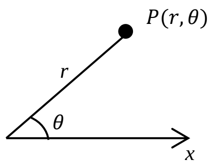
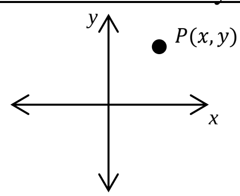
Polar Coordinates

Definitions

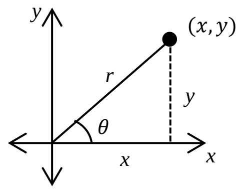
Polar Coordinate System



Cartesian Coordinate System



Converting between Cartesian and Polar



$cos\theta = \frac{x}{r} \;\rightarrow\; x = rcos\theta$

$sin\theta = \frac{y}{r} \;\rightarrow\; y = rsin\theta$

$r^2 = x^2 + y^2$ (Pythagorean)

$tan\theta = \frac{y}{x}$

Polar to Cartesian Conversion

$x = rcos\theta$

$y = rsin\theta$

Cartesian to Polar Conversion

$r^2 = x^2 + y^2$

$tan\theta = \frac{y}{x}$

Tangents to Polar Curves

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dy}{d\theta}} = \frac{\frac{dr}{d\theta}sin\theta + rcos\theta}{\frac{dr}{d\theta}cos\theta - rsin\theta}$$

Found using $x = rcos\theta$ and $y = rsin\theta$

Problems

I. Write the Cartesian equations from the given Polar equations

a) $r = 3sin\,\theta$

$r^2 = 3rsin\theta$
$x^2 + y^2 = 3y$
$x^2 + y^2 - 3y = 0$
$x^2 + \left(y - \frac{3}{2}\right)^2 = \frac{9}{4}$

b) $r^2 = sin2\theta$

$r^2 = 2sin\theta cos\theta$
$r^4 = 2rsin\theta rcos\theta$
$(x^2 + y^2)^2 = 2yx$

c) $r^2 = \theta$

$\tan(r^2) = tan\theta$
$\tan(x^2 + y^2) = \frac{y}{x}$

II. Write the Polar equations from the given Cartesian equations

a) $x^2 = 4y$

$r^2\cos^2\theta = 4rsin\theta$
$r\cos^2\theta = 4sin\theta$
$r = \frac{4sin\theta}{\cos^2\theta}$
$r = 4tan\theta sec\theta$

b) $x^2 - y^2 = 1$

$r^2(\cos^2\theta - \sin^2\theta) = 1$
$r^2(cos2\theta) = 1$
$r^2 = sec2\theta$

c) $y = 2x - 1$

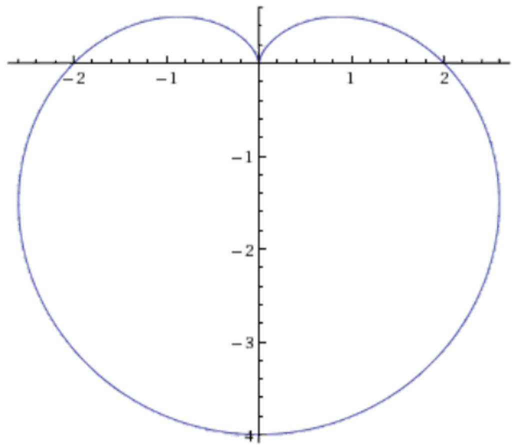
$rsin\theta = 2rcos\theta - 1$
$r(2cos\theta - sin\theta) = 1$
$r = \frac{1}{(2cos\theta - sin\theta)}$

Problems (continued)

III. Sketch the Curve by first converting to Cartesian

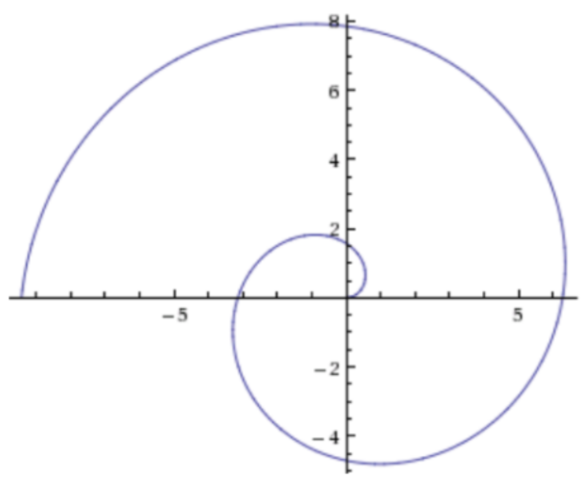
a) $r = 2(1 - sin\theta)$ (Cardioid)

θ	$sin\theta$	$2(1 - sin\theta)$
0	0	2
$\frac{\pi}{2}$	1	0
π	0	2
$\frac{3\pi}{2}$	-1	4
2π	0	2



