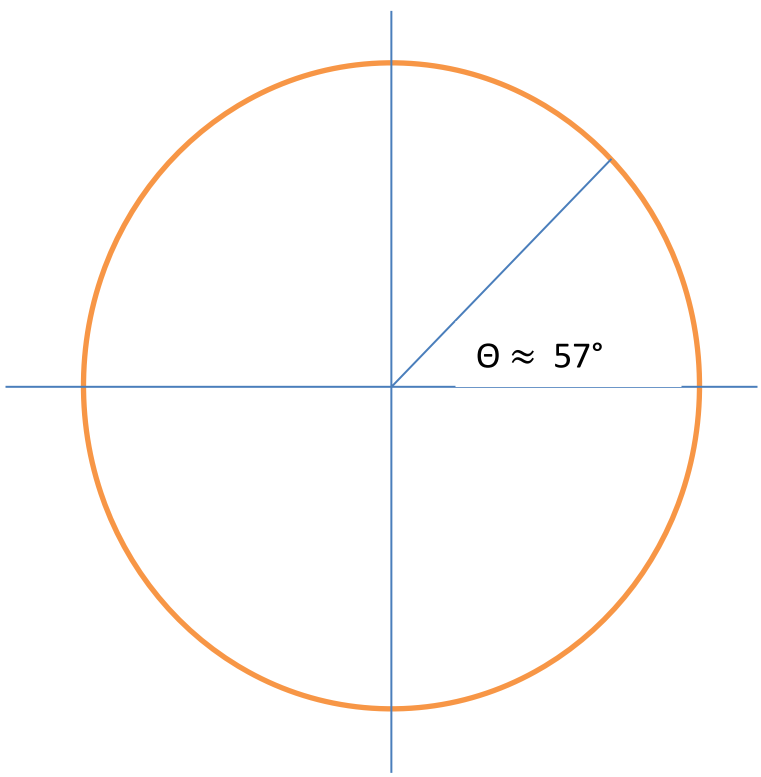
6.1 Basic Right Triangle Trigonometry

MEASURING ANGLES IN RADIANS

First, let's introduce the units you will be using to measure angles, radians.

A radian is a unit of measurement defined as the angle at the center of the circle made when the arc length equals the radius.

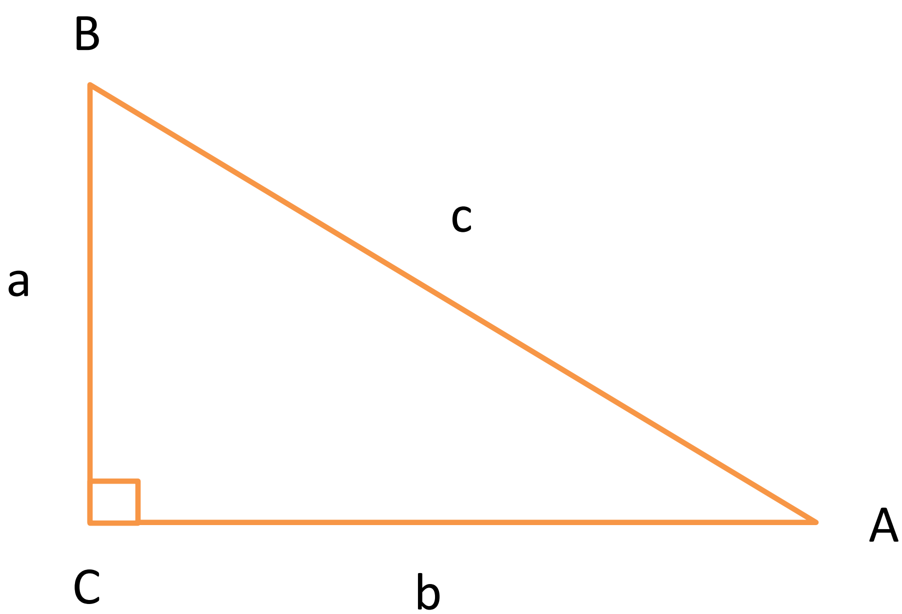
If this definition sounds abstract we define the radian pictorially below. Assuming the radius drawn below equals the arc length between the x-axis and where the radius intersects the circle, then the angle Θ is 1 radian. Note that 1 radian is approximately 57°.



