Differential Equations Formulas and Table of Laplace Transforms

REDUCTION OF ORDER:

Given differential equation in standard form $y'' + p(x)y' + q(x)y = 0$ and one known solution $y_1(x)$, then the second solution $y_2(x)$ is given by:

$$y_2 = y_1(x) \cdot \int \frac{e^{-\int p(x)dx}}{y_1^2(x)} dx$$

WRONSKIAN:

$$W(y_1, y_2)(x) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$$

VARIATION OF PARAMETERS for $y'' + p(x)y' + q(x)y = g(x)$

$$y_p(x) = -y_1(x) \int \frac{y_2(x)g(x)}{W(y_1, y_2)(x)} dx + y_2(x) \int \frac{y_1(x)g(x)}{W(y_1, y_2)(x)} dx$$

FIRST TRANSLATION THEOREM (FTT)

$$\mathcal{L}\left\{e^{at} f(t)\right\} = \mathcal{L}\left\{f(t)\right\}_{s \to s-a} = F(s-a)$$

SECOND TRANSLATION THEOREM (STT)

$$\mathcal{L}\left\{u(t-a) f(t)\right\} = e^{-as} \mathcal{L}\left\{f(t+a)\right\}, \quad \text{(to transform } f(t) \text{ into } F(s)\text{)}$$

or equivalently:

$$\mathcal{L}^{-1}\left\{e^{-as} F(s)\right\} = u(t-a) f(t-a), \quad \text{(to transform } F(s) \text{ into } f(t)\text{)}$$

LAPLACE TRANSFORMS: **Def:** $F(s) = \mathcal{L}\left\{f(t)\right\} = \int_0^\infty f(t) e^{-st} dt$

	$f(t) = \mathcal{L}^{-1}\left\{F(s)\right\}$	$F(s) = \mathcal{L}\left\{f(t)\right\}$
1	1	$\frac{1}{s}, \quad s > 0$
2	e^{at}	$\frac{1}{s-a}, \quad s > a$
3	t	$\frac{1}{s^2}, \quad s > 0$
4	t^n, n is a positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$
5	$t^\alpha, \quad \alpha > -1$	$\frac{\Gamma(\alpha+1)}{s^{\alpha+1}}, \quad s > 0$

6	$\sin(kt)$	$\frac{k}{s^2+k^2}$, $s>0$
7	$\cos(kt)$	$\frac{s}{s^2+k^2}$, $s>0$
8	$\sinh(kt)$	$\frac{k}{s^2-k^2}$, $s>\lvert k\rvert$
9	$\cosh(kt)$	$\frac{s}{s^2-k^2}$, $s>\lvert k\rvert$
10	te^{at}, FTT	$\frac{1}{(s-a)^2}$, $s>a$
11	$t^n e^{at}$, n is a positive integer , FTT	$\frac{n!}{(s-a)^{n+1}}$, $s>a$
12	$e^{at}\sin(kt)$, FTT	$\frac{k}{(s-a)^2+k^2}$, $s>a$
13	$e^{at}\cos(kt)$, FTT	$\frac{s-a}{(s-a)^2+k^2}$, $s>a$
14	$e^{at}f(t)$, FTT	$F(s-a)$
15	$t\sin(kt)$	$\frac{2ks}{(s^2+k^2)^2}$,
16	$t\cos(kt)$	$\frac{s^2-k^2}{(s^2+k^2)^2}$,
17	$t^n f(t)$	$(-1)^n\frac{d^n}{ds^n}F(s)$
18	$u(t-a)f(t)$, STT	$e^{-as}\,\mathcal{L}\{f(t+a)\}$
19	$u(t-a)f(t-a)$, STT	$e^{-as}F(s)$
20	$u(t-a)$	$\frac{e^{-as}}{s}$
21	$\delta(t)$	1
22	$\delta(t-t_0)$	e^{-st_0}
23	$f'(t)$	$sF(s)-f(0)$
24	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
25	$\int_0^t f(\tau)g(t-\tau)d\tau$	$F(s)G(s)$