Introducing WLab

About Wpt

A normalization methodology was proposed and demonstrated that linearly transforms sensor excitations (or linear transforms of sensor excitations) into a material color equivalency representation that can be used as a waypoint for defining Material Adjustment Transforms. The normalization process adjusts for the white point and independently preserves the perceptive aspects of lightness, chroma, and hue resulting in an opponent like coordinate system designated by the axes W, p, and t.

A basic overview of the sequence of linear operations that are concatenated to form a Wpt normalization matrix is as follows. First, sensor excitation coordinates are rotated for a perfect reflecting diffuser so that they are co-linear with the first axis (to become W). Then these excitations are scaled so that their W value is 100. Then a plane of sensor excitation coordinates for reflectances of Munsell colors having constant Munsell value and chroma is fitted and sheared to be orthogonal to the W axis. And finally, rotation and scaling is performed around the W axis so that differences in distances from the W axis of the same set of Munsell reflectances are minimized as well as their angular alignment with axes p and t are roughly similar to those used by CIELAB a* and b* axes. A Wpt normalization matrix therefore represents a transformation of sensor excitations to a coordinate system that preserves the independent perceptive concepts of Munsell value (lightness), chroma and hue.

However, equal distances of Wpt coordinates do not correspond to equal perceptual differences. This is because Wpt normalization is defined using a linear transformation of sensor excitations, and uniform color spaces like CIELAB, CIECAM02, and CAM02-UCS require non-linear transformations of tristimulus values (which are linear transforms of sensor excitations) to achieve some form of perceptual uniformity.

WLab - Getting More Uniform

A set of invertible non-linear transforms was derived that adjusts Wpt (Waypoint) coordinates to and from a more perceptually uniform coordinate system (WLab or Waypoint-Lab) that allows for the advantageous features of Wpt to be directly applied to situations where other standard color spaces are typically used.

The proposed transformations were found by optimizing for perceptual uniformity over both large and small color difference scales. The Munsell color order system was used as a representation for optimizing large-scale perceptual uniformity.

Correlations with Munsell value, chroma and hue along with comparisons with CIELAB and CAM02-UCS were used to correct and assess overall large-scale uniformity. The ΔE*94 color difference equation was used for optimization purposes for small-scale uniformity.

A STRESS analysis was applied to compare color differences based on Euclidean WLab distances with color differences based upon ΔE*ab, ΔE*94, ΔE00, DIN99o, CIECAM02, and CAM02-UCS. It was found that WLab represents a reasonably uniform material color equivalency space with small Euclidean WLab distances not being statistically different from ΔE*94, ΔE00, and Euclidean distances of DIN99o and CAM02-UCS under reference observing conditions.

WLab distances were also found to predict supra-threshold color difference experiment results for another observing condition (under Illuminant A).

WLab Implementation Files (Downloads)

The following zip file contains MATLAB/Octave code that can be used to perform Wpt normalization. The files dumpWptMATs.m and plotWpts.m can be used as an references for creating and using structures containing Wpt normalization matrices.

Wpt_WLab_MatLab.zip

The following Excel file can be used to determine a Wpt normalization matrix specific to an observer and illuminant.

WLab_Converter.xlsx

Steps to convert from X, Y, Z to L_W, a_W, b_W:

1. Determine Wpt normalization matrix for observing conditions (see Introducing Wpt)

$$\mathbf{A} = T(\mathbf{C},\mathbf{I})$$

2. Convert from XYZ to Wpt using Wpt normalization matrix

$$\begin{bmatrix} W \\ p \\ t \end{bmatrix} = \mathbf{A}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$W_n = W/100$$

3. Convert W to L_w

$$W_n = \tfrac{W}{100}$$

$$v = \begin{cases} W_n^{1/3}, & \text{if } W_n > \left(\tfrac{6}{29}\right)^3 \\ \left(\tfrac{1}{3}\right)\left(\tfrac{29}{6}\right)^2 W_n + \tfrac{4}{29}, & \text{otherwise} \end{cases}$$