Many people are more familiar with a *degree* measurement of an angle. Below is a quick formula for converting between degrees and radians. You may use this in order to gain a more intuitive understanding of the magnitude of a given radian measurement, but for most classes at R.I.T. you will be using radians in computation exclusively.

$$radians = degrees\ \frac{\pi}{180}$$

Now consider the right triangle pictured below with sides a,b,c and angles A,B,C. We will be referencing this generic representation of a right triangle throughout the packet.



BASIC FACTS AND DEFINITIONS

1. **Right angle:** angle measuring $\frac{\pi}{2}$ radians (example: angle C above)
2. **Straight angle:** angle measuring π radians
3. **Acute angle:** angle measuring between 0 and $\frac{\pi}{2}$ radians (examples: angles A and B above)
4. **Obtuse angle:** angle measuring between $\frac{\pi}{2}$ and π radians
5. **Complementary angles:** Two angles whose sum is $\frac{\pi}{2}$ radians. Note that A and B are complementary angles since C = $\frac{\pi}{2}$ radians and all triangles have a sum of π radians between the three angles.
6. **Supplementary angles:** two angles whose sum is π radians

7. **Right triangle:** a triangle with a right angle (an angle of $\frac{\pi}{2}$ radians)
8. **Isosceles triangle:** a triangle with exactly two sides of equal length
9. **Equilateral triangle:** a triangle with all three sides of equal length
10. **Hypotenuse:** side opposite the right angle, side c in the diagram above
11. **Pythagorean Theorem:** $c^{\mathbf{2}} = a^2 + b^2$

<u>Example 1:</u> A right triangle has a hypotenuse length of 5 inches. Additionally, one side of the triangle measures 4 inches. What is the length of the other side?

Solution:

$c^{\mathbf{2}} = a^2 + b^2$

$5^{\mathbf{2}} = 4^2 + b^2$

$25 = 16 + b^2$

$9 = b^2$

b **= 3 inches**

<u>Example 2:</u> In the right triangle pictured above, if $A = \frac{\pi}{6}$ radians, what is the measure of angle B?

Solution:

The two acute angles in a right triangle are complementary, so:

$A + B = \frac{\pi}{2}$

$\frac{\pi}{6} + B = \frac{\pi}{2}$

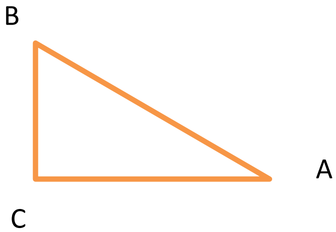
$B = \frac{\pi}{3}$

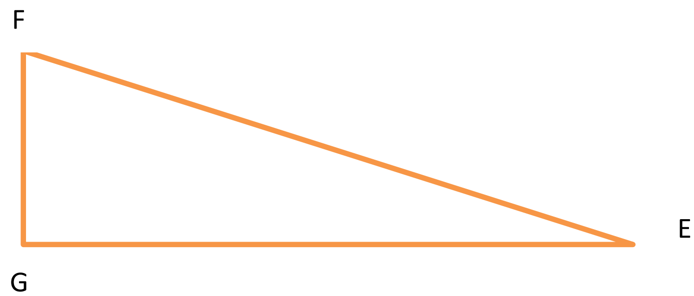
SIMILIAR TRIANGLES

Two triangles are said to be **similar** if the angles of one triangle are equal to the corresponding angles of the other. That is, we say triangles ABC and EFG are similar if *A = E*, *B = F,* and *C = G =* $\frac{\pi}{2}$ radians and we write $ABC \sim EFG$.

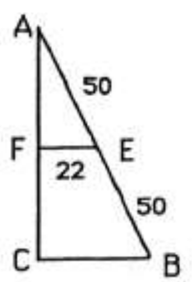
Further, the ratio of corresponding sides are equal, that is;

$$\frac{AC}{EG} = \frac{AB}{EF} = \frac{BC}{FG}$$





Example 3:



Let AE = 50 meters, EF = 22 meters and AB = 100 meters. Find the length of side BC.

