



Comprehensive Summary of Trig Formulas and Graphs

- Trig Functions
- Right triangle and unit circle definitions
- Domain, range, and period
- Formulas and identities
- Inverse trig functions

Trig Cheat Sheet

Definition of the Trig Functions

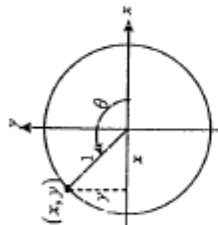
Right triangle definition
For this definition we assume that $0 < \theta < \frac{\pi}{2}$ or $0^\circ < \theta < 90^\circ$.



$$\begin{aligned} \sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} & \csc \theta &= \frac{\text{hypotenuse}}{\text{opposite}} \\ \cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} & \sec \theta &= \frac{\text{hypotenuse}}{\text{adjacent}} \\ \tan \theta &= \frac{\text{opposite}}{\text{adjacent}} & \cot \theta &= \frac{\text{adjacent}}{\text{opposite}} \end{aligned}$$

Unit circle definition

For this definition θ is any angle.



$$\begin{aligned} \sin \theta &= \frac{y}{1} = y & \csc \theta &= \frac{1}{y} \\ \cos \theta &= \frac{x}{1} = x & \sec \theta &= \frac{1}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y} \end{aligned}$$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

$\sin \theta$, θ can be any angle
 $\cos \theta$, θ can be any angle

$\tan \theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi$, $n = 0, \pm 1, \pm 2, \dots$

$\csc \theta$, $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \dots$

$\sec \theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi$, $n = 0, \pm 1, \pm 2, \dots$

$\cot \theta$, $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \dots$

Range

The range is all possible values to get out of the function.

$-1 \leq \sin \theta \leq 1$ $\csc \theta \geq 1$ and $\csc \theta \leq -1$

$-1 \leq \cos \theta \leq 1$ $\sec \theta \geq 1$ and $\sec \theta \leq -1$

$-\infty \leq \tan \theta \leq \infty$ $-\infty \leq \cot \theta \leq \infty$

Period

The period of a function is the number, T , such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\sin(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cos(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\tan(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

$$\csc(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\sec(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cot(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

Formulas and Identities

Half Angle Formulas

$$\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$\tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

Product to Sum Formulas

$$\sin \alpha \sin \beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

Sum to Product Formulas

$$\sin \alpha + \sin \beta = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin \alpha - \sin \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha + \cos \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha - \cos \beta = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

Tangent and Cotangent Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\text{Reciprocal Identities}$$

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\text{Even/Odd Formulas}$$

$$\sin(-\theta) = -\sin \theta \quad \csc(-\theta) = -\csc \theta$$

$$\cos(-\theta) = \cos \theta \quad \sec(-\theta) = \sec \theta$$

$$\tan(-\theta) = -\tan \theta \quad \cot(-\theta) = -\cot \theta$$

Periodic Formulas

$$\text{If } n \text{ is an integer.}$$

$$\sin(\theta + 2\pi n) = \sin \theta \quad \csc(\theta + 2\pi n) = \csc \theta$$

$$\cos(\theta + 2\pi n) = \cos \theta \quad \sec(\theta + 2\pi n) = \sec \theta$$