$$L_w = 116v - 16$$

4. Convert p, t to polar notation cpt, h

$$c_{pt} = \sqrt{p^2 + t^2}$$

$$h = \tan^{-1}(t,p)\frac{180}{\pi}$$

$$\text{if } h < 0 \text{ then } h = h + 360$$

5. Calculate chroma-lightness dependency coefficient based on L_w

$$s = \frac{e^{0.0267(L_w - 51.5762)}}{0.4589}$$

6. Divide chroma-lightness dependency coefficient

$$c_{mod} = \frac{c_{pt}}{s}$$

7. Determine chroma adjustment coefficients based on h_{pt} and L_w

$$r_1 = \begin{cases} [\sin(h-1)]^3, & \text{if } h > 1 \text{ and } h < 181 \\ 0, & \text{otherwise} \end{cases}$$

$$r_2 = \begin{cases} 0.39[\cos(h-14)]^2, & \text{if } h > 104 \text{ and } h < 284 \\ 0, & \text{otherwise} \end{cases}$$

$$d_0 = 62(r_1 + r_2 + 0.31)$$

$$d_1 = 0.36(r_1 + r_2 + 0.19)$$

8. Adjust chroma to have small-scale perceptual uniformity using chroma adjustment coefficients

$$c_w = d_0\left[\ln\left(\frac{c_{mod}}{80} + d_1\right) - \ln(d_1)\right]$$

9. (Optional if LSWLab coordinates are desired) The following large-scale chroma difference conversion is performed

$$c_w = 80\left[e^{\left(\frac{c_w}{24.9523} + \ln(0.1104)\right)} - 0.1104\right]$$

10. Convert from Polar coordinates to Cartesian coordinates

$$a_w = c_w \cos\left(h\frac{\pi}{180}\right)$$

$$b_w = c_w \sin\left(h\frac{\pi}{180}\right)$$

Go Back To Top

Steps to convert from L_w, a_w, b_w to X, Y, Z:

1. Convert from Cartesian coordinates to Polar coordinates

$$c_w = \sqrt{a_w^2 + b_w^2}$$

$$h = \tan^{-1}(t,p)\frac{180}{\pi}$$

$$\text{if } h < 0 \text{ then } h = h + 360$$

2. (Optional if LSWLab coordinates being provided) The following inverse large-scale chroma difference conversion is performed

$$c_w = 24.9523\left[\ln\left(\frac{c_w}{80} + 0.1104\right) - \ln(0.1104)\right]$$

3. Determine chroma adjustment coefficients based on h_{pt} and L_w (same as step 7 above)

$$r_1 = \begin{cases} [\sin(h-1)]^3, & \text{if } h > 1 \text{ and } h < 181 \\ 0, & \text{otherwise} \end{cases}$$

$$r_2 = \begin{cases} 0.39[\cos(h-14)]^2, & \text{if } h > 104 \text{ and } h < 284 \\ 0, & \text{otherwise} \end{cases}$$

$$d_0 = 62(r_1 + r_2 + 0.31)$$

$$d_1 = 0.36(r_1 + r_2 + 0.19)$$

4. Apply inverse of chroma adjustment to get small-scale perceptual uniformity using chroma adjustment coefficients

$$c_{mod} = 80\left[e^{\left(\frac{c_w}{d_0} + \ln(d_1)\right)} - d_1\right]$$

5. Calculate chroma-lightness dependency coefficient based on L_w (same as step 5 above)

$$s = \frac{e^{0.0267(L_w - 51.5762)}}{0.4589}$$

6. Multiply chroma-lightness dependency coefficient

$$c_{pt} = sc_{mod}$$

7. Convert L_w to W

$$v = \frac{L_w + 16}{116}$$

$$W = \begin{cases} 100v^3, & \text{if } v > \left(\frac{6}{29}\right) \\ 300\left(\frac{6}{29}\right)^2\left(v - \frac{4}{29}\right), & \text{otherwise} \end{cases}$$

8. Convert from Polar coordinates to Cartesian coordinates

$$\mathrm{p} = \mathrm{c}_{\mathrm{pt}} \cos\left(h\frac{\pi}{180}\right)$$

$$\mathrm{t} = \mathrm{c}_{\mathrm{pt}} \sin\left(h\frac{\pi}{180}\right)$$

9. Determine inverse of Wpt normalization matrix for observing conditions (see Introducing Wpt)

$$\mathbf{A}^{-1} = \left[T(\mathbf{C},\mathbf{l})\right]^{-1}$$

10. Convert from Wpt to XYZ using inverse of Wpt normalization matrix

$$\begin{bmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \end{bmatrix} = \mathbf{A}^{-1}\begin{bmatrix} W \\ p \\ t \end{bmatrix}$$

