Important Discrete Random Variables

1016-351 Prob & Stat I
D Mathiason

Important discrete random variables

variable	characterization	pmf	mean (μ)	variance (σ^2)
Bernoulli	0-1, failure or success	p(1)=p, p(0)=q=1-p	p	pq
binomial	# successes in n Bernoulli trials, p=P(success)	$\binom{n}{x}p^x(1-p)^{n-x}$ x=0, 1, 2, ..., n	np	np(1-p) = npq
hypergeometric	# successes, sampling w/o replacement N=pop size, n=sample size, M=# successes in population	$\dfrac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$ max(0, n-N+M)≤x x≤min(n,M)	$n\dfrac{M}{N} = np$ with $p = \dfrac{M}{N}$	$\dfrac{N-n}{N-1}n\dfrac{M}{N}\left(1-\dfrac{M}{N}\right)$ or $np(1-p)\left(\dfrac{N-n}{N-1}\right)$
geometric	# failures preceding the first success in repeated Bernoulli trials	$(1-p)^x p$ x=0, 1, 2, ...	$\dfrac{1-p}{p}$	$\dfrac{1-p}{p^2}$
negative binomial	# failures preceding the r^{th} success	$\binom{x+r-1}{r-1}p^r(1-p)^x$ x=0, 1, 2, 3, ...	$\dfrac{r(1-p)}{p}$	$\dfrac{r(1-p)}{p^2}$
Poisson	# occurrences	$\dfrac{e^{-\lambda}\lambda^x}{x!}$, x=0, 1, 2, ...	λ	λ
Poisson process	# occurrences by time t, at rate α per unit time	$\dfrac{e^{-\alpha t}(\alpha t)^x}{x!}$, x=0, 1, 2, ...	αt	αt