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Binary Numbers

Understanding the Binary System:

Decimal or Base-10 numbers uses ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 (count them!) and position plays a major role in expressing their meaning. For example, $53{,}802_{10}$ means

$$\underbrace{5 \times 10^4}_{\text{Ten-thousands}} + \underbrace{3 \times 10^3}_{\text{Thousands}} + \underbrace{8 \times 10^2}_{\text{Hundreds}} + \underbrace{0 \times 10^1}_{\text{Tens}} + \underbrace{2 \times 10^0}_{\text{Units}}$$

Binary or Base-2 numbers uses only two symbols: 0 and 1 and position again plays a major role in expressing their meaning. For example, 10110_2 means

$$\underbrace{1 \times 2^4}_{\text{Sixteens}} + \underbrace{0 \times 2^3}_{\text{Eights}} + \underbrace{1 \times 2^2}_{\text{Fours}} + \underbrace{1 \times 2^1}_{\text{Twos}} + \underbrace{0 \times 2^0}_{\text{Ones (Units)}}$$

You should know the "twos places" to *at least* 2^{10}:

2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1024	512	256	128	64	32	16	8	4	2	1

Changing a Binary Number to a Decimal Number

Example: Rewrite the binary number 101101_2 as a decimal number.

1	0	1	1	0	1
32	~~16~~	**8**	**4**	~~2~~	**1**

and 32 + 8 + 4 + 1 = $\mathbf{45_{10}}$

Now you try one:

(perform all work on a separate page)

$10011101_2 = \_\_\_\_\_\_\_\__{10}$

Changing a Decimal Number to a Binary Number

Repeatedly divide by two and record the remainder for each division – read "answer" upwards.

Example: Rewrite the decimal number 21_{10} as a binary number.

```
2 | 21
2 | 10   R=1
2 |  5   R=0
2 |  2   R=1
2 |  1   R=0
     0   R=1  ↑read↑
```

2 divides into 21 ten times with a remainder of 1; then 2 divides into 10 exactly five times with a remainder of 0; and so forth...

$21 = 2 \cdot 10 + \mathbf{1}$
$10 = 2 \cdot 5 + \mathbf{0}$
$5 = 2 \cdot 2 + \mathbf{1}$
$2 = 2 \cdot 1 + \mathbf{0}$
$1 = 2 \cdot 0 + \mathbf{1}$

The binary result is read upwards↑, therefore $\mathbf{21_{10} = 10101_2}$

Now you try one:

$68_{10} = \_\_\_\_\_\_\_\_\_\__2$

Adding Binary Numbers

Example: Find the sum of 1011_2, 1101_2, and 1001_2.

$$\begin{array}{r} {\scriptstyle 1\,1\,1} \\ 1011 \\ 1101 \\ +\ 1001 \\ \hline 100001 \end{array}$$

Think 1+1+1=3 and translate $3_{10} = 11_2$. Enter the 1 and carry one.

Think 1 + 1 = 2 and translate $2_{10} = 10_2$. Enter the 0 and carry the one.

"4_{10}" is 100_2

Now you try one:

$$\begin{array}{r} 10101 \\ 1111 \\ +\ \ 1010 \\ \hline \end{array}$$

Subtracting Binary Numbers

Example 1: Find the difference between 101_2 and 100_2.

$$\begin{array}{r} 101 \\ -\ \ 100 \\ \hline 001 = 1_2 \end{array}$$

Check:

$$\begin{array}{r} 100 \\ +\ \ \ \ 1 \\ \hline 101 \end{array}$$

Example 2: Find the difference of 1010_2 and 111_2.

$$\begin{array}{r} {\scriptstyle 0\,1\,{}^{1}0\,{}^{1}0} \\ \cancel{101}0 \\ -\ \ \ 111 \\ \hline 11 \end{array}$$

You can't take 1 from 0, so you have to borrow 1 from the first 1 you come to as you move to your left - all zeroes preceding this 1 changes from 0 to a 1.

$10_2 - 1_2 = 1$ (think 2 minus 1 in base 10, and then translate back to base 2.