Worked Examples

Worked example converting from X, Y, Z to L_w, a_w, b_w:

Starting values:
X = 23.33, Y = 18.92, Z = 8.37 for D50 illuminant and 2° Standard Observer

1.

$$\mathbf{A} = \begin{bmatrix} -0.06265 & 1.03839 & 0.02669 \\ 4.68561 & -4.82563 & 0.37293 \\ 0.28350 & 1.50053 & -2.15101 \end{bmatrix}$$

(Found using Wpt-Normalization.xlsx)

2. W = 18.40, p = 21.14, t = 17.00
3. L_w = 49.99
4. c_{pt} = 27.12, h = 38.81
5. s = 2.0885
6. c_{mod} = 12.99
7. d_0 = 33.5053, d_1 = 0.1513
8. c_w = 24.42
9. (LSWLab c_w = 14.67)
10. a_w = 19.03, b_w = 15.31; (LSWLab a_w = 11.43, b_w = 9.19)

Resulting Values:
L_w = 44.99, a_w = 19.03, b_w = 15.31; (LSWLab L_w = 44.99, a_w = 11.43, b_w = 9.19)

Worked example converting from L_w, a_w, b_w to X, Y, Z:

Starting Values:
L_w = 44.99, a_w = 19.03, b_w = 15.31; (LSWLab L_w = 44.99, a_w = 11.43, b_w = 9.19)

1. c_w = 24.42, h = 38.81; (LSWLab c_w = 14.67, h = 38.81)
2. (From LSWLab c_w = 24.42)
3. d_0 = 33.5053, d_1 = 0.1513
4. c_{mod} = 12.99
5. s = 2.0885
6. c_{pt} = 27.12
7. W = 18.41

8. p = 21.14, t = 17.00

9.

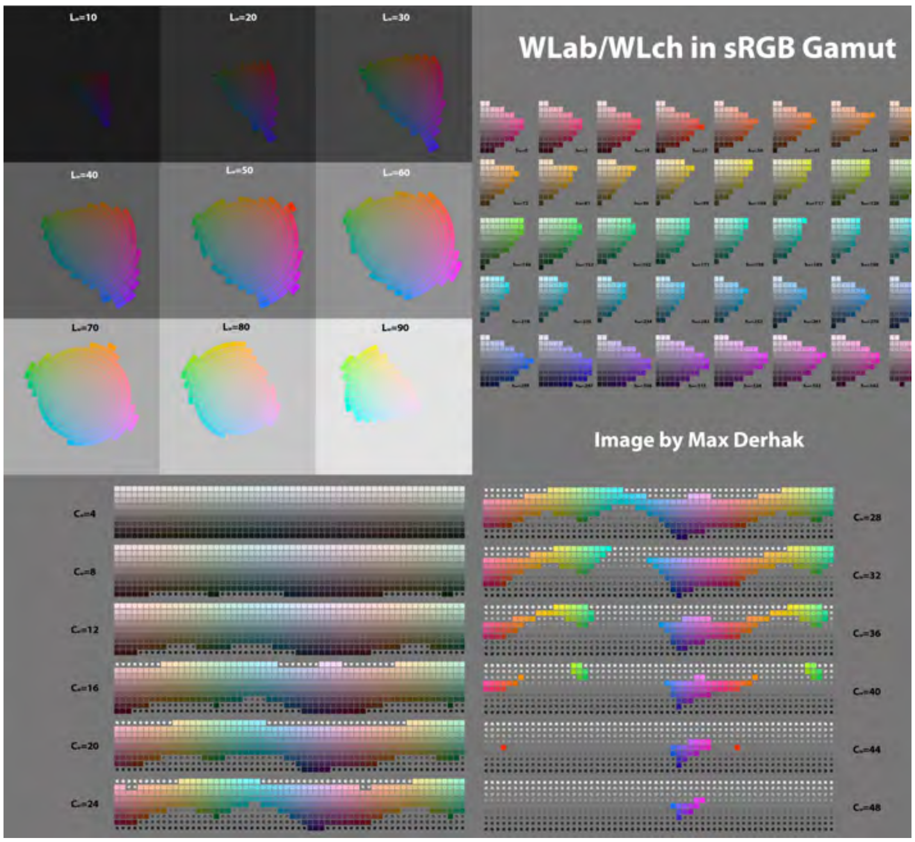
$$\mathbf{A}^{-1} = \begin{bmatrix} 0.96425 & 0.22325 & 0.05067 \\ 1.00000 & 0.10249 & 0.01458 \\ 0.82468 & 0.03814 & -0.44805 \end{bmatrix}$$