Notice that ABC and AEF are similar since corresponding angles are equal. (There is a right angle at both F and C, angle A is the same in both triangles and angle B equals angle E).

Solution: $\frac{AE}{AB} = \frac{EF}{BC}$ thus $\frac{50}{100} = \frac{22}{BC}$

So 50(BC) = (22)(100)

BC = 44 meters

THE SIX TRIGONOMETRIC RATIOS FOR ACCUTE ANGLES

The trigonometric ratios give us a way of relating the angles to the ratios of the sides of a right triangle. These ratios are used pervasively in both physics and engineering (especially the introductory phyiscs sequence at RIT). Below we define the six trigonometric functions and then turn to some examples in which they must be applied.

$$\text{sine } A = \sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c}$$

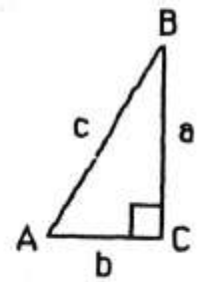
$$\text{cosine } A = \cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\text{tangent } A = \tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b}$$

$$\text{cosecant } A = \csc A = \frac{\text{hyp}}{\text{opp}} = \frac{1}{\sin A} = \frac{c}{a}$$

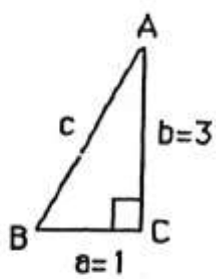
$$\text{secant } A = \sec A = \frac{\text{hyp}}{\text{adj}} = \frac{1}{\cos A} = \frac{c}{b}$$

$$\text{cotangent } A = \cot A = \frac{\text{adj}}{\text{opp}} = \frac{1}{\tan A} = \frac{b}{a}$$



Example 4

In the right triangle ABC, a = 1 and b = 3. Determine the six trigonometric ratios for angle B.



Solution: $c^2 = a^2 + b^2$ Pythagorean Theorem

$c^2 = 1^2 + 3^2$

$c^2 = 10$

$c = \sqrt{10}$ or $-\sqrt{10}$

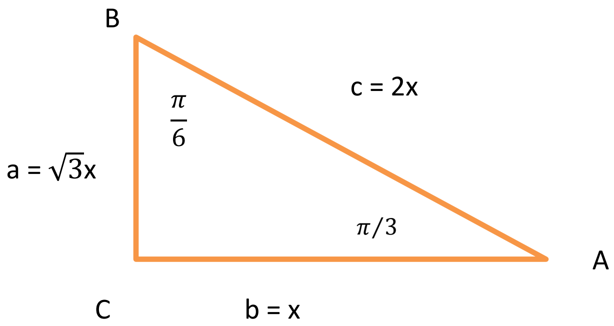
$c = \sqrt{10}$ only, since length is positive

$$\sin B = \frac{\text{opp}}{\text{hyp}} = \frac{3}{\sqrt{10}} \qquad \cos B = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{10}} \qquad \tan B = \frac{\text{opp}}{\text{adj}} = \frac{3}{1}$$

$$\csc B = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{10}}{3} \qquad \sec B = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{10}}{1} \qquad \cot B = \frac{\text{adj}}{\text{opp}} = \frac{1}{3}$$

SPECIAL TRIANGLES

There are certain right triangles with characteristic side angles that appear so often in problems we make note of two of them here. The first has accute angles of $\frac{\pi}{6}$ and $\frac{\pi}{3}$, note that all such traingles are similar. Thus for some length *x*, the side lengths and trigonometric ratios of such a triangle are shown below.



$$sin\frac{\pi}{6} = \frac{1}{2} \qquad sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \qquad cos\frac{\pi}{3} = \frac{1}{2}$$

