



Octal and Hexadecimal Number Systems

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OCTAL or **BASE-8** numbers uses eight symbols: 0, 1, 2, 3, 4, 5, 6, and 7 (count them!) and position plays a major role in expressing their meaning. For example $53,702_8$ means

$$\begin{array}{cccccc} \underline{5 \times 8^4} & + & \underline{3 \times 8^3} & + & \underline{7 \times 8^2} & + & \underline{0 \times 8^1} & + & \underline{2 \times 8^0} \\ 4096's & & 512's & & \text{Sixty-fours} & & \text{Eights} & & \text{Ones (Units)} \end{array}$$

To change this number to base 10, multiply each placeholder by the amount its location represents and add: $(5 \times 4096) + (3 \times 512) + (7 \times 64) + (0 \times 8) + (2 \times 1) = 20,480 + 1536 + 448 + 0 + 1 = 22,466_{10}$

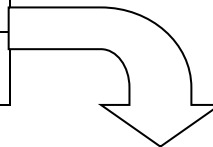
Now you try some:

$436_8 = \underline{\hspace{2cm}}$ (base 10)

$1234_8 = \underline{\hspace{2cm}}$ (base 10)

$524_8 = \underline{\hspace{2cm}}$ (base 10)

Base 16	A	B	C	D	E	F
Base 10	10	11	12	13	14	15



HEXADECIMAL or **BASE-16** numbers uses sixteen symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, and E (count them!) and position plays a major role in expressing their meaning. For example $537CA_{16}$ means

$$\begin{array}{cccccc} \underline{5 \times 16^4} & + & \underline{3 \times 16^3} & + & \underline{7 \times 16^2} & + & \underline{C \times 16^1} & + & \underline{A \times 16^0} \\ 65,536's & & 4096's & & 256's & & \text{Sixteens} & & \text{Ones (Units)} \end{array}$$

To change this number to base 10, multiply each placeholder by the amount its location represents and add: $(5 \times 65,536) + (3 \times 4096) + (7 \times 256) + (12 \times 8) + (10 \times 1) = 327,680 + 12,288 + 1792 + 96 + 10 = 341,866_{10}$

Now you try some:

$4B6_{16} = \underline{\hspace{2cm}}$ (base 10)

$1234_{16} = \underline{\hspace{2cm}}$ (base 10)

$EDA_{16} = \underline{\hspace{2cm}}$ (base 10)

Changing a Decimal Number to an Octal Number

Repeatedly divide by eight and record the remainder for each division – read “answer” upwards.

Example: Rewrite the decimal number 215_{10} as an octal number.

$$\begin{array}{r|l}
 8 & 215 \\
 \hline
 8 & 26 \quad R=7 \\
 8 & 3 \quad R=2 \\
 8 & 0 \quad R=3 \quad \uparrow \text{read} \uparrow \\
 \hline
 & 0
 \end{array}$$

8 divides into 215 twenty-six times with a remainder of 7; then 8 divides into 26 three times with a remainder of 2; and so forth...

The octal result is read upwards \uparrow , therefore $215_{10} = 327_8$

Now you try one:

$$682_{10} = \frac{\quad}{8}$$

Changing a Decimal Number to a Hexadecimal Number

Repeatedly divide by sixteen and record the remainder for each division – read “answer” upwards.

Example: Rewrite the decimal number 215_{10} as an octal number.

$$\begin{array}{r|l}
 16 & 215 \\
 \hline
 16 & 13 \quad R=7 \\
 16 & 0 \quad R=13_{10} = D \\
 \hline
 & 0 \quad \uparrow \text{read} \uparrow
 \end{array}$$

16 divides into 215 thirteen times with a remainder of 7; then 16 divides into 13 zero times with a remainder of 13, which is represented in base

The octal result is read upwards \uparrow , therefore $215_{10} = D7_{16}$

Now you try one:

$$1682_{10} = \frac{\quad}{16}$$

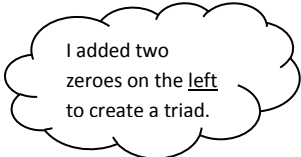
Note how the above algorithms can be adapted to change a decimal number to any chosen base.

Changing Bases Back and Forth between Binary, Octal, and Hexadecimal Systems: An Easy Task!

1. From Binary to Octal – Count off from right to left by three and translate each triad into base 10. These digits will be the base-8 symbols to express this binary number in octal.
2. From Binary to Hexadecimal - Count off from right to left by four and translate each quad into base 10. These digits will be the base-16 symbols to express this binary number in hexadecimal.
3. From Hexadecimal OR Octal to Binary – Change each symbol to binary and you are done!
4. From Octal to Hexadecimal OR from Hexadecimal to Octal – Change the higher base to binary and then use #1 or #2 above to change the binary number to the base desired.

EXAMPLES:

a) Change 1101001010_2 to an octal number.



001 101 001 010
 ↓ ↓ ↓ ↓
 1 5 1 2

therefore, the octal number is **1512₈**

b) Change 1001011101_2 to a hexadecimal number.

0010 0101 1101

2 5 13/D

therefore, the hexadecimal number is **25D₁₆**

c) Change $A3D9_{16}$ to a binary number.

A 3 D 9

1010 0011 1101 1001

therefore, the binary number is **1010001111011001₂**

d) Change 630076_8 to a binary number.

6 3 0 0 7 6

110 011 000 000 111 110

therefore, the binary number is

110011000000111110₂

e) Change $A45_{16}$ to octal.

A 4 5
 1010 0100 0101
 101 001 001 101
 5 1 1 5

(rewritten in binary)

(regrouped the binary digits into groups of three)

therefore the octal number is **5115₈**

f) Change 5401_8 to hexadecimal.

5	4	0	1	
101	100	000	001	(rewritten in binary)
1011	0000	0001		(regrouped the binary digits into groups of four)
B	0	1		therefore the hexadecimal number is B01₁₆

Further Exercises

- Express each number as a decimal number.
 - 263_8
 - $B21_{16}$
 - 5100_8
 - $100E_{16}$
 - 100332_8
 - 10011_{16}
- Express each number as a binary number.
 - 2524_8
 - $BAC9_{16}$
 - 332210_8
 - $4009D_{16}$
- Express each number as an octal number.
 - 101001001_2
 - 1001010000100010_2
 - $B78_{16}$
 - 1234_{16}
- Express each number as a hexadecimal number.
 - 1010100000010101010_2
 - 1010101010_2
 - 2526_8
 - 50004734_8

ANSWERS

'Now your try some' answers:

Octal to Decimal a) 286 b) 664 c) 340	Hexadecimal to Decimal a) 1206 b) 4660 c) 3802	Decimal to Octal b) 1252	Decimal to Octal a) 692
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'Further Exercises' answers:

Exercise Set #1 a. 179 b. 2849 c. 2624 d. 4110 e. 32,986 f. 65,553	Exercise Set #2 a. 010101010100 b. 1011101011001001 c. 011011010010001000 d. 01000000000010011101	Exercise Set #3 a. 511 b. 112042 c. 5564 d. 11064	Exercise Set #4 a. 540AA b. 2AA c. 556 d. A009DC
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