the log of both sides.
$x \ln e = \ln \dfrac{7}{4}$	Use exponent rule.
$x(1) = \ln \dfrac{7}{4}$	Use identity that $\ln e = 1$.
$x = \ln \dfrac{7}{4}$ or $x = \ln 7 - \ln 4$	Simplify.

Practice Problems:
Write in exponential form:

1. $\log_3 27 = 3$

2. $\log_2 32 = 5$

3. $\log_{17} 1 = 0$

Write it logarithmic form:

4. $6^2 = 36$

5. $10^3 = 1000$

6. $9^{1/2} = 3$

Solve for x:

7. $\log_x 49 = 2$

8. $\log_5 125 = x$

9. $\log_8 x = 2$

10. $e^x = 11$

11. $\frac{1}{4}e^{x} = 20$

12. $5^{x} = 26$

13. $\log_{16} x = \frac{1}{2}$

14. $\log_{7} x = -1$

15. $\log_{3} \frac{1}{9} = x$

Use the properties of logs to write in expanded form:

16. $\log_{8}(xz)$

17. $\log_{7}\left(\frac{u^{3}}{v^{4}}\right)$

18. $\log_{5}\left(\frac{xy^{2}}{z^{4}}\right)$

19. $\ln\left(x^2yz\right)$

20. $\log_7\sqrt{\frac{x^3}{y}}$

Solve the following equations using logarithms:

21. $5^x = 6$

22. $e^x = 3$

23. $2^{x-1} = 6$

24. $3^{2x-1} = 4$

Answers to Logarithms:

1. $3^3 = 27$

2. $2^5 = 32$

3. $17^0 = 1$

4. $\log_6 36 = 2$

5. $\log 1000 = 3$

6. $\log_9 3 = \frac{1}{2}$

7. $x = 7$

8. $x = 3$

9. $x = 64$

10. $x = \ln 11$

11. $x = \ln 80$

12. $x = \frac{\ln 26}{\ln 5}$

13. $x = 4$

14. $x = \frac{1}{7}$

15. $x = -2$

16. $\log_8 x + \log_8 z$

17. $3\log_7 u + 4\log_7 v$

18. $\log_5 x + 2\log_5 y - 4\log_5 z$

19. $2\ln x + \ln y + \ln z$

20. $\frac{3}{2}\log_7 x - \frac{1}{2}\log_7 y$

21. $x = \frac{\ln 6}{\ln 5}$ or $\log_5 6$

22. $x = \ln 3$

23. $x = \frac{\ln 6}{\ln 2} + 1$

24. $x = \frac{1}{2}\left(\frac{\ln 4}{\ln 3} + 1\right)$