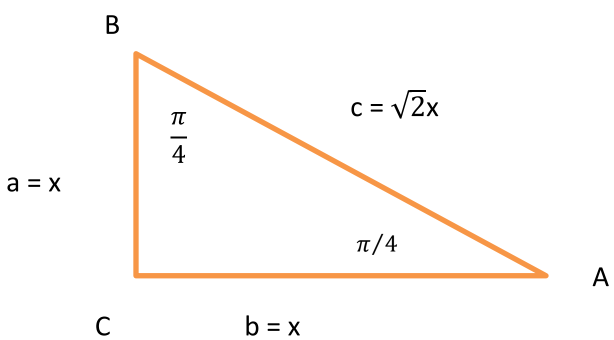
$$tan\frac{\pi}{6} = \frac{\sqrt{3}}{3} \qquad tan\frac{\pi}{3} = \sqrt{3}$$

$$csc\frac{\pi}{6} = 2 \qquad csc\frac{\pi}{3} = \frac{2\sqrt{3}}{3}$$

$$sec\frac{\pi}{6} = \frac{2\sqrt{3}}{3} \qquad sec\frac{\pi}{3} = 2$$

$$cot\frac{\pi}{6} = \sqrt{3} \qquad cot\frac{\pi}{3} = \frac{\sqrt{3}}{3}$$

The second special right triangle we will look at has both acute angles to be $\frac{\pi}{4}$. Then for some length *x*, the side lengths and trigonometric ratios of such a triangle are shown below.



$$sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$tan\frac{\pi}{4} = 1$$

$$csc\frac{\pi}{4} = \sqrt{2}$$

$$sec\frac{\pi}{4} = \sqrt{2}$$

$$cot\frac{\pi}{4} = 1$$

Example 5:

If the hypotenuse of a $\frac{\pi}{6}$ - $\frac{\pi}{3}$ - $\frac{\pi}{2}$ triangle is 10 meters, what are the lengths of the triangles legs?

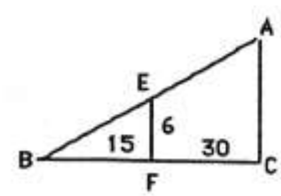
Solution:

If 10 = 2x, then **x = 5 meters** and $\sqrt{3}x = \sqrt{3}(5) = \mathbf{5\sqrt{3}}$ **meters**

Problems

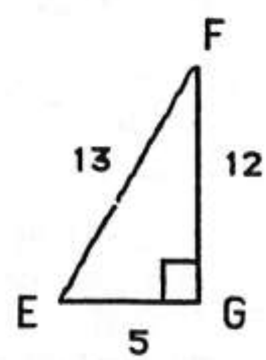
1. In right triangle ABC, if side c = 39 (hypotenuse) and side b = 36, find a.

2. Given the lengths defined in the right triangle pictured below, find the length of side AC.



3. Evauluate:

   a. sin E = ________ b. tan E = _______
   c. cos F = ________ d. sec F = _______



4. Evaluate:

$\sin\frac{\pi}{6} =$ ________

$\sec\frac{\pi}{3} =$ ________

$\csc\frac{\pi}{4} =$ ________

$\tan\frac{\pi}{3} =$ ________

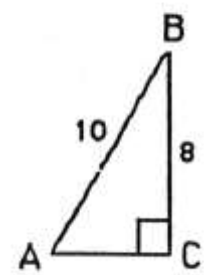
$\tan\frac{\pi}{4} =$ ________

$\cot\frac{\pi}{6} =$ ________

5. Evaluate:

a. tan A =__________ b. csc B = ___________

c. cot A = __________ d. sec B = ___________



Answers

1. a = 15 (pythagorean theorem)
2. AC = 18 (properties of similar triangles)
3.
    a. $\frac{12}{13}$
    b. $\frac{12}{5}$
    c. $\frac{12}{13}$
    d. $\frac{13}{12}$
4.
    a. $\frac{1}{2}$
    b. 2
    c. $\sqrt{2}$
    d. $\sqrt{3}$
    e. 1
    f. $\sqrt{3}$
5.
    a. $\frac{4}{3}$
    b. $\frac{5}{3}$
    c. $\frac{3}{4}$
    d. $\frac{5}{4}$