Mathematics for Intelligence

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Toyota Applied Mathematics Initiative

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Introduction to MIT Lincoln Laboratory

Established 1951
Lexington, MA

“Technology in Support of National Security”
Intelligence, Surveillance, and Reconnaissance (ISR) Systems and Architectures

High-Resolution Imaging

Novel laser radar (ladar) sensors are being developed and fielded to provide wide area mapping and high-resolution imaging capabilities.

Airborne Sensing

Lincoln Laboratory is developing airborne radar and wide area motion imaging technology for current and future unmanned platforms.

Processing, Exploitation, and Dissemination

Advanced sensors require new approaches to processing, exploitation, and dissemination (PED) to handle large, nuanced, and high-bandwidth data.

Advanced Algorithms and Discrimination

The Laboratory is at the forefront of advanced algorithm development for detection, location, and clutter discrimination across a broad array of missions, from airborne RF to undersea acoustics.
Lincoln Laboratory assists the Department of Defense in improving the acquisition and employment of various tactical air and counterterrorist systems.
Composition of Professional Staff

Degrees

- No Degree
- Bachelor's
- Master's
- Doctorate

Academic Disciplines

- No Degree
- Physics
- Electrical Engineering
- Aerospace/Astronautics
- Mechanical Engineering
- Mathematics
- Computer Science, Computer Engineering, Computer Information Systems
- Biology, Chemistry, Meteorology, Materials Science
- Other
Outline

• ISR overview

• Educational Background
  – College internships/projects

• Career at MIT LL

• Sample Weather Project

• Summary
Education Experiences

• Ponaganset High School:
  – AP Calculus (Single Variable), AP Physics (Mechanics)

• RIT:
  – B.S. in Applied Mathematics
1. NMR Laboratory at RIT, studied the binding time of Copper and Gadodiamide, a common MRI contrast agent
2. Dr. Ross at RIT, characterized the binding polynomial for proteins forming cataracts
3. Information Assurance Internship, in Rome, NY studied security protocols for cloud computing
College Internships/Projects

4. Multidisciplinary Vision Research Lab, at RIT, de-interlaced and calibrated videos from eye tracking headsets

5. Oral Microbiology Laboratory, at RIT, computed volume and density statistics of biofilms from 3D stacks of confocal microscopy images

6. MIT Lincoln Laboratory, in Lexington, MA studied material identification from an airborne platform using hyperspectral imagery

Image courtesy of NASA/JPL-Caltech
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Projects at MIT Lincoln Labs

• Weather Analysis for Radar Systems:
  – What percent of the time is it raining/cloudy and which published model best represents real weather?
  – How will weather effect radar performance?

• Synthetic Aperture Radar Modeling:
  – Can we model the surveillance abilities of an aircraft in an unknown environment?
  – What assumptions do we have to make about what the aircraft is looking at, who/what is looking at it?
  – What is the probability that a given ground tracking device can geolocate the aircraft?
Projects at MIT Lincoln Labs

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Mathematics for Intelligence

- Radar System: Air Force
  - Need a new intelligence gathering platform (plane, satellite, etc)
  - Robust to weather
  - Maximum Operability
Mathematics for Intelligence

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- **Communication: Netflix**
  - Want a new way to connect viewers (balloon, satellite, etc)
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System Analysis – Set Up

• Options:
  – Satellite vs Airplane
  – X-band (~10 GHz)
  – K-band (~20 GHz)

• Questions:
  – How much better is one frequency over another?
  – What does the geometry look like?
  – How does frequency relate to rain and cloud loss?
  – How does gas/atmosphere effect signal loss?
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More advanced?

Search for other commonly used frequencies
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K-band is 2x better than X-band at picture quality**

** Does not reflect real data but rather frequency trend
Geometry

Low Earth Orbit
300km

40°

10km

0-2 km

2-5 km

100km
More advanced?
Realistic ray bending and curved earth.
Signal Loss

Attenuation is loss due to signal scattering, absorption or fading

- How does frequency relate to rain and cloud loss?
- How does gas/atmosphere effect signal loss?

**dB Loss = \int_{C} \gamma \, ds**

### Specific Attenuation in dB/km

<table>
<thead>
<tr>
<th></th>
<th>Gas*</th>
<th>Rain**</th>
<th>Clouds***</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-Band</td>
<td>0.001</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>K-Band</td>
<td>0.002</td>
<td>2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

**More Advanced?**

- Let rain rate and cloud density vary so specific attenuation becomes a function that can be integrated
- See appendix for more information on variation of atmospheric parameters and specific attenuation

A decibel is a logarithmic unit used to express the ratio between two values, often power or intensity

*Typical dry air  **Light Rain (2mm/hr)  ***Moderate clouds (0.5 g/m³)*
Airborne Loss

More advanced? Consider different plane altitudes.
Airborne Loss – Gas X-band

- Gas Loss = 0.001 \times d_a
  
  \[= 0.001 \times 100.5 = 0.1 \text{ dB}\]

- Pythagorean
• Rain Loss = 0.02\* \(d_r\)
  
  \[= 0.02 \times 1.005\text{km} = 0.02 \text{ dB}\]

• Trigonometry

More Advanced?
Add multiple rain cells, or different rain rates.
Airborne Loss – Cloud X-band