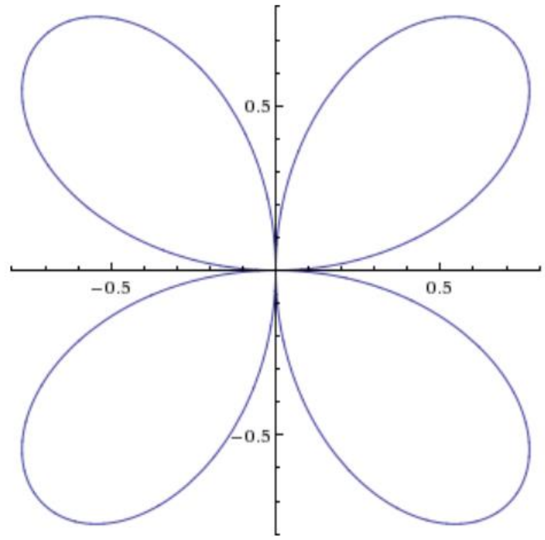
b) $r = \theta$

θ	r
0	0
$\frac{\pi}{2}$	$\frac{\pi}{2}$
π	π
$\frac{3\pi}{2}$	$\frac{3\pi}{2}$
2π	2π



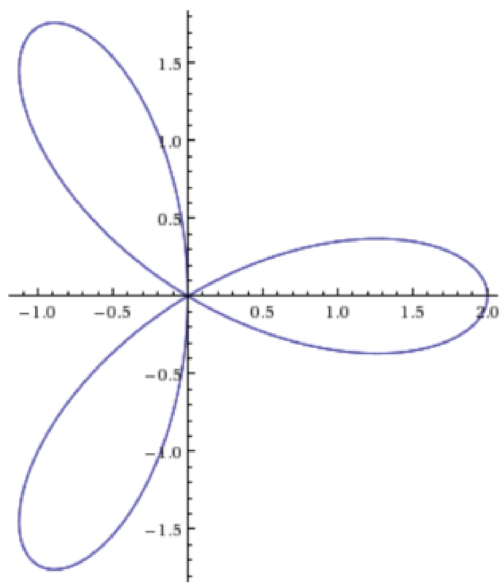
c) $r = \sin(2\theta)$

θ	2θ	$sin2\theta$
0	0	0
$\frac{\pi}{4}$	$\frac{\pi}{2}$	1
$\frac{\pi}{2}$	π	0
$\frac{3\pi}{4}$	$\frac{3\pi}{2}$	-1
π	2π	0
$\frac{5\pi}{4}$	$\frac{5\pi}{2}$	1
$\frac{3\pi}{2}$	3π	0
$\frac{7\pi}{4}$	$\frac{7\pi}{4}$	-1
2π	4π	0



c) $r = 2\cos(3\theta)$

θ	3θ	$2\cos(3\theta)$
0	0	1
$\frac{\pi}{6}$	$\frac{\pi}{2}$	0
$\frac{\pi}{3}$	π	-1
$\frac{\pi}{2}$	$\frac{3\pi}{2}$	0
$\frac{2\pi}{3}$	2π	1
$\frac{5\pi}{6}$	$\frac{5\pi}{2}$	0
π	3π	-1



IV. Find the slope of the tangent line to the following polar curve

$r = 3cos\theta \quad \left(at\ \theta = \frac{\pi}{3}\right)$

Given:

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}sin\theta + rcos\theta}{\frac{dr}{d\theta}cos\theta - rsin\theta}$$

$$\frac{dy}{dx} = \frac{-3sin\theta sin\theta + 3cos\theta cos\theta}{-3sin\theta cos\theta - 3cos\theta sin\theta} = \frac{3(\cos^2\theta - \sin^2\theta)}{-3(2sin\theta cos\theta)} = \frac{-cos2\theta}{sin2\theta} = -cot2\theta$$

$$\frac{dy}{dx}\left(at\ \theta = \frac{\pi}{3}\right) = -\cot\left(\frac{2\pi}{3}\right) = \frac{1}{\sqrt{3}}$$

Alternate Approach:

$r = 3cos\theta \quad \left(at\ \theta = \frac{\pi}{3}\right)$

Then: $x = rcos\theta = (3cos\theta)cos\theta = 3\cos^2\theta$

$y = rsin\theta = (3cos\theta)sin\theta$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dy}{d\theta}} = \frac{(3\cos^2\theta - 3\sin^2\theta)}{-(6sin\theta cos\theta)} = \frac{-cos2\theta}{sin2\theta} = -cot2\theta$$

$$\frac{dy}{dx}\left(at\ \theta = \frac{\pi}{3}\right) = -\cot\left(\frac{2\pi}{3}\right) = \frac{1}{\sqrt{3}}$$