Practice with Trigonometric Identities

Complete the following practice exercise using the Trigonometry Identities reference page handout.

Practice Exercises:

1. Given the double angle formula for cosine: $\cos 2\theta = \cos^2\theta - \sin^2\theta$
   a. Use the trig identity $\sin^2\theta + \cos^2\theta = 1$ to rewrite $\cos 2\theta$ in terms of $\sin^2\theta$ only.
   b. Use the trig identity $\sin^2\theta + \cos^2\theta = 1$ to rewrite $\cos 2\theta$ in terms of $\cos^2\theta$ only.

2. Consider the following identity: $\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$
   a. Draw the graph of the cosine function on the domain $0 \le \theta \le 2\pi$.
   b. Extend the graph of the cosine function to show the graph for $-\frac{\pi}{2} \le \theta \le 0$.
   c. How does the graph of the cosine function for $-\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$ compare to the sine function for $0 \le \theta \le 2\pi$?
   d. Verify that the identity above: $\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$ is correct using the addition identity for $\sin(x+y)$.

3. Derive the double angle formula for sine by using the addition formula for sine. That is, find $\sin(2x) = \sin(x+x) =$etc. Derive the double angle formula for cosine using a similar technique.

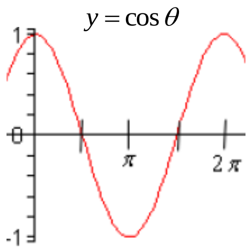
4. Derive the half angle formula for $\sin^2 x$ by starting with the cosine double angle formula $\cos 2x = 1 - 2\sin^2 x$ and by solving for $\sin^2 x$ in terms of $\cos 2x$. Derive the other half angle formula using a similar technique.

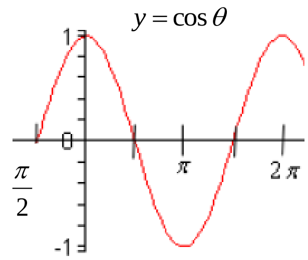
5. The Law of Sines and Cosines are applicable to **all** triangles. Find the length of side "a" of triangle ABC if:
   a. $A = 40°$, $B = 100°$, $b = 20$ (Use Law of Sines)
   b. $A = 40°$, $c = 12$, $b = 20$ (Use Law of Cosines)

SOLUTIONS:

1. a) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= (1-\sin^2\theta) - \sin^2\theta$
$= 1 - 2\sin^2\theta$

b) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \cos^2\theta - (1-\cos^2\theta)$
$= 2\cos^2\theta - 1$

2. a) 

b) 

c) The graph is the same as one period of sine (this identity states that the cosine function is the same as the sine function shifted $\frac{\pi}{2}$ units to the left).

d) $\sin(x+y) = \sin x \cos y + \cos x \sin y$

$$\sin\left(\theta + \frac{\pi}{2}\right) = \sin\theta\cos\frac{\pi}{2} + \cos\theta\sin\frac{\pi}{2}$$
$$= (\sin\theta)(0) + (\cos\theta)(1)$$
$$= \cos\theta$$

3. $\sin 2x = \sin(x+x) = \sin x\cos x + \cos x \sin x = 2\sin x \cos x$

$\cos 2x = \cos(x+x) = \cos x \cos x - \sin x \sin x = \cos^2 x - \sin^2 x$

4. $\cos 2x = 1 - 2\sin^2 x$
$2\sin^2 x = 1 - \cos 2x$
$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$\cos 2x = 2\cos^2 x - 1$
$2\cos^2 x = 1 - \cos 2x$
$\cos^2 x = \frac{1}{2}(1-\cos 2x)$

5. a) $\frac{\sin 40°}{a} = \frac{\sin 100°}{20} \longrightarrow a = 13.05$

b) $a^2 = 20^2 + 12^2 - (20)(12)\cos 40°$
$a = 18.98$