- Cloud Loss = 0.03 * \( d_c \)
  
  \[ = 0.03 \times 30 \text{ km} = 0.9 \text{ dB} \]

- Trigonometry

More Advanced?
Add multiple cloud layers
Airborne Loss – Total X-band

- Gas Loss = 0.101 dB
- Rain Loss = 0.020 dB
- Cloud Loss = 0.905 dB
- Total = $2 \times (0.101 + 0.020 + 0.905)$
  \[= 2.052 \text{ dB 2-way loss}\]
- Gas Loss = 0.016 dB
- Rain Loss = 0.031 dB
- Cloud Loss = 0.118 dB
- Total = 2 \times (0.016 + 0.031 + 0.118) = 0.33 \text{ dB 2-way loss}
## Summary Table 2-Way Loss

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>X-Band</strong></td>
<td>0.20</td>
<td>0.04</td>
<td>1.81</td>
<td>2.05</td>
<td>37%</td>
</tr>
<tr>
<td><strong>X-Band</strong></td>
<td>0.03</td>
<td>0.06</td>
<td>0.23</td>
<td>0.33</td>
<td>7%</td>
</tr>
<tr>
<td><strong>K-Band</strong></td>
<td>0.40</td>
<td>4.02</td>
<td>12.06</td>
<td>16.48</td>
<td>97%</td>
</tr>
<tr>
<td><strong>K-Band</strong></td>
<td>0.06</td>
<td>6.22</td>
<td>1.57</td>
<td>7.85</td>
<td>84%</td>
</tr>
</tbody>
</table>

### More advanced?
- Search for power output of typical systems
- Compare cost of a plane vs. a satellite

**dB Loss** = \(-10 \log_{10}\left(\frac{P_{\text{received}}}{P_{\text{transmit}}}\right)\)

\[% \text{ Loss} = 100 \times \left(1 - 10^{\text{dB Loss}/10}\right)\]
More Advanced?

- What percent of the time do you want to operate?
- How often does it rain?
- How often is it cloudy?
- How would this change the analysis?

<table>
<thead>
<tr>
<th>Tropics</th>
</tr>
</thead>
<tbody>
<tr>
<td>~15% of the time it rains from NASA TRMM satellite</td>
</tr>
<tr>
<td>~30% of the time it is cloudy from NASA CloudSat satellite</td>
</tr>
<tr>
<td>~100% of the time there is an atmosphere (gas)</td>
</tr>
</tbody>
</table>

Still More advanced?
- Search for statistics about your local area
- Look in appendix for more advanced ideas
Weather Data

NASA Satellites
- Tropical Rainfall Measurement Mission (TRMM)
- CloudSat

Other available data
- European Center for Mid-Range Weather Forecasting
- NOAA

Statistical Models
- International Telecommunications Union
Weather Analysis for Radar Systems

- Evaluated current ITU models
- Created a statistical model of rain and cloud distributions from NASA’s TRMM and CloudSat data
- Mapped signal attenuation into mission relevant parameters (ex. Revisit rate)
- Informed program managers of risk due to weather
- Will present work at radar conference in July
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Summary

• Gave high level overview of college internships and projects
• Summarized my weather analysis and ongoing radar projects with MIT Lincoln Laboratory
• Introduced sample weather modeling project using high school geometry

Review of Mathematics Used:
• Pythagorean Theorem
• Trigonometry (Soh, Cah, Toa)
• Calculus Ideas
• Logarithmic Ideas
• Critical Thinking / Trade-off Analysis
Questions: Ask Me About

• Internships + Projects
  – De-interlacing/Calibration
  – Confocal Microscopy
  – Hyperspectral
  – Cloud Computing/Cyber Security

• MIT Lincoln Labs
  – Weather Modeling
    • Sample Weather Project
  – SAR Radar Modeling

• Other
  – Swing Dancing
  – Life in Boston
Appendix

- Documentation References
- Gas Specific Attenuation plot by altitude
- Cloud Specific Attenuation Plots
- MIT LLRISE Program: [https://www.ll.mit.edu/outreach/LLrise.html](https://www.ll.mit.edu/outreach/LLrise.html)
International Telecommunications Union: www.itu.int/en
- ITU P676-10 Gas Specific Attenuation
- ITU P838-3 Rain Specific Attenuation
- ITU P840-6 Cloud Specific Attenuation

NASA:
- TRMM: http://pmm.nasa.gov/TRMM
- CloudSat: www.cloudsat.cira.colostate.edu

Other Weather Information:
- NOAA, ERA-40 by ECMWF

More advanced?
- Interpret and code algorithms outlined by ITU papers.
- Download and investigate TRMM/CS weather. (Matlab has built-in functions to read data files.)
Gas Specific Attenuation by Altitude

Specific Attenuation [dB/km] vs. Altitude [km]

- X-Band
- K-Band

Summer conditions, Disney World, Orlando, FL
ITU P676.10: http://www.itu.int/rec/R-REC-P.676-10-201309-I/en
Rain Specific Attenuation

Summer conditions, Disney World, Orlando, FL
Calculated with horizontal polarization.
ITU P838.3: http://www.itu.int/rec/R-REC-P.838
Cloud Specific Attenuation

Summer conditions, Disney World, Orlando, FL
Calculated using a temperature profile of the atmosphere.
ITU P840.6: http://www.itu.int/rec/R-REC-P.840-6-201309-I