Check:

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 111 \\ +\ \ \ 11 \\ \hline 1010 \end{array}$$

Example 3: Here is another example where borrowing is necessary:

$$\begin{array}{r} {\scriptstyle 0\,1\,1\,1\,1\,{}^{1}0} \\ \cancel{10000}0 \\ -\ \ \ \ \ \ 1 \\ \hline 011111 \end{array}$$

Now you try one:

$$\begin{array}{r} 10010 \\ -\ \ \ 101 \\ \hline \end{array}$$

Multiplying Binary Numbers

Example: Find the product of 1011_2 and 101_2. [Note that multiplying by 1 leaves the number unchanged and multiplying by 0 always results in zero.]

```
      1011
  x    101
  --------
      1011  → multiplying by 1 leaves the number unchanged
     0000X  → X is a placeholder and multiplying by 0 is 0000 (zero)
 +  1011XX  → two place holders (XX) and multiplying by 1 again
  --------    leaves the number unchanged
    110111
```

Check by translating the factors and product into base ten:

$1011_2 = 11_{10}$

$101_2 = 5_{10}$

So, 110111_2 should equal 55_{10}. You should verify this.

Now you try one:

```
   10011
 x  1011
 -------
```

Dividing Binary Numbers

Example: Divide of 10010_2 by 11_2. [Note that a binary number "goes into" another binary number either no times or one time. There are no other possibilities.] Follow the division algorithm learned in grammar school

```
      110
 11 |10010
    -11
      11
     -11
      00
     -00
       0
```

11 does not go into 1, nor does it go into 10, but it will go into 100 – one time! (You can "think" in base 10 here.)

Multiply 1 times 11 and subtract it from 100_2 - borrowing will be necessary and the result in this case is 1.

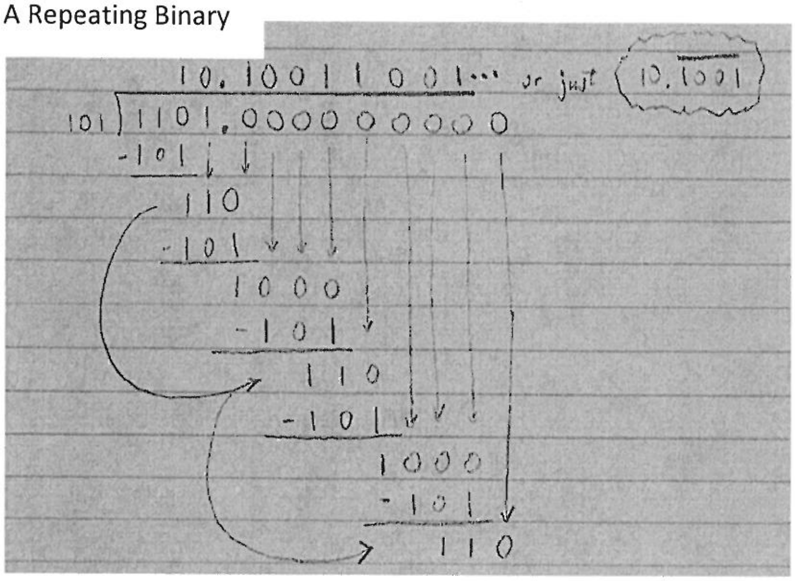
Bring down the one and begin the division algorithm all over again, (i.e., divide, multiply, subtract, and bring down the next bit.

Hence the quotient will be 110_2. To check, multiply the divisor, 11_2, by the quotient, 110_2. The result should be 10010_2. Verify.

