I. Finding Roots of Quadratic Equations

a. The Standard Form of a quadratic equation is: \( ax^2 + bx + c = 0 \).

b. We can use the Quadratic Formula to solve equations in standard form:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

c. Discriminant – The radical portion of this formula \( \sqrt{b^2 - 4ac} \), determines the nature of the roots. This quantity under the radical sign \( b^2 - 4ac \), is called the discriminant.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

d. Three things may occur regarding the discriminant:

i. If \( b^2 - 4ac > 0 \)
   We can take the square root of this positive amount and there will be two different real answers (or roots) to the equation.

ii. If \( b^2 - 4ac < 0 \)
   We cannot take the square root of a negative number, so there will be no real roots.

iii. If \( b^2 - 4ac = 0 \)
    The amount under the radical is zero and since the square root of zero is zero, we will get only 1 distinct real root.
II. Examples

a. \(x^2 - 6x + 9 = 0\)
   \[a = 1 \quad b = -6 \quad c = 9\]
   \[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]
   \[x = \frac{+6 \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}\]
   \[x = \frac{6 \pm \sqrt{36 - 36}}{2}\]
   \[x = \frac{6 \pm 0}{2}\]
   \[x = \frac{6}{2} = 3\] (There is only 1 real root.)

b. \(x^2 + 3x + 1 = 0\)
   \[a = 1 \quad b = 3 \quad c = 1\]
   \[x = \frac{-3 \pm \sqrt{9 - 4(1)(1)}}{2(1)}\]
   \[x = \frac{-3 \pm \sqrt{5}}{2}\]
   \[x = \frac{-3 + \sqrt{5}}{2}\] and \[x = \frac{-3 - \sqrt{5}}{2}\]
   Since the discriminant is positive (it equals +5) there are two real roots.
c. \( x^2 + x + 3 = 0 \)
\[ a = 1 \quad b = 1 \quad c = 3 \]
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
x = \frac{-1 \pm \sqrt{1 - 4(1)(3)}}{2(1)}
\]
\[
x = \frac{-1 \pm \sqrt{-11}}{2}
\]

The discriminant is negative. The discriminant is -11. Since we cannot take the square root of a negative number we have no real roots.

### III. Practice Problems

By examining the discriminant \( b^2 - 4ac \), determine how many real roots, if any, the following quadratic equations have.

1. \( x^2 - 4x + 4 = 0 \)

2. \( x^2 + 4 = 0 \)

3. \( x^2 - 2x + 4 = 0 \)

4. \( x^2 - 4x = 0 \)
5. \( 5r^2 - 3r + 2 = 0 \)

6. \( 7x^2 - 10x - 5 = 0 \)

7. \( x^2 - 4 = 0 \)

8. \( 25r^2 - 10r = -1 \)

9. \( 6y^2 - 5y = 21 \)

10. \( 2y^2 - 19y = 3 \)
Answers: Roots of Quadratic Equations
1. 1 real root
2. no real roots
3. no real roots
4. 2 real roots
5. no real roots
6. 2 real roots
7. 2 real roots
8. 1 real root
9. 2 real roots
10. 2 real roots