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Karnaugh Maps

A PICTORIAL MEANS TO MINIMIZE DISJUNCTIVE FORMS

Examples involving just 2 variables:

The setup for two variables looks like this:

	y	y’
x	a	b
x’	c	d

where square
“a” represents xy
“b” represents xy’
“c” represents x’y
“d” represents x’y’

a. Simplify xy + xy’

	y	y’
x	✓✓	✓✓
x’		

Step 1: Draw the Karnaugh map that represents the expression by placing a mark in each appropriate square.

Step 2: “Cover” the marked boxes as best you can with ovals. Allowed coverings for this setup are limited to 2x1 rectangles. This example is covered by one 2x1 rectangle.

Step 3: Expressions/rectangles that are covered can be reduced. Note that in this example the y and y’ will cancel out across the top – only the x at the left side of the map remains.
Hence, xy + xy’ will be reduces to just x.

Step 4: So, the ***minimial disjunctive form***, sometimes called the “***minimal sum***,” equals x. The expression xy + xy’ will be logically equivalent to x. (Verify!)

b. Simplify xy + x’y + x’y’

	y	y’
x	✓✓	
x’	✓✓	✓✓

Step 1: Draw the Karnaugh map.

Step 2: Attempt to cover all marked squared with as few rectangles as possible. All marked squares are covered here using just two ovals.

Step 3: Two reductions can be made. (1) The horizontal oval eliminates y and y’ found across the top and leaves the x’ found at the left (2) The vertical oval eliminated the x and x’ found along the left and leaves only the y found at the top. **Hence xy + x’y + x’y’ can be reduced to just x’ + y.**

Step 4: So, the minimal disjunctive form equals x' + y. The expression xy + x'y + x'y' will be logically equivalent to x' + y. (Verify!)

c. Simplify xy + x'y'

	y	y'
x	✓✓	
x'		✓✓

n the Karnaugh map.

Step 2: Attempt to cover all marked squared with as few rectangles as possible. Note that in this case it is impossible to cover the two rectangles with a 2x1 rectangle.

Step 3: Consequently, in this case no reductions can be made. **Hence it is already in minimal form.**

Step 4: So, the minimal disjunctive form is xy + x'y' and cannot be reduced further.

Examples involving just 3 variables:

The setup for three variables looks like this:

	yz	yz'	y'z'	y'z
x	a	b	c	d
x'	e	f	g	H

where square

"a" represents xyz "e" represents x'yz

"b" represents xy'z "f" represents x'y'z

"c" represents x'yz' "g" represents x'y'z'

"d" represents x'y'z' "h" represents x'yz'

a. Simplify xyz + xyz' + x'yz' + x'y'z

	yz	yz'	y'z'	y'z
x	✓✓	✓✓		
x'		✓✓		✓✓

Step 1: Draw the Karnaugh that represents the expression by *carefully* placing a mark in each appropriate square.

Step 2: Cover the marked boxes as best you can with ovals, using as few rectangles as possible. Allowed coverings for this expanded setup is now limited to 2x2, 1x4, and 1x2 rectangles. A good plan is to try to cover using a 2x2 first, then 1x4, and finally 2x1.

Step 3: Two reductions can be made. (1) The horizontal oval eliminates z and z' across the top; y has nothing to cancel it. On the left is x. Hence, the reduction here leaves xy. (2) The vertical oval eliminates the x and x' along the left side leaving yz' at the top. Hence the reduction for this oval leaves yz'. No rectangle is able to incorporate the lower rightmost square. **This cannot be reduced.**

Step 4: So, the minimal disjunctive form is xy + yz' + x'y'z and this will be logically equivalent to the given expression, xyz + xyz' + x'yz' + x'y'z.

b. Simplify xyz + xyz’ + xy’z + x’yz + x’y’z

Step 1: Draw the Karnaugh map.

	yz	yz’	y’z’	y’z
x	✓✓	✓✓		✓✓
x’	✓✓			✓✓

Step 2: Note that Karnaugh maps are displayed as 3-dimensional objects cut and laid flat. Thus the leftmost and rightmost edges can be connected to form a cylinder and as a consequence, a 2x2 rectangle can be used to cover the four connecting squares (in red). If helpful, the top and bottom of the map can be connected as well. To include the one square left out, a 1x2 rectangle can be used (in blue).

[Note that if, instead, you use more than two rectangles to cover the marked squares, you will obtain a simplified, logically equivalent expression, but it will not be the *minimal* simplification.]

Step 3: The 2x2 reduction eliminates the x and x’ along the left side and the y and y’ across the top, leaving only the z. The 1x2 reduction eliminates the z and z’, leaving the x found on the left and the y on top – and upon multiplying these, what remains is xy. **Hence, xyz + xyz’ + xy’z + x’yz + x’y’z can be minimally reduced to just z + xy.**

Step 4: So, the minimal disjunctive form is z +xy and this will be logically equivalent to the given expression, xyz + xyz’ + xy’z + x’yz + x’y’z.

c. Simplify xyz + xyz’ + x’yz’ + x’y’z’ + x’y’z

	yz	yz’	y’z’	y’z
x	✓✓	✓✓		
x’		✓✓	✓✓	✓✓

	yz	yz’	y’z’	y’z
x	✓✓	✓✓		
x’		✓✓	✓✓	✓✓

Step 1: Draw the Karnaugh map.

Step 2: Note that there are two different ways to cover all of the marked squares. Neither is *more* minimal than the other. This example will have two, slightly different, minimal forms.

Step 3: Thus this expression can be reduced to either xy + yz’ + x’y’ **or** xy + x’z’ + x’y’.
(Verify!)