Now you try one:

11011 ÷ 11

A Repeating Binary



EXERCISE SETS:

1. **Change each binary number to a decimal number.**
   - **a.** 101011_2
   - **b.** 1000011_2
   - **c.** 11011_2
   - **d.** 111111_2

2. **Change each decimal number to a binary number.**
   - **a.** 78
   - **b.** 133
   - **c.** 500
   - **d.** 222

3. **Add:**
   - **a.** $1111_2 + 10101_2$
   - **b.** $10011_2 + 1000_2 + 110011_2$
   - **c.** $1111.101_2 + 111.001_2 + 11.000_2 + 1.111_2$
   - **d.** $1011_2 + 1001_2 + 1111_2 + 1010_2 + 11_2$

4. **Subtract:**
   - **a.** $1011_2 - 101_2$
   - **b.** $10001_2 - 11_2$
   - **c.** $10101_2 - 1010_2$
   - **d.** $10000_2 - 1_2$

5. **Multiply:**
   - **a.** $101_2 \times 11_2$
   - **b.** $1010_2 \times 101_2$
   - **c.** $1001_2 \times 1001_2$
   - **d.** $111_2 \times 111_2$

6. **Divide:**
   - **a.** $1111 \div 100$
   - **b.** $1000 \div 111$
   - **c.** $1010 \div 11$
   - **d.** $11101 \div 110$

ANSWERS

You Try One:

Page 1: $10011101_2 = 57$
$68_{10} = 1000100_2$

Page 2: 10101 + 1111 + 1010 = 101110
10010 – 101 = 1101

Page 3: 10011 x 1011 = 11010001

Solution Set #1
a. 43
b. 67
c. 27
d. 63

Solution Set #2
a. 1001110
b. 10000101
c. 111110100
d. 11011110

Solution Set #3
a. 100100
b. 1001110
c. 11011.101
d. 110000

Solution Set #4
a. 110
b. 1110
c. 1011
d. 1111

Solution Set #5
a. 1111
b. 110010
c. 1010001
d. 110001

Solution Set #6
a. 11.11
b.
c.
d.

Binary Numbers: Solution Set

You Try One:

Page 1:

$$100111 01_2 = 1\times2^7 + 0\times2^6 + 0\times2^5 + 1\times2^4 + 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$$
$$= 1\times(128) + 1\times(16) + 1\times(8) + 1\times(4) + 1\times(1)$$
$$= 157_{10}$$

$68_{10} = 1000100_2$

Method 1

2	68	
2	34	R=0
2	17	R=0
2	8	R=1
2	4	R=0
2	2	R=0
2	1	R=0
	0	R=1

Method 2

$68 = 2\cdot34 + 0$
$34 = 2\cdot17 + 0$
$17 = 2\cdot8 + 1$
$8 = 2\cdot4 + 0$
$4 = 2\cdot2 + 0$
$2 = 2\cdot1 + 0$
$1 = 2\cdot0 + 1$

Page 2: $10101_2 + 1111_2 + 1010_2 = 101110_2$

```
 1 1 1
 10101
  1111
+ 1010
------
101110
```

check:

```
 21
 15
+10
---
 46 ✓
```

$10010_2 - 101_2 = 1101_2$

```
 0 1 10 0 10
 1 0 0 1 0
 -   1 0 1
 ---------
   1 1 0 1
```

Check:

```
  101
+1101
-----
10010 ✓
```

Check

```
 18
- 5
---
 13 ✓
```

Page 3: $10011_2 \times 1011_2 = 11010001_2$

```
      10011
    x  1011
    -------
      10011
     10011x
    00000xx
 + 10011xxx
 ----------
   11010001
```

Check:

```
  19
 x11
 ---
 209 ✓
```

Page 5: $11011_2 \div 11_2 = 1001_2$ (continued)

```
      1001
   -------
11 | 11011
    -11
     ---
     000
     -00
     ---
      001
      -00
      ---
       11
      -11
      ---
        0
```

Check:
$27 \div 3 = 9$ ✓

Solution Set #1

$$101011_2 = 1\times2^5 + 0\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0$$
$$= 1\times(32) + 1\times(8) + 1\times(2) + 1\times(1)$$
$$= 43_{10}$$

$$1000011_2 = 1\times2^6 + 0\times2^5 + 0\times2^4 + 0\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0$$
$$= 1\times(64) + 1\times(2) + 1\times(1)$$
$$= 67_{10}$$

$$11011_2 = 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0$$
$$= 1\times(16) + 1\times(8) + 1\times(2) + 1\times(1)$$
$$= 27_{10}$$

$$111111_2 = 1\times2^5 + 1\times2^4 + 1\times2^3 + 1\times2^2 + 1\times2^1 + 1\times2^0$$
$$= 1\times(32) + 1\times(16) + 1\times(8) + 1\times(4) + 1\times(2) + 1\times(1)$$
$$= 63_{10}$$

