I. **Fractions**
   a. **Definition**: A fraction (also called a rational number) is a number that represents the quotient (or division) of two integers.
   
b. \[ \frac{a}{b} = \text{Numerator} \]
   \[ \frac{b}{a} = \text{Denominator} \]
   
c. The denominator tells how many equal parts there are.
   d. The numerator tells how many of these parts are taken or used.
   e. **Simplest form**: A fraction is said to be written in simplest form when there are no common factors in the numerator and denominator. The fraction has been reduced completely.

II. **Zero**
   a. \( \frac{0}{c} = 0 \) where \( c \neq 0 \). In other words, zero in the numerator of a fraction yields zero for an answer as long as the denominator does not equal zero.
   
b. \( \frac{c}{0} = \text{undefined} \). In other words, division by zero is impossible!
   
   \[ \frac{0}{0} = \text{undefined} \]

III. **Mixed Numbers & Improper Fractions**
   a. You can change a mixed number to an improper fraction:
   
b. \[ \frac{3}{8} = \frac{2 + 3}{8} = \frac{(8 \times 2) + 3}{8} = \frac{19}{8} \]
   
   **Steps:**
   1. Multiply the denominator by the whole number.
   2. Add to the numerator.
   3. The product becomes the new numerator.
   4. Denominator remains the same.
IV. Multiplication and Division of Fractions

a. 1. \( \frac{2}{3} \times \frac{7}{5} = \frac{14}{15} \)  
   Multiply across the numerator and across the denominator.

2. \( \frac{2}{3} \times \frac{6}{5} = \frac{4}{5} \)  
   You may cancel before multiplying.

b. 1. \( \frac{2}{3} \div \frac{7}{5} = \frac{2}{3} \times \frac{5}{7} = \frac{10}{21} \)  
   To divide, multiply by the reciprocal. DO NOT cancel before inverting!!!

2. \( \frac{2}{3} \div \frac{19}{6} = \frac{2}{3} \times \frac{6}{19} = \frac{4}{19} \)  

V. Equivalent Fractions

a. You may multiply or divide both numerator and denominator of a fraction by the same non-zero number without changing the fraction.

1. \( \frac{2}{3} \times \frac{7}{7} = \frac{14}{21} \) and \( \frac{14}{21} = \frac{2}{3} \)  
   These are Equivalent Fractions.

2. \( \frac{2}{3} \div \frac{5}{5} = \frac{2}{3} \times \frac{5}{5} = \frac{10}{15} \) and \( \frac{10}{15} = \frac{2}{3} \)

b. Your answer to a problem may be \( \frac{2}{5} \), by the selection of answers may look like:

   A. \( \frac{5}{30} \)  
   B. \( \frac{13}{30} \)  
   C. \( \frac{10}{50} \)  
   D. \( \frac{12}{30} \)  
   E. \( \frac{18}{30} \)

   The correct answer is D because \( \frac{12}{30} = \frac{2}{5} \).
VI. **Addition and Subtraction of Fractions**

a. You may add or subtract fractions with the same denominator by adding (or subtracting) the numerators and putting this answer over the common denominator.

\[
\begin{align*}
\text{Ex)} & \quad \frac{2}{5} + \frac{1}{5} = \frac{3}{5} \\
\text{Ex)} & \quad \frac{7}{10} - \frac{1}{10} = \frac{6}{10} = \frac{3}{5}
\end{align*}
\]

b. To add (or subtract) fractions with different denominators you must first find the **Least Common Denominator** (LCD) and convert each fraction to an equivalent fraction whose denominator is the LCD.

1. **Definition**: The LCD is the smallest number that each denominator will divide into evenly. (The LCD is the smallest multiple that the given denominators have in common.)

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**Example – Finding Common Denominators:**

\[
\begin{align*}
\frac{1}{8} & + \frac{5}{6} - \frac{7}{48} \\
\end{align*}
\]

Examine each of the given denominators and break them down into their prime factors:

- \(8 = 2 \times 2 \times 2\)
- \(6 = 2 \times 3\)
- \(48 = 2 \times 2 \times 2 \times 2 \times 3\)

The LCD is equal to every factor of the first denominator multiplied by any new factor that appears in a subsequent denominator.

\[
\text{LCM} = 2 \times 2 \times 2 \times 3 \times 2
\]

We begin with every factor of the first denominator.

| 3 is a new factor from the 2nd denominator. (The 2 of the 2nd denominator does not have to be repeated since there are already 3 factors of 2. |
| There are 4 two’s in the last denominator and only 3 so far in the LCD so we need one more factor of two. The factor of 3 in the last denominator does not need to be repeated. |

...so our LCD is 48.
Now that we have our LCD, let’s finish the problem:

Change each of the given fractions to an equivalent fraction whose denominator is the LCD.

\[
\frac{1}{8} + \frac{5}{6} - \frac{7}{48} = \frac{6}{48} + \frac{40}{48} - \frac{7}{48}
\]

Add the equivalent fractions and reduce the answer.

\[
\frac{6}{48} + \frac{40}{48} - \frac{7}{48} = \frac{39}{48} = \frac{13}{16}
\]
Practice Problems:

1. \( \frac{3}{4} (20) = \)

2. \( \frac{5}{8} + \frac{1}{8} = \)

3. \( \frac{5}{8} - \frac{1}{8} = \)

4. \( \frac{1}{2} + \frac{1}{3} = \)

5. \( \frac{2}{5} + \frac{5}{6} = \)

6. \( \frac{5}{8} + \frac{3}{4} = \)

7. \( \frac{3}{4} - \frac{1}{2} = \)

8. \( \frac{5}{6} - \frac{2}{3} = \)

9. \( \frac{15}{16} - \frac{7}{18} = \)

10. \( \frac{9}{7} - \frac{3}{35} = \)

11. \( \frac{2}{3} \times \frac{3}{8} = \)

12. \( \frac{3}{5} \times \frac{1}{3} \times \frac{5}{8} = \)

13. \( \frac{3}{4} \times 8 = \)

14. \( \frac{2}{3} \div 4 = \)

15. \( \frac{2}{3} \div 4 = \)

16. \( \frac{15}{16} \div \frac{3}{4} = \)

17. \( \frac{11}{12} + \frac{21}{4} = \)

18. \( 7 \div \frac{1}{2} = \)

19. \( \frac{14}{0} = \)

20. \( 0 \div 14 = \)

21. \( \frac{1}{13} \div 0 = \)

22. \( 0 - \frac{1}{13} = \)

23. \( \frac{4}{3} + \frac{3}{10} - \frac{5}{6} = \)
Answers to Fractions:

1. 15  
2. $\frac{3}{4}$  
3. 5  
4. $\frac{5}{6}$  
5. $3\frac{7}{30}$  
6. $\frac{11}{8}$ or $1\frac{3}{8}$  
7. $\frac{1}{4}$  
8. $2\frac{1}{2}$  
9. $3\frac{79}{144}$  
10. $\frac{6}{5}$ or $1\frac{1}{5}$  
11. $\frac{1}{4}$  
12. $\frac{1}{8}$  
13. 26  
14. $\frac{1}{2}$  
15. $\frac{1}{6}$  
16. $\frac{5}{4}$ or $1\frac{1}{4}$  
17. $\frac{11}{63}$  
18. 14  
19. Undefined.  
20. 0  
22. $-\frac{1}{13}$  
23. $\frac{24}{30} = \frac{4}{5}$