(Found using Wpt-Normalization.xlsx)

10. X = 24.33, Y = 18.92, Z = 8.37

Resulting Values:
X = 24.33, Y = 18.92, Z = 8.37

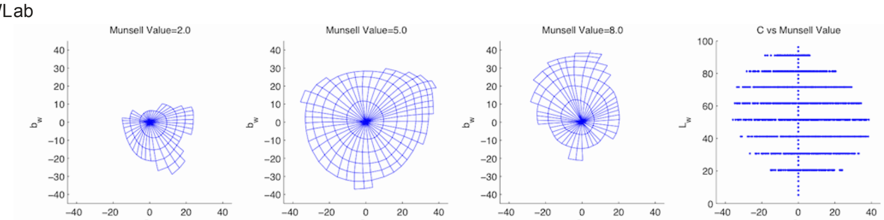
A View of sRGB colors in WLab



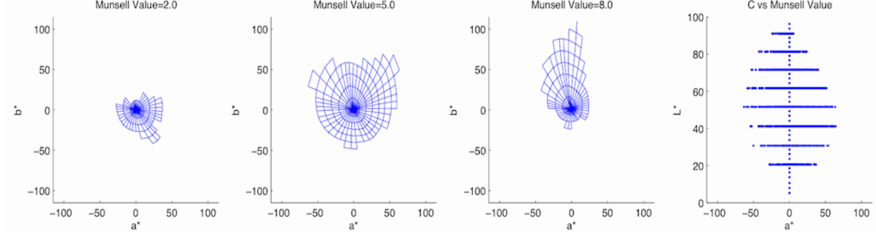
Munsell Uniformity Plots

The following plots show Munsell Renotation coordinates for the Standard 1931 Observer under Illuminant C in various color spaces.