Solution Set #2

$78_2 = 1001110_2$

78	2	
39	2	R=0
19	2	R=1
9	2	R=1
4	2	R=1
2	2	R=0
2	1	R=0
	0	R=1

$78 = 2\cdot39 + 0$
$39 = 2\cdot19 + 1$
$19 = 2\cdot9 + 1$
$9 = 2\cdot4 + 1$
$4 = 2\cdot2 + 0$
$2 = 2\cdot1 + 0$
$1 = 2\cdot0 + 1$

$133_{10} = 10000101_2$

133	2	
66	2	R=1
33	2	R=0
16	2	R=1
8	2	R=0
4	2	R=0
2	2	R=0
1	2	R=0
	0	R=1

$133 = 2\cdot66 + 1$
$66 = 2\cdot33 + 0$
$33 = 2\cdot16 + 1$
$16 = 2\cdot8 + 0$
$8 = 2\cdot4 + 0$
$4 = 2\cdot2 + 0$
$2 = 2\cdot1 + 0$
$1 = 2\cdot0 + 1$

Solution Set #2 (continued)

$500_{10} = 111110100_2$

500	2	
250	2	R=0
125	2	R=0
62	2	R=1
31	2	R=0
15	2	R=1
7	2	R=1
3	2	R=1
1	2	R=1
0		R=1

$500 = 2\cdot 250 + 0$
$250 = 2\cdot 125 + 0$
$125 = 2\cdot 62 + 1$
$62 = 2\cdot 31 + 0$
$31 = 2\cdot 15 + 1$
$15 = 2\cdot 7 + 1$
$7 = 2\cdot 3 + 1$
$3 = 2\cdot 1 + 1$
$1 = 2\cdot 0 + 1$

$222_{10} = 11011110_2$

222	2	
111	2	R=0
55	2	R=1
27	2	R=1
13	2	R=1
6	2	R=1
3	2	R=0
1	2	R=1
0		R=1

$222 = 2\cdot 111 + 0$
$111 = 2\cdot 55 + 1$
$55 = 2\cdot 27 + 1$
$27 = 2\cdot 13 + 1$
$13 = 2\cdot 6 + 1$
$6 = 2\cdot 3 + 0$
$3 = 2\cdot 1 + 1$
$1 = 2\cdot 0 + 1$

Solution Set #3

$1111_2 + 10101_2 = 100100_2$

$$\begin{array}{r} 10101 \\ +1111 \\ \hline 100100 \end{array}$$

Check: $\begin{array}{r} 21 \\ +15 \\ \hline 36 \end{array}$ ✓

$10011_2 + 1000_2 + 110011_2 = 1001110_2$

$$\begin{array}{r} 110011 \\ 1000 \\ +10011 \\ \hline 1001110 \end{array}$$

Check: $\begin{array}{r} 51 \\ 8 \\ +19 \\ \hline 78 \end{array}$ ✓

$1111.101_2 + 111.001_2 + 11.000_2 + 1.111_2 = 11011.101$

$$\begin{array}{r} 1111.101 \\ 111.001 \\ 11.000 \\ +\ 1.111 \\ \hline 11011.101 \end{array}$$

Check: $\begin{array}{r} 15.625 \\ 7.125 \\ 3.000 \\ +1.875 \\ \hline 27.625 \end{array}$ ✓

$1011_2 + 1001_2 + 1111_2 + 1010_2 + 11_2 = 110000_2$

$$\begin{array}{r} 1011 \\ 1001 \\ 1111 \\ 1010 \\ +\ 11 \\ \hline 110000 \end{array}$$

