Factoring

I. Common Factoring

A. Definition: "Greatest Common Factor" (GCF) or "common factor" is a number or algebraic expression that appears in every term of the expression.

B. Examples:

i. $3x^2 + 6x + 12$

a. Common Factor = 3

b. Factored Form: $3(x^2 + 2x + 4)$

ii. $12a^2bc^3 - 10ab^3c^3 - 6a^3b^3c^3$

a. Common Factor = $2abc^3$

b. Factored Form: $2abc^3(6a - 5b^2 - 3a^2b^2)$

II. Trinomials

A. A trinomial may have a common factor, or may factor into the product of two binomials, or both.

B. Standard form: $ax^2 + bx + c$

C. To factor: Try to find possible combinations of the factors of "*a*" and "*c*" that will result in "*b*".

D. Example:

i. $3x^3 - 6x^6 + 3x^3y = 3x^3(1 - 2x^3 + y)$

ii. $x^2 + 12x - 28 = (x + 14)(x - 2)$

iii. $x^2 - 6x + 9 = (x - 3)(x - 3)$

$= (x - 3)^2$

iv. $3x^2 + 10x - 8 = (3x - 2)(x + 4)$

v. $x^6 + 4x^3 + 3 = (x^3 + 3)(x^3 + 1)$

III. Difference of Two Squares

A. Special Formula:

i. $x^2 - a^2 = (x - a)(x + a)$

B. The difference of two perfect squares can always be factored at least once.

C. The sum of perfect squares can never be factored. They result in imaginary roots.

D. Examples:

i. $1 - 81b^4 = (1 + 9b^2)(1 - 9b^2)$

$= (1 + 9b^2)(1 - 3b)(1 + 3b)$

ii. $4x^2y^4 - 16z^6 = 4(x^2y^4 - 4z^6)$

$= 4(xy^2 + 2z^3)(xy^2 - 2z^3)$

iii. $x^2 + 9$ Cannot be factored

IV. Sum & Difference of Two Cubes

A. Special Formulas:

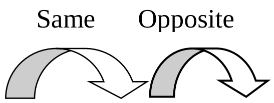
i. $x^3 - a^3 = (x-a)(x^2 + ax + a^2)$

ii. $x^3 + a^3 = (x+a)(x^2 - ax + a^2)$

B. Hint for factoring:

i. The sign between *"x"* and *"a"* in the first factor of the product will be the same sign as the sign between the two cubes. The sign between "x^2" and *"ax"* in the second factor will be the opposite sign.

C. Examples:



i. $x^3 - 8 = x^3 - 2^3 = (x-2)(x^2 + 2x + 4)$

ii. $x^3 + 27 = x^3 + 3^3 = (x+3)(x^2 - 3x + 9)$

iii. $a^3 + 8b^3 = a^3 + (2b)^3 = (x+2b)(a^2 - 2ab + 4b^2)$

iv. $27x^3 - 64y^3z^6 = (3x)^3 + (4yz^2)^3$

$= (3x - 4yz^2)(9x^2 + 12xyz^2 + 16y^2z^4)$

V. Factoring by Grouping

A. Method: Arrange the four terms in two groups of two terms each. Choose any two terms that have a common factor as a group.

B. Remove any common factor from each group.

C. Remove the common factor from the two terms that result in step 2.

D. Warning: Not all expressions can be factored by grouping. In fact, some polynomials cannot be factored by any method.

E. Step-by-Step:

i. $ab - b + ac - c$	1. Two groups of two terms
ii. $b(a-1) + c(a-1)$	2. Remove common factors
iii. $(a-1)(b+c)$	3. Remove resulting common factors

F. Examples:

i. $3ax + 2ay + 3bx + 2by = 3x(a+b) + 3y(a+b)$

$= (3x + 3y)(a+b)$

ii. $6x^3y - 4xy^3 + 12yx^2 - 8y^3 = -4y^3(x+2) + 6yx^2(x+2)$

$= (6yx^2 - 4y^3)(x+2)$

VI. FACTOR COMPLETELY – Some problems may require more than one factoring step.

1. $x^2 - 5x - 24$
2. $x^2 - 5xy - 24y^2$
3. $8c^3d^3e^3 + 4c^2d^4e + 6c^2e^5$
4. $7xy^2 + 14x^2y^3 + 21x^4y^3$
5. $x^2 - 9$
6. $6x^2 - 8x - 8$
7. $30x^3y - 40x^2y - 40xy$
8. $x^4 - 81$
9. $x^2 + 3x - 18$
10. $5x^2 + 11x + 4$
11. $t^3 - 27$
12. $x^2 + 4$
13. $3x^2 + 12$
14. $2x^8y - 2y$
15. $24a^4 + 8a^2 - 80$
16. $10a^3 - 25a^2$
17. $4c^2 - 1$
18. $16e^2 - 15m^6$
19. $9x^4y^4 - 1$
20. $81x^4y^4 - 1$
21. $64x^3 - y^6$
22. $z^6 + 125$

Factoring: Answers

1. $(x-8)(x+3)$

2. $(x-8y)(x+3y)$

3. $2c^2e(4cd^3e^2+2d^4+3e^4)$

4. $7xy^2(1+2xy+3x^3y)$

5. $(x+3)(x-3)$

6. $2(3x+2)(x-2)$

7. $10xy(3x+2)(x-2)$

8. $(x-3)(x+3)(x^2+9)$

9. $(x-3)(x+6)$

10. Cannot be factored.

11. $(t-3)(t^2+3t+9)$

12. Cannot be factored.

13. $3(x^2+4)$

14. $2y(x^4+1)(x^2+1)(x+1)(x-1)$

15. $8(3a^2-5)(a^2+2)$

16. $5a^2(2a-5)$

17. $(2c-1)(2c+1)$

18. Cannot be factored.

19. $(3x^2y^2-1)(3x^2y^2+1)$

20. $(9x^2y^2+1)(3x^2y^2+1)(3x^2y^2-1)$

21. $(4x-y^3)(x^2+4xy^3+y^6)$

22. $(z^3+5)(z^2-5z+25)$