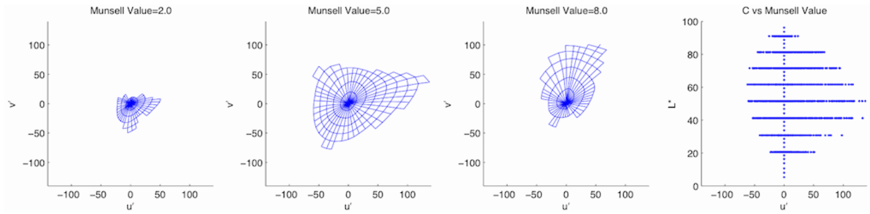
WLab



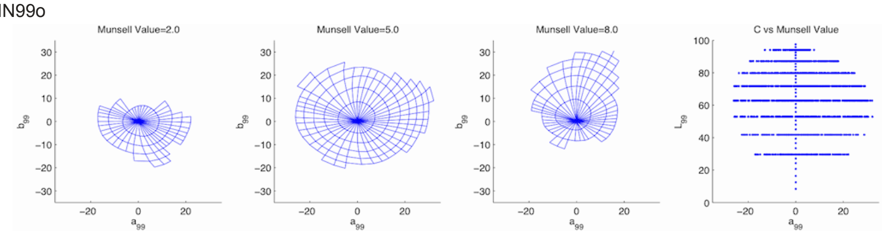
CIELAB



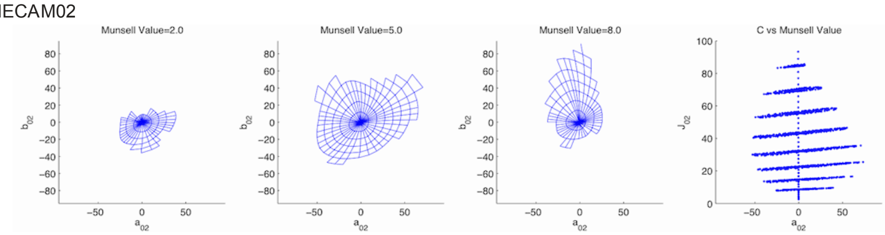
CIELUV



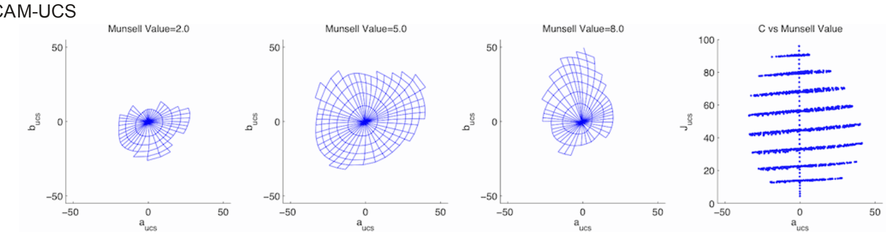
DIN99o



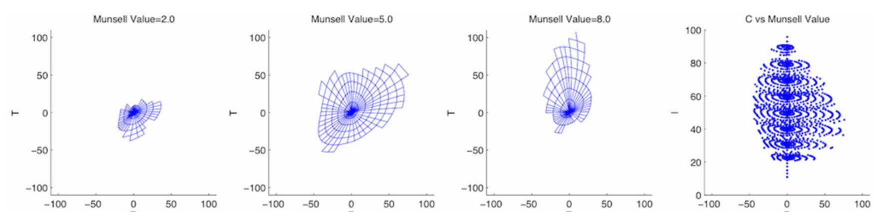
CIECAM02



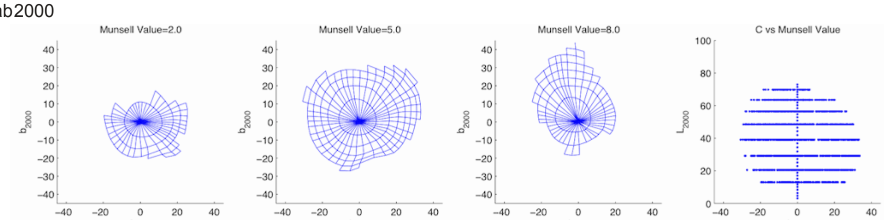
CAM-UCS



IPT



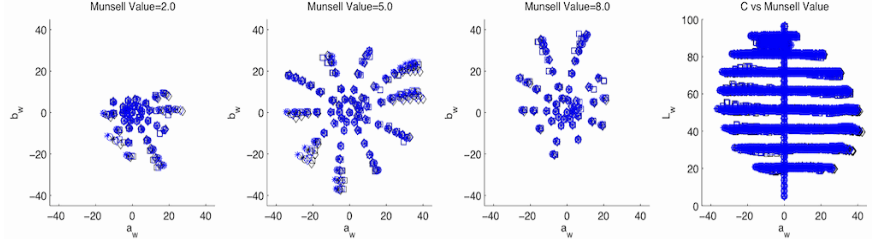
Lab2000



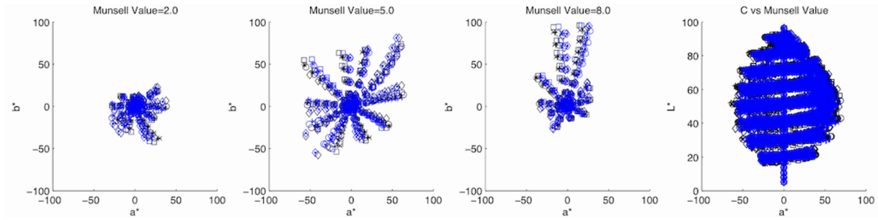
Munsell Scatter Plots

The following plots show Munsell Renotation points for 10 hues for combinations of the Standard 1931 (black) Observer and Standard 1965 Observer (blue) with Illuminant A, Illuminant C, Illuminant E, D50, D65, and F11 (12 total combinations) using the color equivalency normalizaiton of various color spaces.

