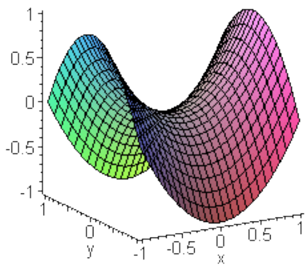
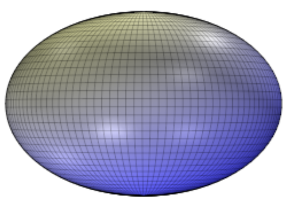
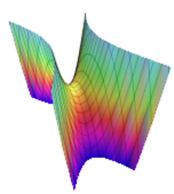
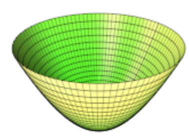
Quadratic Surfaces in Space

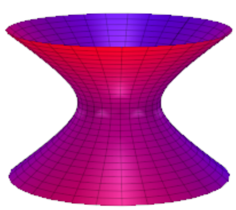


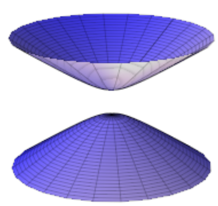
Pairing a 3-dimensional surface with its algebraic representation takes some practice. The following flash cards and practice sheets are available here to encourage and help with that practice. Quadratic surfaces, the 3-dimensional analogues of the conic sections, are only six of many such space surfaces. Not included here are space surfaces such as planes, cylinders, spheres, or surfaces created through rotations of lines in space.

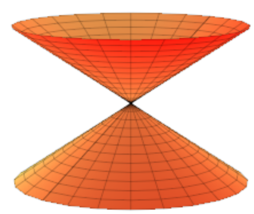

Ellipsoid


Hyperbolic paraboloid

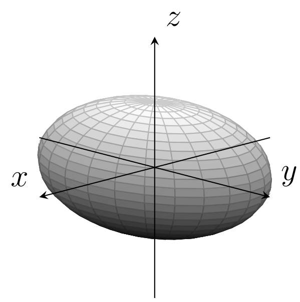

Elliptic paraboloid

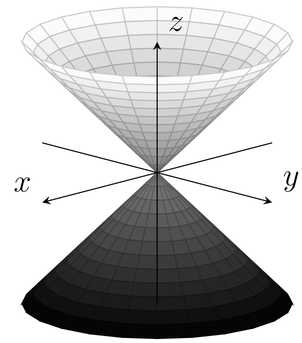

Hyperboloid of one sheet

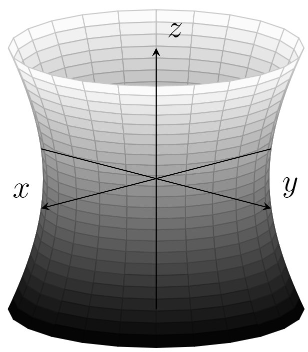

Hyperboloid of two sheets

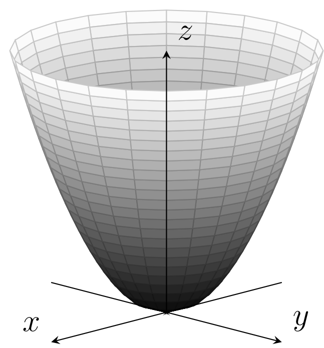

Cone

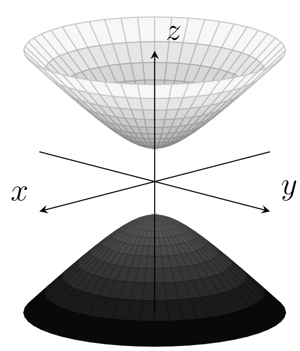
Once the basic quadratic surfaces, all centered about the origin, are mastered, they can be stretched, squeezed and/or translated to other locations in space. Examples are provided to help you to picture and identify such augmented quadratic surfaces.

