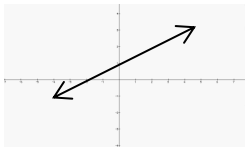
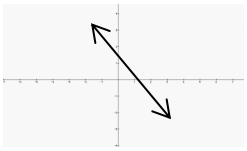
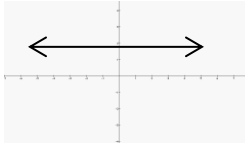
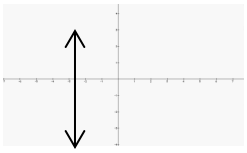
Graphing Linear Equations

I. Graphing Linear Equations

a. The graphs of first degree (linear) equations will always be straight lines.

b. Graphs of lines can have…

Positive Slope	Negative Slope	Zero slope (horizontal line)	Undefined slope (vertical line)

c. The methods of graphing linear equations that have slope are

i. Substitution
ii. Intercepts
iii. Slope – Intercept
iv. Point – Slope

d. Substitution Method

To generate points (x, y)…

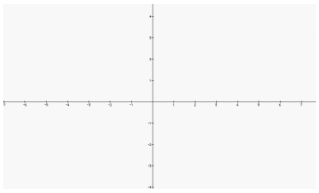
1. arbitrarily choose (at least 3) values for x
2. substitute each of these values of x into the equations and solve for the corresponding y value
3. Plot the 3 (or more) points on the graph and connect with a straight line. (All the points will fall on the same straight line. If this is not the case, check your work for error.)

Example 1: Graph $y = 3x + 2$

Choose 3 values for *x*:

Substitute each value into the equation and find y:

Plot the 3 points (0, 2), (1, 5), and (-1, -1) on graph and connect with a line:



e. Intercepts Method

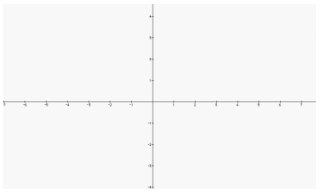
i. X-intercept – the point where the line crosses the x, axis (x, 0)
ii. Y-intercept – the point where the line crosses the y-axis (0, y)
iii. To graph a line using this method…
    1. Find the x-intercept by letting y = 0 and solving for x
    2. Find the y-intercept by letting x = 0 and solving for y
    3. Plot both intercepts on the graph and connect with a straight line

Example 2: Graph $3x + 4y = 2$

Find the x-intercept

Find the y-intercept

Plot and connect

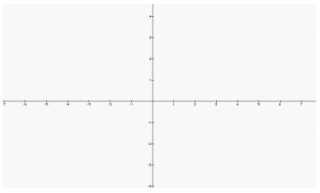


f. Slope: the slant or tilt of a line; the ratio of $\frac{vertical\ \ change}{horizontal\ \ change}$

In algebraic notation, slope is represented by the letter *m* for 2 points represented by (x_1, y_1) and (x_2, y_2); $m = \frac{y_2 - y_1}{x_2 - x_1}$

Example 3: Find the slope of the line passing through (4, 3) and (3, 7)

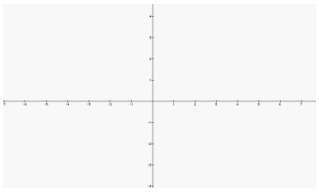
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



Notice that because the line has negative slope, the line drops when reading it from left to right.

Example 4: Find the slope of the line through the points (-3, -1) and (-1, 3)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



Notice that because the line has positive slope, the line climbs when reading it from left to right.

g. Slope – Intercept Method

i. If the equation of a line is in the form $y = mx + b$, then m is the slope and b is the y-intercept.

ii. To graph equations using the Slope – Intercept Method…

1. Determine the slope (m) and the y-intercept (b)
2. Plot the y-intercept (0, b)
3. Generate additional points on the line by starting at the y-intercept and moving the rise and run of the slope.