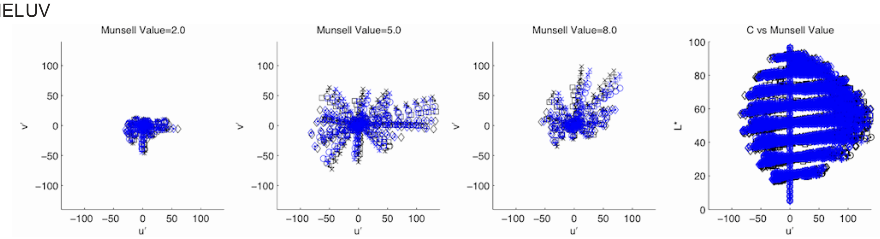
WLab



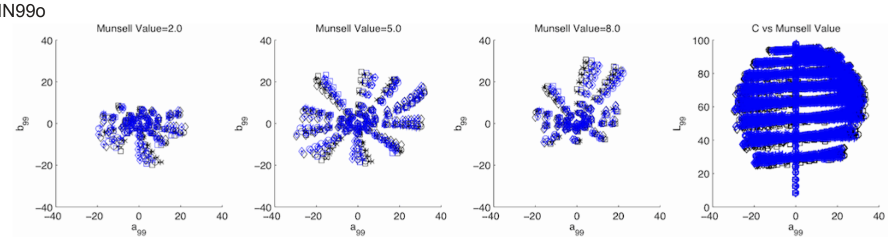
CIELAB



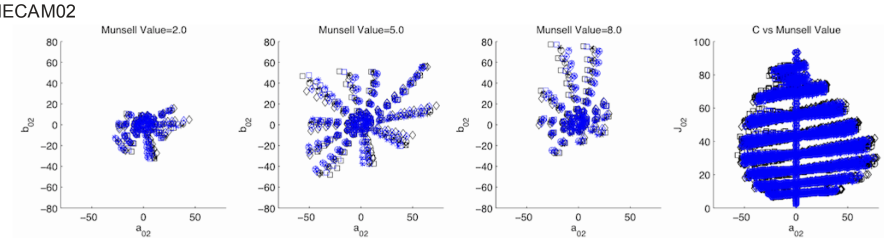
CIELUV



DIN99o



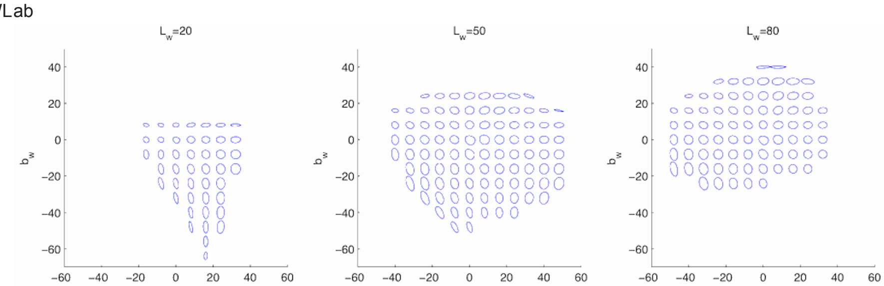
CIECAM02



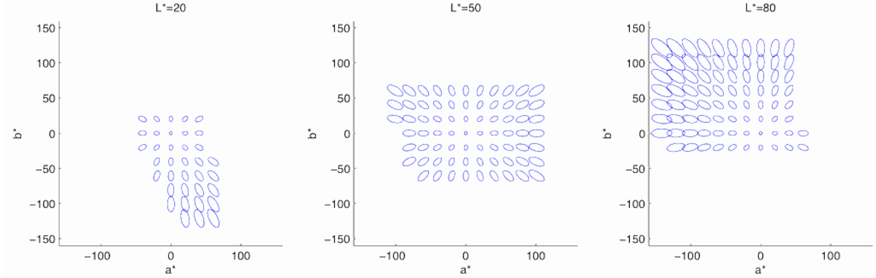
Color Difference Ellipse Plots

The following plots show ellipses of constant ΔE^*_{94} in various color spaces.