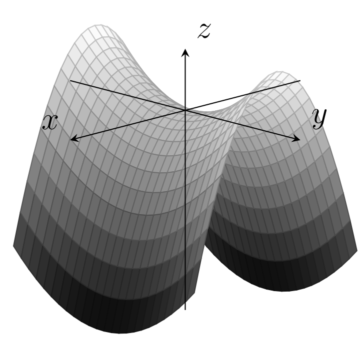

z
x
y


z
x
y


z
x
y


z
x
y


z
x
y


z
x
y

Elliptic Cone

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

$$\begin{cases} x(u,v) = au\cos v \\ y(u,v) = bu\sin v \\ z(u,v) = cu \end{cases}$$

$$u \in \mathbb{R}, \quad v \in [0, \pi)$$

Ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\begin{cases} x(u,v) = a\cos u\sin v \\ y(u,v) = b\sin u\sin v \\ z(u,v) = c\cos v \end{cases}$$

$$u \in [0, 2\pi), \quad v \in [0, \pi)$$

Elliptic Paraboloid

$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$\begin{cases} x(u,v) = a\sqrt{u}\cos v \\ y(u,v) = b\sqrt{u}\sin v \\ z(u,v) = cu \end{cases}$$

$$u \in \mathbb{R}^+, \quad v \in [0, 2\pi)$$

Hyperboloid of One Sheet

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\begin{cases} x(u,v) = a\sqrt{1+u^2}\cos v \\ y(u,v) = b\sqrt{1+u^2}\sin v \\ z(u,v) = cu \end{cases}$$

$$u \in \mathbb{R}, \quad v \in [0, 2\pi)$$

Hyperbolic Paraboloid

$$z = \frac{y^2}{a^2} - \frac{x^2}{b^2}$$

$$\begin{cases} x(u,v) = au \\ y(u,v) = bv \\ z(u,v) = cuv \end{cases}$$

$$u \in \mathbb{R}^+, \quad v \in [0, 2\pi)$$

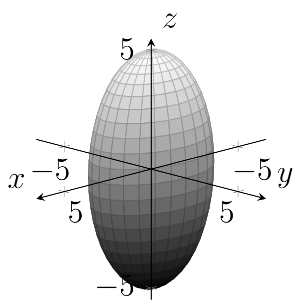
Hyperboloid of Two Sheet

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$

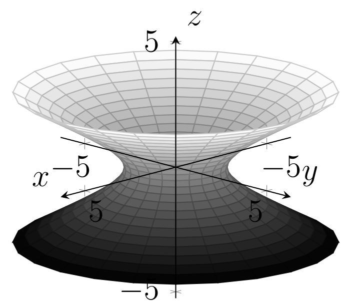
$$\begin{cases} x(u,v) = a\sinh u\cos v \\ y(u,v) = b\sinh u\sin v \\ z(u,v) = c\cosh u \end{cases}$$

$$u \in \mathbb{R}, \quad v \in [0, \pi)$$

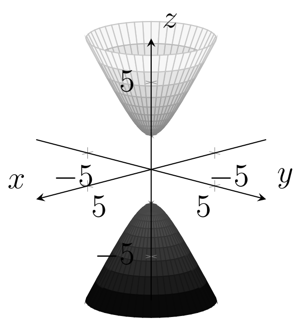
1.)



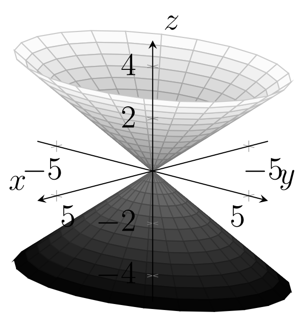
2.)



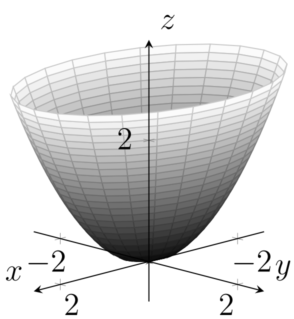
3.)



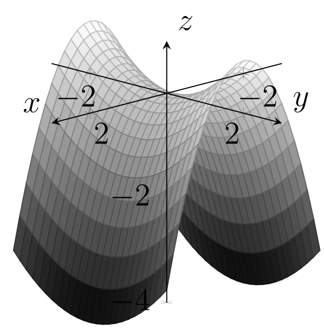
4.)



5.)