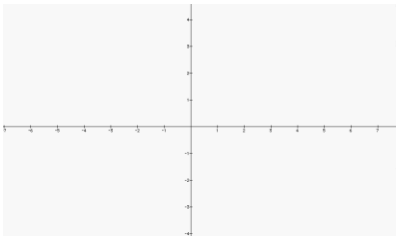
Example 5: Graph $4x + 5y = 10$

First put the equation into $y = mx + b$ form:

State the y-intercept (b) and the slope (m)

Plot the y-intercept

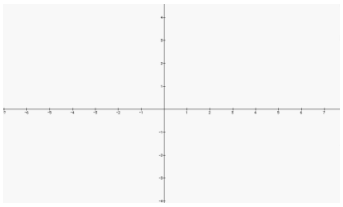
Starting at the y-intercept, count down 4 units and right 5 units *or* up 4 units and left 5 units



h. Point – Slope Method
   i. The method outlined in the previous Slope – Intercept Method can be used to graph a line starting at any given point.

Example 6: Graph the line which passes through the point (-1, 1) and has a slope of ½ .

Begin at (-1, 1); then using $m = \frac{1}{2}$, move up 1 unit and right 2 units

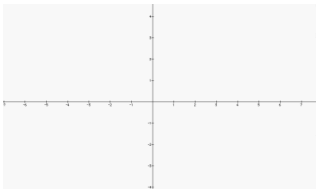


i. Graphing Vertical and Horizontal Lines
   i. The preceding methods will not work for equations in the form x = a or y = b.

   ii. Vertical Lines

1. Equation: x = *a* 2. The graph is a line parallel to the y-axis and passes through the x-intercept (*a*, 0). 3. All the x-coordinates on the line are *a*. 4. The slope of a vertical line is undefined.

Example 7: Graph $x = -2$

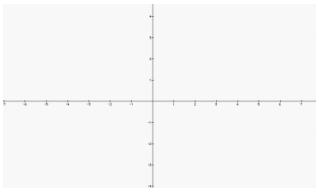


   iii. Horizontal Lines

1. Equation: y = *b* 2. The graph is a line parallel to the x-axis and passes through the y-intercept (0, *b*). 3. All the y-coordinates on the line are *b*. 4. The slope of a horizontal line is zero.

Example 8: Graph the line which has a slope of zero and passes through the point (-2, 3).

If m = 0, then the line is horizontal. Its equation reads y = 3.



j. Graphing Parallel and Perpendicular Lines

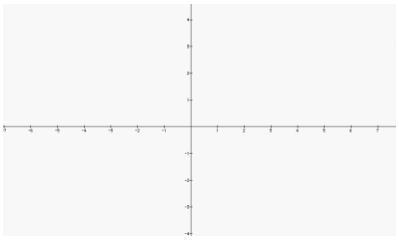
i. Parallel lines: Slopes are equal $(m_1 = m_2)$

Example 9: Graph the line which is parallel to $3x + 2y = 4$ and has a y-intercept of 4.

Put in $y = mx + b$ form and find *m*.

Use the same slope.

Begin at (0, 4) and move down 3 units and right 2 units.



ii. Perpendicular lines: Slopes are negative reciprocals of each other

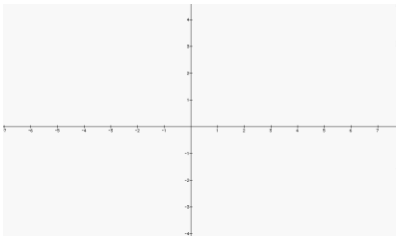
$$m_1 = \frac{-1}{m_2}$$

Example 10: Graph the line which is perpendicular to $3x + 2y = 4$ and passes through the point (-1, 2).

Put in $y = mx + b$ form and find *m*.

Use the negative reciprocal of the slope.

Begin at (-1, 2) and move up 2 units and right 3 units.



Graphing Linear Equations Problems

Graph the following using the method (if given)

1. $y = 3x - 7$ (using substitution method)

2. $3x + 4y = 12$ (using intercepts method)

3. $2x + 4y = 8$ (using slope-intercept method)

4. The line passing through (-2, -1) and has a slope of $\frac{-2}{3}$.

5. $x = 0$

6. $y = -2$

7. Choose any two points on the line x = 7 and find the slope.

8. Choose any two points on the line y = -3 and find the slope.

9. Graph together on the same set of axes: $y = \frac{1}{2}x + 4$ and $y = -2x + 1$

10. Graph by any method: $5y - 2x = 10$