Step 4: So, the minimal disjunctive form sought is either xy + yz’ + x’y’ **or** xy + x’z’ + x’y’ and each of these will be logically equivalent to the given expression, xyz + xyz’ + x’yz’ + x’y’z’ + x’y’z, as well as to each other.

xyzt	xyzt’	xyz’t’	xyz’t
xy’zt	xy’zt’	xy’z’t’	xy’z’t
x’y’zt	x’y’zt’	x’y’z’t’	x’y’z’t
x’yzt	x’yzt’	x’yz’t’	x’yz’t

Examples involving 4 variables:

The setup for four variables looks like this, where the rep **zt** **zt’** **z’t’** **z’t** s indicated within the square itself.

a. xy’x’t’ + xy’z’t

	zt	**zt’**	**z’t’**	**z’t**
xy				
xy’				
x’y’			✓✓	✓✓
x’y				

Step 1: Carefully draw the Karnaugh map, making sure the headings are exactly like those in the “setup” diagram.

Step 2: Cover the marked squares with as few rectangles as possible. In addition to the 1x2, 1x4 and 2x2 rectangles already considered, we now include 2x4 rectangles as well. Here one 1x2 rectangle covers both terms.

Step 3: From this expression we can eliminate the t’ and t, leaving only z’ on top. To the left we find x’y’. **Thus, the reduced expression is x’y’z’.**

Step 4: So, the minimal disjunctive form is x’y’z’ and this will be logically equivalent to the given expression, xy’x’t’ + xy’z’t.

a. xyzt + xyzt’ + x’yzt + x’yzt’

	zt	**zt’**	**z’t’**	**z’t**
xy	✓✓	✓✓		
xy’				
x’y’				
x’y	✓✓	✓✓		

Step 1: Carefully draw the Karnaugh map.

Step 2: Cover the marked squares with as few rectangles as possible. Recalling that the map can be rolled into a cylinder, a 2x2 rectangle can be drawn to cover all four marked squares.

Step 3: Along the top we can eliminate the t’ and t, leaving only the z. Along the left we find x and x’ can be eliminated, leaving only the y. **Thus, the reduced expression is y + z.**

Step 4: So, the minimal disjunctive form is y + z and this will be logically equivalent to the much more complicated expression, xyzt + xyzt’ + x’yzt + x’yzt’.

b. xyzt + xyz’t + xy’zt + xy’z’t + x’y’zt + x’y’z’t + x’yzt + x’yz’t

	zt	**zt’**	**z’t’**	**z’t**
xy	✓✓			✓✓
xy’	✓✓			✓✓
x’y’	✓✓			✓✓
x’y	✓✓			✓✓

Step 1: Carefully draw the Karnaugh map.

Step 2: Cover the marked squares with as few rectangles as possible. Recalling that the map can be rolled into a cylinder, a 2x4 rectangle can be drawn in this case to cover all eight marked squares.

Step 3: Along the top we can eliminate the z and z’, leaving the t only. Along the left we find that everything will be eliminated. **Thus, the reduced expression is just t.**

Step 4: So, the minimal disjunctive form is just t and this will be logically equivalent to the much more complicated expression, xyzt + xyz’t + xy’zt + xy’z’t + x’y’zt + x’y’z’t + x’yzt + x’yz’t .

b. xy’ + xyz + x’y’z’ + x’yzt’

	zt	**zt’**	**z’t’**	**z’t**
xy	✓✓	✓✓		
xy’	✓✓	✓✓	✓✓	✓✓
x’y’			✓✓	✓✓
x’y		✓✓		

Step 1: Carefully draw the Karnaugh map. Note how the term x’y marks all squares beginning with x’y (highlighted in blue). These all like along the x’y row. Further note how the term xyz will mark all squares that begin with xyz (highlighted in green), and the term x’y’z’ will mark all quares that begin with x’y’z’ (highlighted in red).

Step 2: Cover the marked squares with as few rectangles as possible. Two 2x2 rectangles cover most of the marked squares. Note how the square along the bottom can be covered by wrapping around to the top of the map.

Step 3: The 2x2 along the top will eliminate the t and t’, leaving the z, while along the left we find the elimination of y and y’, leaving x. The simplification here yields xz.

The 2x2 found in the middle right will also eliminate the t and t’, leaving the z’, while along the left we find the elimination of x and x’, leaving y. The simplification here yields y’z’.

Finally, the 1x2 that connects the top and bottom of the map eliminates along the left x and x’, leaving y from the left and zt’ from the top, yielding yzt’.

Thus, the reduced expression is xz + y’z’ + yzt’.

Step 4: So, the minimal disjunctive form is xz + y’z’ + yzt’ and this will be logically equivalent to the more complicated expression, xy’ + xyz + x’y’z’ + x’yzt’.

Exercises

Use Karnaugh maps to find the minimal form for each expression.

1. $xy + xy'$
2. $xy + x'y + x'y'$
3. $xy' + x'y'$
4. $xyz + xyz' + xy'z$
5. $xyz' + xy'z + xy'z' + x'yz + x'yz' + x'y'z$
6. $xyz + xyz' + x'yz + x'y'z$
7. $xyz + xyz' + xy'z + xy'z' + x'y'z$
8. $xyzt + xyz't + xyzt' + x'yzt + x'y'zt + x'yzt'$
9. $xyz't + xy'zt + xy'z't + x'yzt + x'yz't + x'yzt' + x'y'z't'$
10. $xy'z't + xy'zt' + xy'z't' + x'y'zt + x'y'z't + x'y'zt' + x'y'z't'$

Answers

1. x
2. y + x’
3. y’
4. xy + xz
5. x’z + yz’ + xy’   or   x’y + xz’ + y’z
6. x’z + xy
7. x + y’z
8. yx + xyt + x’zt
9. x’yz + xy’t + yz’t + x’y’z’t’
10. x’y’ + y’t’ + y’z’