Octal and Hexadecimal Number Systems

Octal and Hexadecimal Number Systems

OCTAL or **BASE-8** numbers uses eight symbols: 0, 1, 2, 3, 4, 5, 6, and 7 (count them!) and position plays a major role in expressing their meaning. For example $53{,}702_8$ means

$\underline{5 \times 8^4}$	+	$\underline{3 \times 8^3}$	+	$\underline{7 \times 8^2}$	+	$\underline{0 \times 8^1}$	+	$\underline{2 \times 8^0}$
4096's		512's		Sixty-fours		Eights		Ones (Units)

To change this number to base 10, multiply each placeholder by the amount its location represents and add: $(5 \times 4096) + (3 \times 512) + (7 \times 64) + (0 \times 8) + (2 \times 1) = 20{,}480 + 1536 + 448 + 0 + 1 = 22{,}466_{10}$

Now you try some:

436_8 = ________________(base 10)

1234_8 = ________________(base 10)

524_8 = ________________(base 10)

Base 16	A	B	C	D	E	F
Base 10	10	11	12	13	14	15

HEXADECIMAL or **BASE-16** numbers uses sixteen symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, and E (count them!) and position plays a major role in expressing their meaning. For example $537CA_{16}$ means

$\underline{5 \times 16^4}$	+	$\underline{3 \times 16^3}$	+	$\underline{7 \times 16^2}$	+	$\underline{C \times 16^1}$	+	$\underline{A \times 16^0}$
65,536's		4096's		256's		Sixteens		Ones (Units)

To change this number to base 10, multiply each placeholder by the amount its location represents and add: $(5 \times 65{,}536) + (3 \times 4096) + (7 \times 256) + (12 \times 8) + (10 \times 1) = 327{,}680 + 12{,}288 + 1792 + 96 + 10 = 341{,}866_{10}$

Now you try some:

$4B6_{16}$ = ________________(base 10)

1234_{16} = ________________(base 10)

EDA_{16} = ________________(base 10)

Changing a Decimal Number to an Octal Number

Repeatedly divide by eight and record the remainder for each division – read "answer" upwards.

Example: Rewrite the decimal number 215_{10} as an octal number.

8	215	
8	26	R=7
8	3	R=2
8	0	R=3 ↑read↑
	0	

8 divides into 215 twenty-six times with a remainder of 7; then 8 divides into 26 three times with a remainder of 2; and so forth...

The octal result is read upwards↑, therefore **$215_{10} = 327_8$**

Now you try one:

682_{10} = $\_\_\_\_\_\_\_\_\_\__8$

Changing a Decimal Number to an Hexadecimal Number

Repeatedly divide by sixteen and record the remainder for each division – read "answer" upwards.

Example: Rewrite the decimal number 215_{10} as an octal number.

16	215	
16	13	R=7
16	0	R=13_{10} = D
	0	↑read↑

16 divides into 215 thirteen times with a remainder of 7; then 16 divides into 13 zero times with a remainder of 13, which is represented in in base

The octal result is read upwards↑, therefore **$215_{10} = D7_{16}$**

Now you try one:

1682_{10} = $\_\_\_\_\_\_\_\_\_\__{16}$

Note how the above algorithms can be adapted to change a decimal number to <u>any</u> chosen base.

Changing Bases Back and Forth between Binary, Octal, and Hexadecimal Systems: An Easy Task!

1. From Binary to Octal – Count off from right to left by three and translate each triad into base 10. These digits will be the base-8 symbols to express this binary number in octal.

2. From Binary to Hexadecimal - Count off from right to left by four and translate each quad into base 10. These digits will be the base-16 symbols to express this binary number in hexadecimal.

3. From Hexadecimal OR Octal to Binary – Change each symbol to binary and you are done!

4. From Octal to Hexadecimal OR from Hexadecimal to Octal – Change the higher base to binary and then use #1 or #2 above to change the binary number to the base desired.

EXAMPLES:

a) *Change 1101001010_2 to an octal number.*

I added two zeroes on the left to create a triad.

001 101 001 010

1 5 1 2 therefore, the octal number is $\mathbf{1512_8}$

b) *Change 1001011101_2 to a hexadecimal number.*

0010 0101 1101

2 5 13/D therefore, the hexadecimal number is $\mathbf{25D_{16}}$

c) *Change $A3D9_{16}$ to a binary number.*

A 3 D 9

1010 0011 1101 1001 therefore, the binary number is $\mathbf{1010001111011001_2}$

d) *Change 630076_8 to a binary number.*

6 3 0 0 7 6

110 011 000 000 111 110 therefore, the binary number is

$\mathbf{110011000000111110_2}$

e) *Change $A45_{16}$ to octal.*

A 4 5

1010 0100 0101 (rewritten in binary)

101 001 001 101 (regrouped the binary digits into groups of three)

5 1 1 5 therefore the octal number is $\mathbf{5115_8}$

f) Change 5401_8 to hexadecimal.

5	4	0	1	
101	100	000	001	(rewritten in binary)
1011	0000	0001		(regrouped the binary digits into groups of four)
B	0	1		therefore the hexadecimal number is $\mathbf{B01_{16}}$

Further Exercises

1. Express each number as a decimal number.
   a. 263_8
   b. $B21_{16}$
   c. 5100_8
   d. $100E_{16}$
   e. 100332_8
   f. 10011_{16}

2. Express each number as a binary number.
   a. 2524_8
   b. $BAC9_{16}$
   c. 332210_8
   d. $4009D_{16}$

3. Express each number as an octal number.
   a. 101001001_2
   b. 1001010000100010_2
   c. $B78_{16}$
   d. 1234_{16}

4. Express each number as a hexadecimal number.
   a. 1010100000010101010_2
   b. 1010101010_2
   c. 2526_8
   d. 50004734_8

<u>ANSWERS</u>

'Now your try some' answers:

Octal to Decimal
a) 286
b) 664
c) 340

Hexadecimal to Decimal
a) 1206
b) 4660
c) 3802

Decimal to Octal
b) 1252

Decimal to Octal
a) 692

'Further Exercises' answers:

Exercise Set #1
a. 179
b. 2849
c. 2624
d. 4110
e. 32,986
f. 65,553

Exercise Set #2
a. 010101010100
b. 1011101011001001
c. 011011010010001000
d. 01000000000010011101

Exercise Set #3
a. 511
b. 112042
c. 5564
d. 11064

Exercise Set #4
a. 540AA
b. 2AA
c. 556
d. A009DC