$$\tan(\theta + \pi n) = \tan \theta \quad \cot(\theta + \pi n) = \cot \theta$$

Double Angle Formulas

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

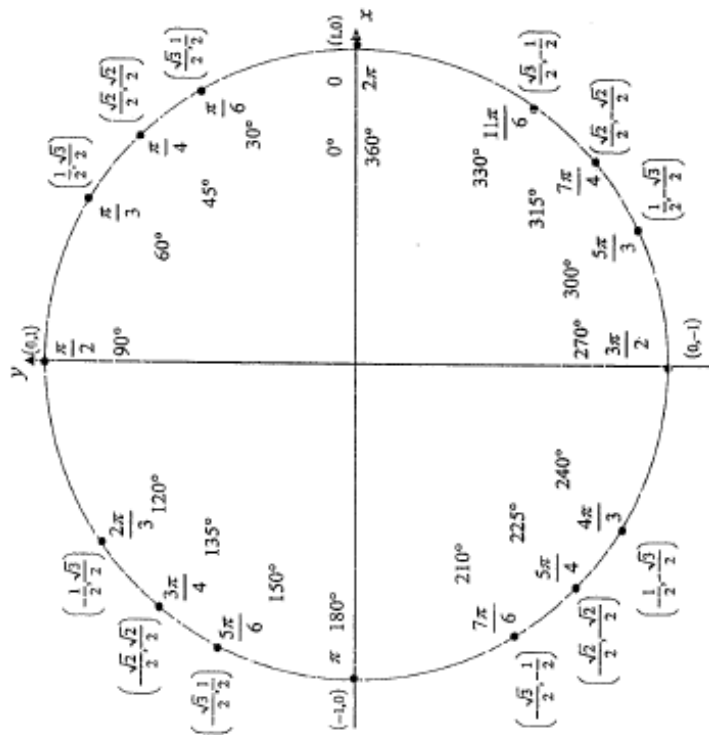
$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Degrees to Radians Formulas

If x is an angle in degrees and r is an angle in radians then

$$\frac{\pi}{180} x = r \Rightarrow r = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180r}{\pi}$$

Unit Circle



For any ordered pair on the unit circle (x, y) : $\cos \theta = x$ and $\sin \theta = y$

Example

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \quad \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Inverse Trig Functions

Definition
 $y = \sin^{-1} x$ is equivalent to $x = \sin y$
 $y = \cos^{-1} x$ is equivalent to $x = \cos y$
 $y = \tan^{-1} x$ is equivalent to $x = \tan y$

Inverse Properties
 $\cos(\cos^{-1}(x)) = x$
 $\sin(\sin^{-1}(x)) = x$
 $\tan(\tan^{-1}(x)) = x$
 $\cos^{-1}(\cos(\theta)) = \theta$
 $\sin^{-1}(\sin(\theta)) = \theta$
 $\tan^{-1}(\tan(\theta)) = \theta$

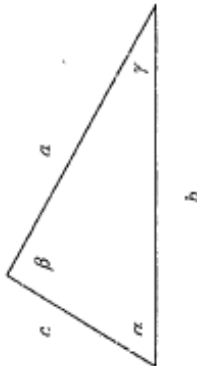
Domain and Range

Function	Domain	Range
$y = \sin^{-1} x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \cos^{-1} x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Alternate Notation

$\sin^{-1} x = \arcsin x$
 $\cos^{-1} x = \arccos x$
 $\tan^{-1} x = \arctan x$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos \frac{1}{2}(\alpha - \beta)}{\sin \frac{1}{2} \gamma}$$

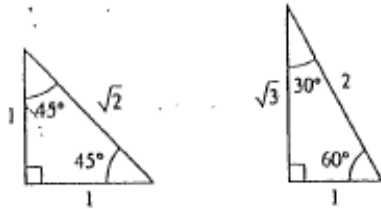
Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan \frac{1}{2}(\alpha - \beta)}{\tan \frac{1}{2}(\alpha + \beta)}$$

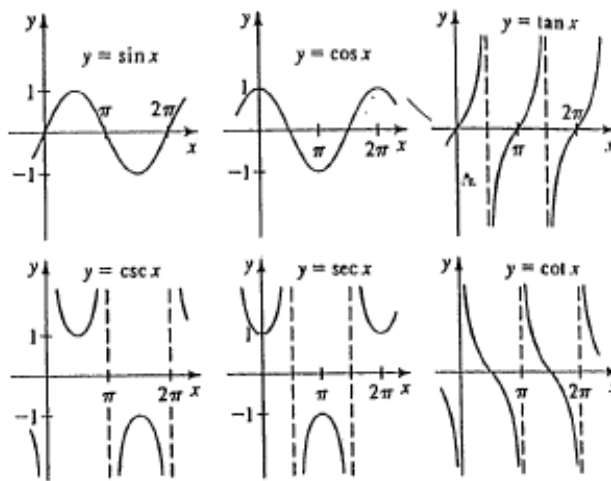
$$\frac{b-c}{b+c} = \frac{\tan \frac{1}{2}(\beta - \gamma)}{\tan \frac{1}{2}(\beta + \gamma)}$$

$$\frac{a-c}{a+c} = \frac{\tan \frac{1}{2}(\alpha - \gamma)}{\tan \frac{1}{2}(\alpha + \gamma)}$$