Check: $\begin{array}{r} 11 \\ 9 \\ 15 \\ 10 \\ +3 \\ \hline 48 \end{array}$ ✓

Solution Set #4

$1011_2 - 101_2 = 110_2$

$$\begin{array}{r} 1011 \\ -101 \\ \hline 110 \end{array}$$

Check: $\begin{array}{r} 11 \\ -5 \\ \hline 6 \end{array}$ ✓

$10001_2 - 11_2 = 1110_2$

$$\begin{array}{r} 10001 \\ -11 \\ \hline 1110 \end{array}$$

Check: $\begin{array}{r} 17 \\ -3 \\ \hline 14 \end{array}$ ✓

$10101_2 - 1010_2 = 1011_2$

$$\begin{array}{r} 10101 \\ -1010 \\ \hline 1011 \end{array}$$

Check: $\begin{array}{r} 21 \\ -10 \\ \hline 11 \end{array}$ ✓

$10000_2 - 1_2 = 1111_2$

$$\begin{array}{r} 10000 \\ -1 \\ \hline 1111 \end{array}$$

Check: $\begin{array}{r} 16 \\ -1 \\ \hline 15 \end{array}$ ✓

Solution Set #5

$101_2 \times 11_2 = 1111_2$

$$\begin{array}{r} 101 \\ \times 11 \\ \hline 101 \\ +101\text{x} \\ \hline 1111 \end{array}$$

Check: $\begin{array}{r} 5 \\ \times 3 \\ \hline 15 \end{array}$ ✓

$1010_2 \times 101_2 = 110010_2$

$$\begin{array}{r} 1010 \\ \times 101 \\ \hline 1010 \\ 0000\text{x} \\ +1010\text{xx} \\ \hline 110010 \end{array}$$

Check: $\begin{array}{r} 10 \\ \times 5 \\ \hline 50 \end{array}$ ✓

$1001_2 \times 1001_2 = 1010001_2$

$$\begin{array}{r} 1001 \\ \times 1001 \\ \hline 1001 \\ 0000\text{x} \\ 0000\text{xx} \\ +1001\text{xxx} \\ \hline 1010001 \end{array}$$

Check: $\begin{array}{r} 9 \\ \times 9 \\ \hline 81 \end{array}$ ✓

$111_2 \times 111_2 = 110001_2$

$$\begin{array}{r} 111 \\ \times 111 \\ \hline 111 \\ 111\text{x} \\ +111\text{xx} \\ \hline 110001 \end{array}$$

Check: $\begin{array}{r} 7 \\ \times 7 \\ \hline 49 \end{array}$ ✓

Solution Set #6

$1111_2 \div 100_2 = 11.11_2$

$$\begin{array}{r} 11.11 \\ 100\overline{)1111} \\ -100 \\ \hline 111 \\ -100 \\ \hline 110 \\ -100 \\ \hline 100 \\ -100 \\ \hline 000 \end{array}$$

Check: $15 \div 4 = 3.75$ ✓

$1000_2 \div 111_2 = 1.\overline{001}_2$

$$\begin{array}{r} 1.001 \\ 111\overline{)1000.000000} \\ -111 \\ \hline 1000 \\ -111 \\ \hline 1000 \\ -111 \\ \hline 1 \end{array}$$

repeating

Check: $8 \div 7 = 1.\overline{142857}$ ✓

$1010_2 \div 11_2 = 11.\overline{01}_2$

$$\begin{array}{r} 11.01 \\ 11\overline{)1010.0000} \\ -11 \\ \hline 100 \\ -11 \\ \hline 100 \\ -11 \\ \hline 100 \\ -11 \\ \hline 1 \end{array}$$

repeating

Check: $10 \div 3 = 3.\overline{3}$ ✓

$11101_2 \div 110_2 = 100.1\overline{10}$

$$\begin{array}{r} 100.110 \\ 110\overline{)11101.0000} \\ -110 \\ \hline 1010 \\ -110 \\ \hline 1000 \\ -110 \\ \hline 1000 \\ -110 \\ \hline 10 \end{array}$$

repeating

Check: $29 \div 6 = 4.8\overline{3}$ ✓