Ambiguous Triangles

Given triangular parts **SSS**, **ASA** or **AAS** always guarantees a single, unique triangle. Given the triangular parts **SSA**, however, is different and leaves the triangle unclear, or *ambiguous*. The "**Ambiguous Case**" (SSA) occurs when we are given two sides and the angle opposite one of these given sides. The triangles resulting from this condition needs to be explored much more closely than the SSS, ASA, and AAS cases, for SSA may result in one triangle, two triangles, or even no triangle at all!

Solving Triangles – The Ambiguous Case (SSA)

Given: angle A sides a, b	Zero Triangles 0	One Triangle 1	Two Triangles 2
The given angle A is **greater** than 90° angle A > 90°	a ≤ b	a > b	
The given angle A is **less** than 90° angle A < 90°	a < h or a < b h (height of triangle) = b sin A	a = h or a ≥ b	h < a or a < b

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

When relying on a < b, must then use the Law of Sines to determine sin *B*

If sin B > 1, NO TRIANGLES

If sin B < 1, TWO TRIANGLES (found in quadrants I and II)

Two Examples

Solving Triangles for the Ambiguous Case (SSA)

Example #1 (No Triangles)

Given A = 42°, a = 3, b = 8

Since A = 42° < 90° and a < b, we calculate the value of sin B using the Law of Sines:

$\frac{3}{\sin 42°} = \frac{8}{\sin B}$ yields that sin B = 1.784 which is greater than one

(recall that -1 < sin B < +1). Hence, there are **no possible triangles** and nothing to solve for.

Example #2 (Two Triangles)

Given A = 34°, a = 2, b = 3

Since A = 34° < 90° and a < b, we again calculate the value of sin B using the Law of Sines:

$\frac{2}{\sin 34°} = \frac{3}{\sin B}$ yields that sin B = 0.839, which is between zero and one. Hence there will be **two** possible triangles to solve for.

First Triangle

$\boldsymbol{B_1} = \sin^{-1} 0.839 = 57.01°$

Therefore $\boldsymbol{C_1} = 180° - 34° - 57.01° = 88.99°$

Finally, again using the Law of Sines,

$\frac{2}{\sin 34°} = \frac{c}{\sin 88.99°}$ and, solving this equation for c, we get c_1 = 3.58.

Triangle #1

$< B_1 = 57.01°$
$< C_1 = 88.99°$
$side\ c_1 = 3.58$

Second Triangle

Angle $\boldsymbol{B_2}$ is found by subtracting $< \boldsymbol{B_1}$ from 180°.

Thus $< \boldsymbol{B_2} = 180° - 57.01° = 122.99°$.

Angle $\boldsymbol{C_2} = 180° - 34° - 122.99° = 23.01°$

And to find the missing side, c_2, we solve the ratio $\frac{2}{\sin 34°} = \frac{c_2}{\sin 23.01°}$.

Hence, side c_2 = 1.40.

Triangle #2

$< B_2 = 122.99°$
$< C_2 = 23.01°$
$side\ c_2 = 1.40$

Now you try some!

Exercises

a) Determine the number of triangle that can be represented given the specified two sides and angle.
b) Then solve for all possible triangles.

1. A = 120°, a = 250, b = 195
2. A = 70°, a = 20 , b = 30
3. A = 10°, a = 10, b = 5
4. A = 40°, a =5 , b = 9
5. A = 40°, a = 270 , b = 580
6. A = 45°, a = 5, b = 6
7. A = 10°, a = 10 , b = 20
8. A = 85°, a = 350, b = 351
9. A = 30°, a = 30 , b = 65
10. A = 120 °, a = 25, b = 10
11. A = 30°, a = 20, c = 28
12. A = 50°, a = 150, c = 100
13. A = 30°, a = 160, b = 120
14. A = 75°, a = 180 , c = 185
15. A = 60°, a = 170, b = 180
16. A = 150°, a = 150 , c = 15
17. A = 170°, a = 12, b = 8
18. A = 120°, a = 120, c = 160

Solutions

		FIRST TRIANGLE		SECOND TRIANGLE	
	Number of Triangles	Other Two Angles	Third Side	Other Two Angles	Third Side
1.	1	B=42.49°, C=17.51°	c=86.84		
2.	0				
3.	1	B=4.98°, C=165.02°	c=14.89		
4.	0				
5.	0				
6.	2	B=58.05°, C=76.95°	c=6.89	B=121.95°, C=13.05°	c=1.60
7.	2	B=20.32°, C=149.68°	c=29.07	B=159.68°, C=10.32°	c=10.32
8.	2	B=87.49°, C=7.51°	c=45.92	B=92.51°, C=2.49°	c=15.26
9.	0				
10.	1	B=20.26°, C=39.73°	c=18.45		
11.	2	B=105.57°, C=44.43°	b=38.53	B=14.43°, C=135.57°	b=9.97
12.	1	B=99.29°, C=30.71°	b=193.24		
13.	1	B=22.02°, C=127.98°	c=252.25		
14.	2	B=21.90°, C=83.10°	b=69.51	B=8.10°, C=96.90°	b=26.26
15.	2	B=66.49°, C=53.51°	c=157.82	B=113.51°, C=6.49°	c=22.18
16.	1	B=2.87°, C=27.13°	c=136.82		
17.	1	B=6.65°, C=3.35°	c=4.04		
18.	0				