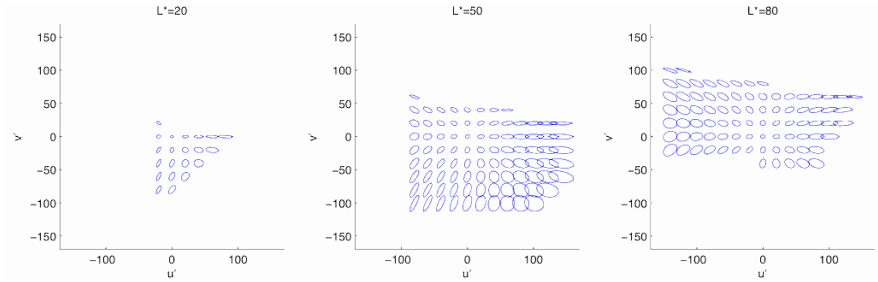
WLab



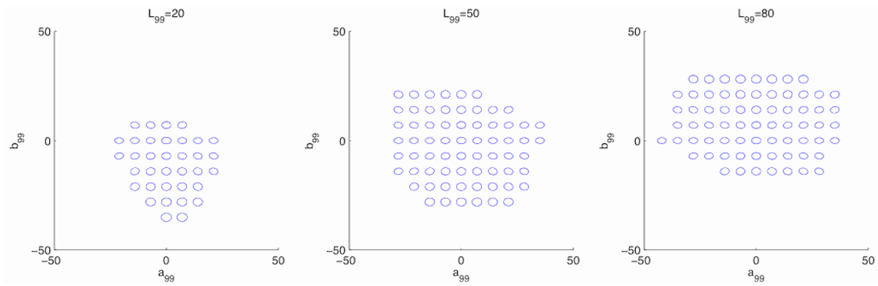
CIELAB



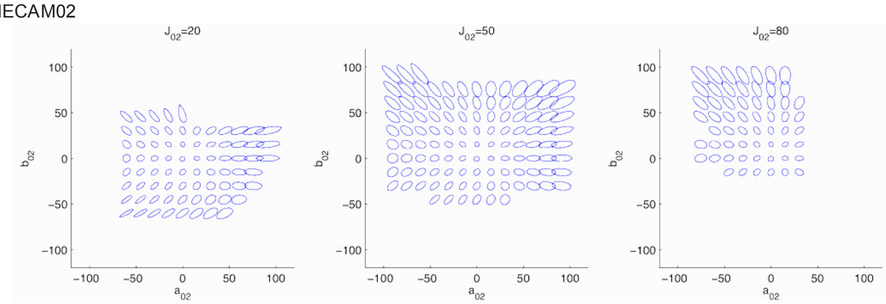
CIELUV



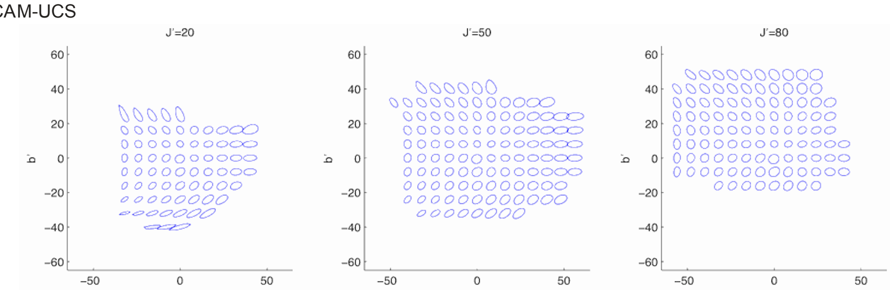
DIN99o



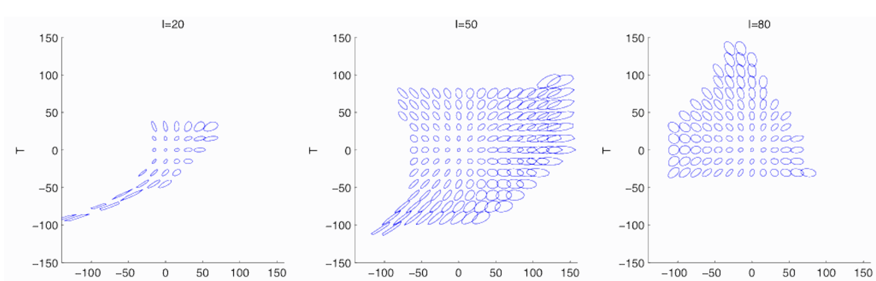
CIECAM02



CAM-UCS



IPT



Lab2000