SPECIAL TRIANGLES

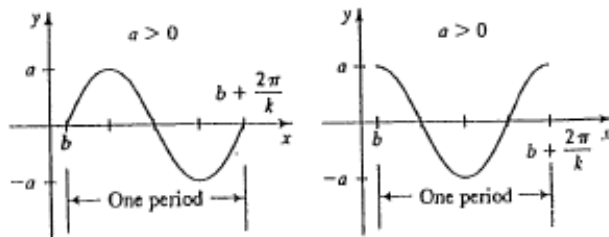


GRAPHS OF THE TRIGONOMETRIC FUNCTIONS



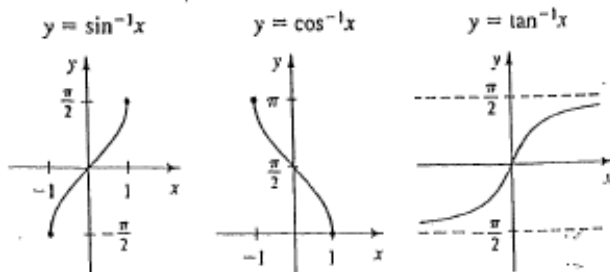
SINE AND COSINE CURVES

$y = a \sin k(x - b) \quad (k > 0)$ $y = a \cos k(x - b) \quad (k > 0)$



amplitude: $|a|$ period: $2\pi/k$ phase shift: b

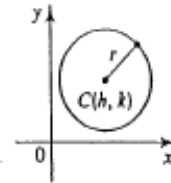
GRAPHS OF THE INVERSE TRIGONOMETRIC FUNCTIONS



CONIC SECTIONS

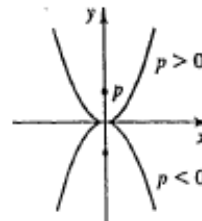
Circles

$(x - h)^2 + (y - k)^2 = r^2$



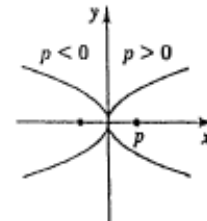
Parabolas

$x^2 = 4py$

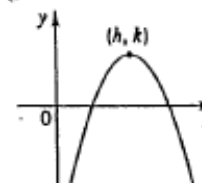


Focus $(0, p)$, directrix $y = -p$

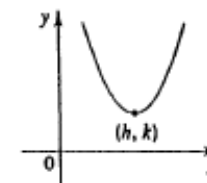
$y^2 = 4px$



Focus $(p, 0)$, directrix $x = -p$



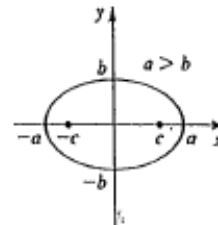
$y = a(x - h)^2 + k$,
 $a < 0, h > 0, k > 0$



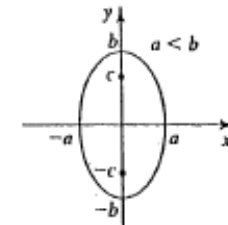
$y = a(x - h)^2 + k$,
 $a > 0, h > 0, k > 0$

Ellipses

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



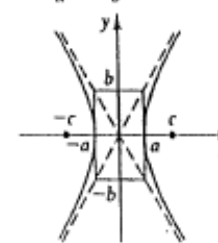
Foci $(\pm c, 0)$, $c^2 = a^2 - b^2$



Foci $(0, \pm c)$, $c^2 = b^2 - a^2$

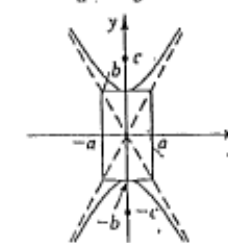
Hyperbolas

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



Foci $(\pm c, 0)$, $c^2 = a^2 + b^2$

$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



Foci $(0, \pm c)$, $c^2 = a^2 + b^2$