6.)



$$\frac{x^2}{1^2} + \frac{y^2}{1.5^2} - \frac{z^2}{4^2} = 0 \qquad \text{(a)}$$

$$\frac{x^2}{1.5^2} + \frac{y^2}{1^2} = z \qquad \text{(b)}$$

$$\frac{x^2}{2^2} + \frac{y^2}{2^2} - \frac{z^2}{1^2} = 1 \qquad \text{(c)}$$

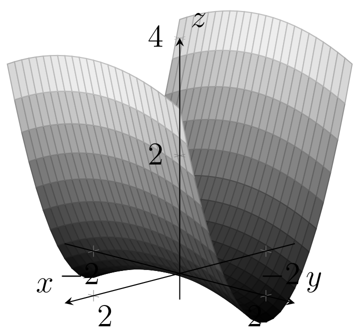
$$\frac{x^2}{3^2} + \frac{y^2}{2^2} + \frac{z^2}{5^2} = 1 \qquad \text{(d)}$$

$$\frac{y^2}{2^2} - \frac{x^2}{1^2} = z \qquad \text{(e)}$$

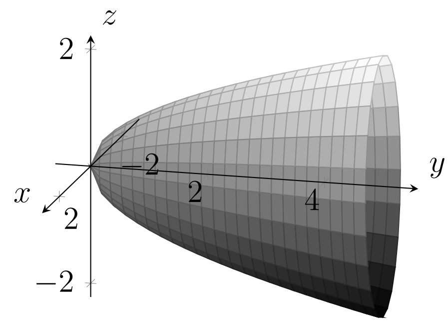
$$\frac{z^2}{2^2} - \frac{x^2}{1^2} - \frac{y^2}{1^2} = 1 \qquad \text{(f)}$$

Answer of P2: $7-i$ $8-l$ $9-h$ $10-j$ $11-g$ $12-k$

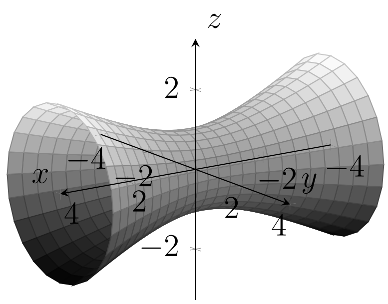
7.)



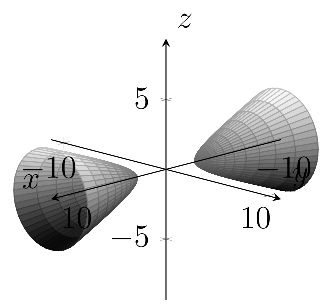
8.)



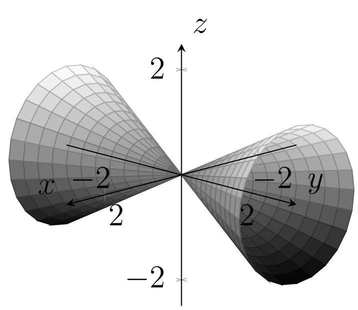
9.)



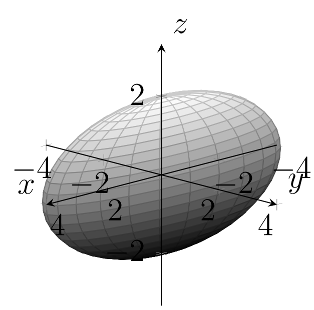
10.)



11.)



12.)



$$\frac{x^2}{0.5^2} - \frac{y^2}{0.5^2} + \frac{z^2}{0.5^2} = 0 \qquad \text{(g)}$$

$$\frac{z^2}{2^2} + \frac{y^2}{1^2} - \frac{x^2}{1^2} = 1 \qquad \text{(h)}$$

$$\frac{x^2}{1^2} - \frac{y^2}{3^2} = z \qquad \text{(i)}$$

$$-\frac{z^2}{1^2} + \frac{x^2}{3^2} - \frac{y^2}{1^2} = 1 \qquad \text{(j)}$$

$$\frac{x^2}{4^2} + \frac{y^2}{1^2} + \frac{z^2}{2^2} = 1 \qquad \text{(k)}$$

$$\frac{x^2}{0.5^2} + \frac{z^2}{1^2} = y \qquad \text{(l)}$$

Answer of P1: $1-d \quad 2-c \quad 3-f \quad 4-a \quad 5-b \quad 6-e$